

Module 11: Direct and Inverse Variation

Topic 1 Content: Exploring Direct Variations Notes

Introduction



Today's Lesson

- You will explore data sets and practical situations that represent direct variations.

Hello there! I'm so glad you could join me for this lesson in Algebra I, where you will explore data sets and practical situations that represent direct variations.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Exploring Direct Variations

An interactive graphic with a dark background featuring faint chalkboard-style numbers and a white 3D geometric shape. At the top is a blue rounded rectangle with the text "EXPLORING DIRECT VARIATIONS" in white. Below it is a yellow text prompt: "Click the Examples Below to Learn More". Three pink rounded rectangles are arranged below: "Example One" (top left), "Self-Check" (top right), and "Example Two" (bottom left).

Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One

EXAMPLE 1

Is the relation a direct variation?
 $\{(8, 12), (10, 15), (12, 18)\}$

Direct Variation

A direct variation models a proportional relationship between two measures.

- y varies directly as x
- y is directly proportional to x

Is the relation a direct variation?

$$\{(8, 12), (10, 15), (12, 18)\}$$

A direct variation models a proportional relationship between two measures. When a direct variation exists, “ y varies directly as x ” or in other words, “ y is directly proportional to x .”

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x} \quad (8, 12)$$

$\frac{12}{8}$ simplifies to....

4 $\frac{3}{2}$ $\frac{2}{3}$

Is the relation a direct variation?

$$\{(8, 12), (10, 15), (12, 18)\}$$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x}$$
$$(8, 12)$$

$\frac{12}{8}$ simplifies to...

- A) 4
- B) $\frac{3}{2}$
- C) $\frac{2}{3}$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$\frac{y}{x} \quad (8, 12)$

$\frac{12}{8}$ simplifies to $\frac{3}{2}$.

$\frac{3}{2}$ [View Work](#) [Next](#)

$\frac{12}{8}$ simplifies to $\frac{3}{2}$.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

$$\frac{12}{8}$$

Simplify the fraction by dividing both the numerator and denominator by 4.

$$\frac{12 \div 4}{8 \div 4} = \frac{3}{2}$$

[Next](#)

$$\frac{12}{8}$$

Simplify the fraction by dividing both the numerator and denominator by 4.

$$\frac{12 \div 4}{8 \div 4} = \frac{3}{2}$$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x} \quad (10, 15)$$

$\frac{15}{10}$ simplifies to....

$\frac{3}{2}$ 5 $\frac{1}{5}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x}$$
$$(10, 15)$$

$\frac{15}{10}$ simplifies to...

- A) $\frac{3}{2}$
- B) 5
- C) $\frac{1}{5}$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x} \quad (10, 15)$$
$$\frac{15}{10} \text{ simplifies to } \frac{3}{2}.$$

$\frac{3}{2}$ [View Work](#) [Next](#)

$\frac{15}{10}$ simplifies to $\frac{3}{2}$.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

$$\frac{15}{10}$$

Simplify the fraction by dividing both the numerator and denominator by 5.

$$\frac{15 \div 5}{10 \div 5} = \frac{3}{2}$$

[Next](#)

$$\frac{15}{10}$$

Simplify the fraction by dividing both the numerator and denominator by 5.

$$\frac{15 \div 5}{10 \div 5} = \frac{3}{2}$$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x} \quad (12, 18)$$

$\frac{18}{12}$ simplifies to....

3
2

1
6

4
3

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x}$$
$$(12, 18)$$

$\frac{18}{12}$ simplifies to...

