Module 2: Properties of Exponents
Topic 3: Zero and Negative Exponents

Introduction

Hi there! In your earlier math studies, you have had practice working with exponents. More likely than not, those exponents were usually positive whole numbers. In this lesson, you will apply your knowledge of exponents to discover a rule that allows you to raise a value to the power of zero, or to a negative power.

Module 2: Properties of Exponents
Topic 3: Zero and Negative Exponents

Table of Exponents

2^4	16
2^3	8
2^2	4
2^1	2
2^0	
2^{-1}	
2^{-2}	

In order to understand how zero and negative exponents work, take a few moments to work through this table.

What is the value of 2^4 ? You could use the calculator or mental math to determine that the value is 16.

2^3 …that's 8.

2^2 …that's 4.

And, 2^1 …that's simply 2.

Table of Exponents (continued)

2^4	16
2^3	8
2^2	4
2^1	2
2^0	
2^{-1}	
2^{-2}	

Now take a moment to analyze the values in the table. Do you notice any patterns? You can determine a value in the right column of the table by dividing the previous value by 2.

For example, the first value in the right column is 16.

$\frac{16}{2} = 8$, which is the second value in the right column.

$\frac{8}{2} = 4$, which is the third value in the right column.

And, $\frac{4}{2} = 2$, which is the fourth value in the right column.

Table of Exponents (continued)

2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$

If you use this pattern to complete the right column, you'll find that…

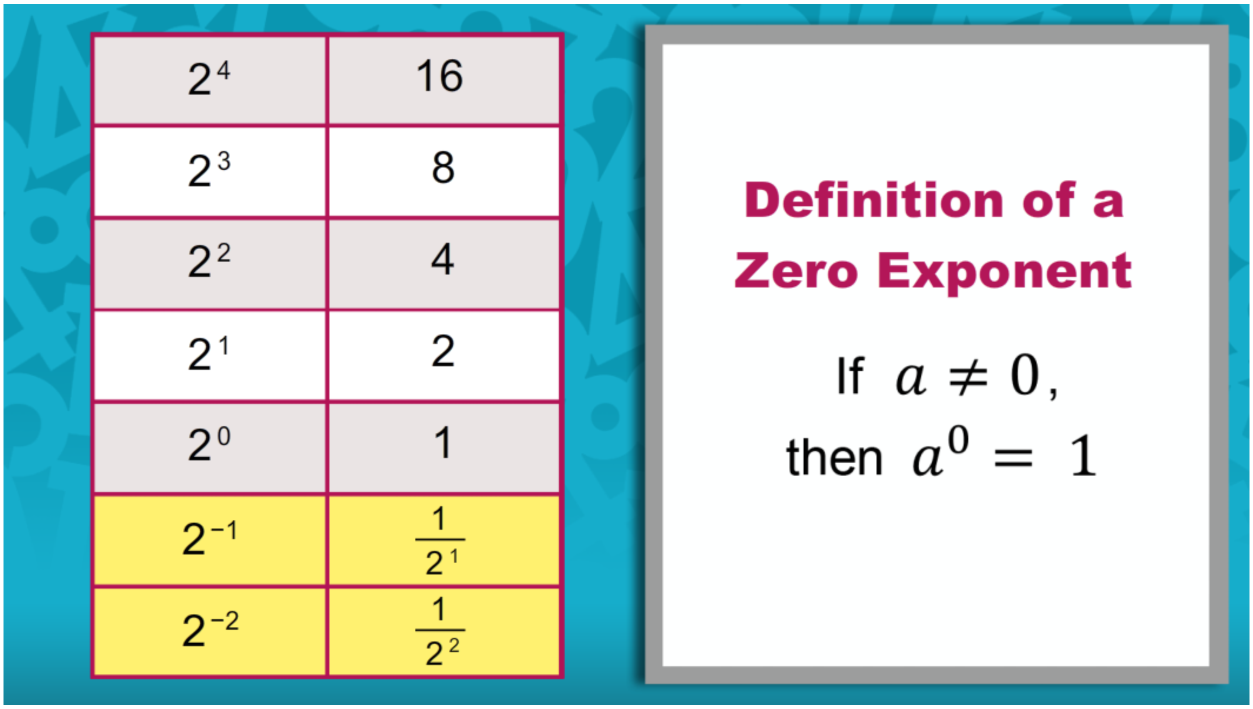
$2^0 = 1$;