- A) $\frac{3}{2}$
- B) $\frac{1}{6}$
- C) $\frac{4}{3}$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

For each ordered pair, set up a ratio of y to x . Then, simplify the ratio.

$$\frac{y}{x} \quad (12, 18)$$
$$\frac{18}{12} \text{ simplifies to } \frac{3}{2}.$$

$\frac{3}{2}$ [View Work](#) [Next](#)

$\frac{18}{12}$ simplifies to $\frac{3}{2}$.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

$$\frac{18}{12}$$

Simplify the fraction by dividing both the numerator and denominator by 6.

$$\frac{18 \div 6}{12 \div 6} = \frac{3}{2}$$

[Next](#)

$$\frac{18}{12}$$

Simplify the fraction by dividing both the numerator and denominator by 6.

$$\frac{18 \div 6}{12 \div 6} = \frac{3}{2}$$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example One (continued)

EXAMPLE 1

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

(x, y)

$$\frac{12}{8} = \frac{3}{2} \qquad \frac{15}{10} = \frac{3}{2} \qquad \frac{18}{12} = \frac{3}{2}$$

$\frac{3}{2}$ is the constant of proportionality or the constant of variation.

Is the relation a direct variation?

$\{(8, 12), (10, 15), (12, 18)\}$

(x, y)

$$\frac{12}{8} = \frac{3}{2} \qquad \frac{15}{10} = \frac{3}{2} \qquad \frac{18}{12} = \frac{3}{2}$$

Notice that each ratio of the dependent variable, y , to the independent variable, x , has the same value, $\frac{3}{2}$. Therefore, you can conclude that the relation does represent a direct variation. The value, $\frac{3}{2}$, is referred to as the constant of proportionality or the constant of variation.

Module 11: Direct and Inverse Variation

Topic 1 Content: Exploring Direct Variations Notes

Example Two

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

Direct Variation

A direct variation exists when the ratio of the dependent variable to the independent variable is constant.

$$\frac{\text{dependent variable}}{\text{independent variable}}$$

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

A direct variation exists when the ratio of the dependent variable to the independent variable is constant.

$$\frac{\text{dependent variable}}{\text{independent variable}}$$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

independent variable =

In the given scenario, the independent variable is...

the number of cupcakes prepared the amount of time

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

In the given scenario, the independent variable is...

- A) the amount of time
- B) the number of cupcakes prepared

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

independent variable = the amount of time

In the given scenario, the independent variable is the amount of time.

the amount of time

In the given scenario, the independent variable is *the amount of time*.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

dependent variable =

In the given scenario, the dependent variable is...

the number of cupcakes prepared the amount of time

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

In the given scenario, the dependent variable is...

- A) the amount of time
- B) the number of cupcakes prepared

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

dependent variable = the number of cupcakes

In the given scenario, the dependent variable is the number of cupcakes prepared.

the number of cupcakes
prepared

In the given scenario, the dependent variable is [the number of cupcakes prepared](#).

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

independent variable = the amount of time
dependent variable = the number of cupcakes

The number of cupcakes prepared depends on the amount of time that passes. Therefore, the independent variable is the amount of time, and the dependent variable is the number of cupcakes prepared.

Next

The number of cupcakes prepared depends on the amount of time that passes. Therefore,

- the independent variable is **the amount of time**; and
- the dependent variable is **the number of cupcakes prepared**.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

$$\frac{\text{dependent variable}}{\text{independent variable}} = \frac{\text{the number of cupcakes}}{\text{the amount of time}}$$

Yesterday, Tony prepared cupcakes at a rate of...

6 cupcakes per minute **5 cupcakes per minute** **4 cupcakes per minute**

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

$$\frac{\text{dependent variable}}{\text{independent variable}} = \frac{\text{the number of cupcakes}}{\text{the amount of time}}$$

Now that you know that the dependent variable is the number of cupcakes and the independent variable is time, you can calculate the ratios.

Yesterday, Tony prepared cupcakes at a rate of...

- A) 6 cupcakes per minute
- B) 5 cupcakes per minute
- C) 4 cupcakes per minute

Module 11: Direct and Inverse Variation

Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

$$\frac{\text{dependent variable}}{\text{independent variable}} = \frac{\text{the number of cupcakes}}{\text{the amount of time}}$$
$$\frac{120}{20} = 6$$

Yesterday, Tony prepared 120 cupcakes in 20 minutes.

6 cupcakes per minute

Yesterday, Tony prepared 120 cupcakes in 20 minutes.

$$\frac{120}{20} = 6$$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

$$\frac{\text{dependent variable}}{\text{independent variable}} = \frac{\text{the number of cupcakes}}{\text{the amount of time}}$$

Today, Tony prepared cupcakes at a rate of...

6 cupcakes per minute 5 cupcakes per minute 4 cupcakes per minute

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes. Does the relationship represent a direct variation?

$$\frac{\text{dependent variable}}{\text{independent variable}} = \frac{\text{the number of cupcakes}}{\text{the amount of time}}$$

Now that you know that the dependent variable is the number of cupcakes and the independent variable is time, you can calculate the ratios.

Today, Tony prepared cupcakes at a rate of...

- A) 6 cupcakes per minute
- B) 5 cupcakes per minute
- C) 4 cupcakes per minute

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

$$\frac{\text{dependent variable}}{\text{independent variable}} = \frac{\text{the number of cupcakes}}{\text{the amount of time}}$$
$$\frac{600}{120} = 5$$

Today, Tony prepared 600 cupcakes in 120 minutes.

5 cupcakes per minute

Today, Tony prepared 600 cupcakes in 120 minutes.

$$\frac{600}{120} = 5$$

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Example Two (continued)

EXAMPLE 2

Tony works as a pastry chef at a popular bakery. Yesterday, he prepared 120 cupcakes in 20 minutes. Today, he prepared 600 cupcakes in 120 minutes.
Does the relationship represent a direct variation?

Yesterday

$$\frac{120}{20} = 6$$

Today

$$\frac{600}{120} = 5$$

The rates are not constant. Therefore, the scenario does not represent a direct variation.

$$\frac{120}{20} = 6$$

Yesterday, Tony prepared 6 cupcakes per minute.


$$\frac{600}{120} = 5$$

Today, Tony prepared 5 cupcakes per minute.

The rates are not constant. Therefore, the scenario does not represent a direct variation.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Self-Check 1

 **Self-Check**

In which table does y vary directly with x ?

- A
- B
- C

SUBMIT

Tables

x	y
3	9
4	12
5	25

A

x	y
3	6
4	16
5	30

B

x	y
3	-12
4	-16
5	-20

C

Solve the problem in the image above to check your understanding of the content.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Self-Check 1: Answer

Correct

That's correct!

x	y
3	9
4	12
5	25

$$\frac{y}{x}$$
$$\frac{9}{3} = 3$$
$$\frac{12}{4} = 3$$
$$\frac{25}{5} = 5$$

The ratios of $\frac{y}{x}$ are not equal. Therefore, y does not vary directly with x.

Table A Table B Table C Continue

SUBMIT c

Correct

That's correct!

x	y
3	6
4	16
5	30

$$\frac{y}{x}$$
$$\frac{6}{3} = 2$$
$$\frac{16}{4} = 4$$
$$\frac{30}{5} = 6$$

The ratios of $\frac{y}{x}$ are not equal. Therefore, y does not vary directly with x.

Table A **Table B** Table C Continue

SUBMIT c

For your reference, the images above show the correct solution to the self-check problem.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Self-Check 1: Answer (continued)

The screenshot shows a digital interface for a self-check. At the top, it says "Self Check" and "Tables". A grey box with a checkmark icon and the word "Correct" is overlaid on the screen. Inside this box, there is a table with two columns: "x" and "y". The table contains three rows of data: (3, -12), (4, -16), and (5, -20). To the right of the table, the ratio $\frac{y}{x}$ is calculated for each row, showing $\frac{-12}{3} = -4$, $\frac{-16}{4} = -4$, and $\frac{-20}{5} = -4$. Below the table, a text box states: "The ratios of $\frac{y}{x}$ are equal. Therefore, y varies directly with x." At the bottom of the grey box, there are four buttons: "Table A", "Table B", "Table C" (which is highlighted with a red border), and "Continue". Below the grey box, there are two large buttons: "SUBMIT" on the left and "C" on the right.

x	y
3	-12
4	-16
5	-20

$\frac{y}{x}$

$$\frac{-12}{3} = -4$$
$$\frac{-16}{4} = -4$$
$$\frac{-20}{5} = -4$$

The ratios of $\frac{y}{x}$ are equal. Therefore, y varies directly with x.

Table A Table B **Table C** Continue


SUBMIT C

For your reference, the image above shows the correct solution to the self-check problem.

Module 11: Direct and Inverse Variation

Topic 1 Content: Exploring Direct Variations Notes

Self-Check 2



Self-Check

Avery manages a local car wash. Last week, she measured the efficiency of her employees on random occasions. On Monday morning, her staff washed 12 cars in 30 minutes. On Thursday morning, the staff washed 18 cars in 45 minutes. Choose each valid statement that describes the scenario.

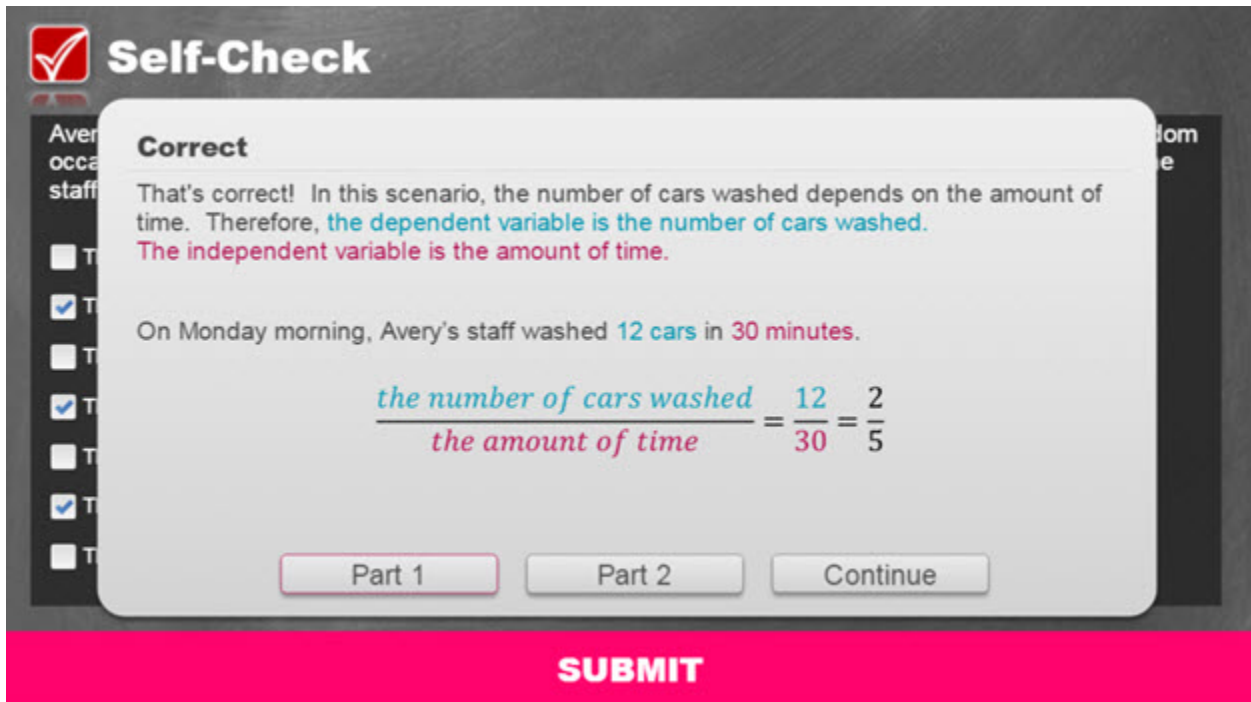
- The scenario does *not* represent a direct variation.
- The constant of variation is $\frac{2}{5}$.
- The constant of variation is $\frac{5}{8}$.
- The scenario represents a direct variation.
- There is no constant of variation.
- The staff washed 2 cars every 5 minutes.
- The staff washed 5 cars every 8 minutes.

SUBMIT

Solve the problem in the image above to check your understanding of the content.

Module 11: Direct and Inverse Variation
Topic 1 Content: Exploring Direct Variations Notes

Self-Check 2: Answer



Self-Check

Correct

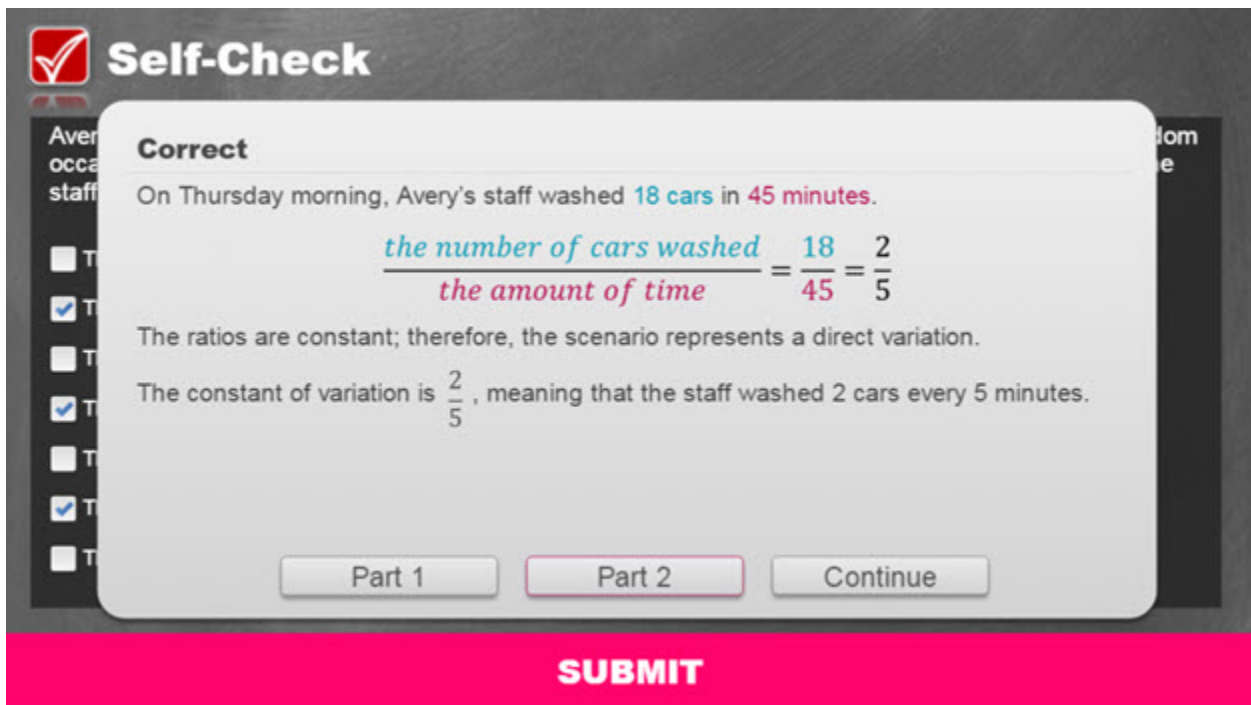
That's correct! In this scenario, the number of cars washed depends on the amount of time. Therefore, **the dependent variable is the number of cars washed.**
The independent variable is the amount of time.

On Monday morning, Avery's staff washed **12 cars** in **30 minutes**.

$$\frac{\text{the number of cars washed}}{\text{the amount of time}} = \frac{12}{30} = \frac{2}{5}$$

Part 1 Part 2 Continue

SUBMIT



Self-Check

Correct

On Thursday morning, Avery's staff washed **18 cars** in **45 minutes**.

$$\frac{\text{the number of cars washed}}{\text{the amount of time}} = \frac{18}{45} = \frac{2}{5}$$

The ratios are constant; therefore, the scenario represents a direct variation.

The constant of variation is $\frac{2}{5}$, meaning that the staff washed 2 cars every 5 minutes.

Part 1 Part 2 Continue

SUBMIT

For your reference, the images above show the correct solution to the self-check problem.

Module 11: Direct and Inverse Variation

Topic 1 Content: Exploring Direct Variations Notes

Conclusion



You have reached the conclusion of this lesson where you explored data sets and practical situations that represent direct variations.