$2^{-1} = \frac{1}{2}$; and

$2^{-2} = \frac{1}{4}$.

The completed table shows the patterns that arise when you raise a number to the power of zero and when you raise a number to a negative power.

Definition of a Zero Exponent



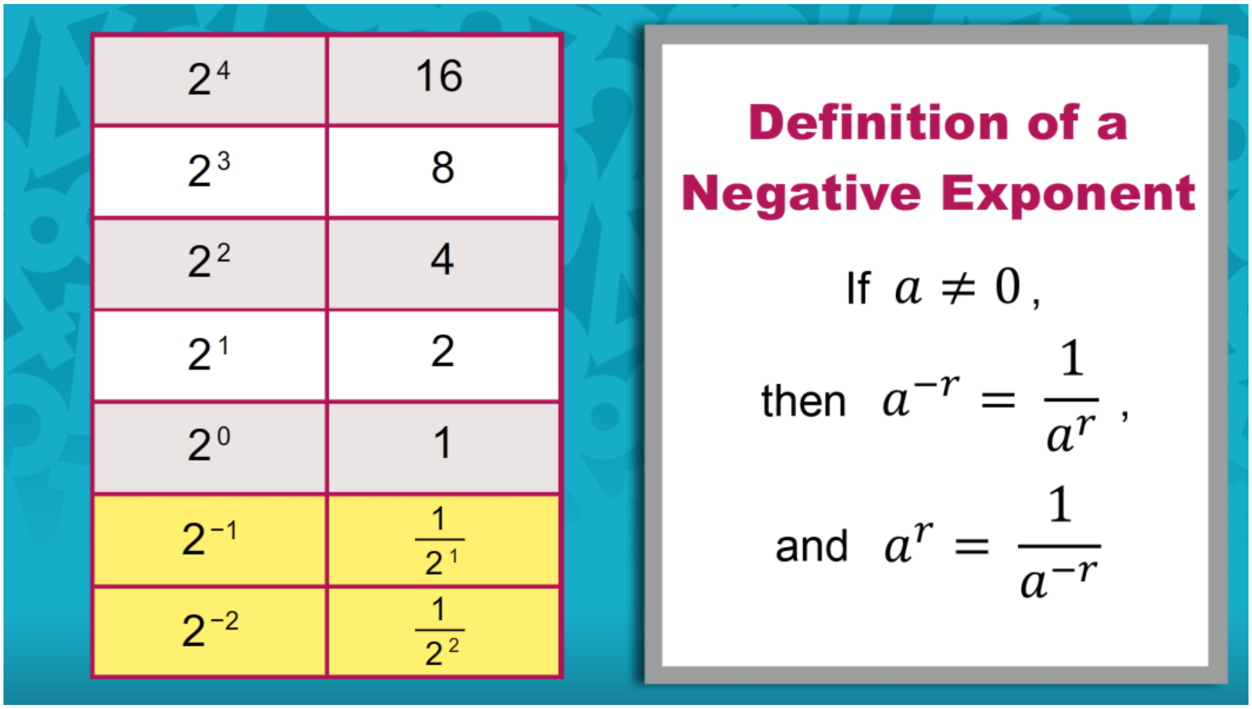
2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2^1}$
2^{-2}	$\frac{1}{2^2}$

In the table, you determined that $2^0 = 1$.

The definition of a zero exponent states that if $a \neq 0$, then $a^0 = 1$.

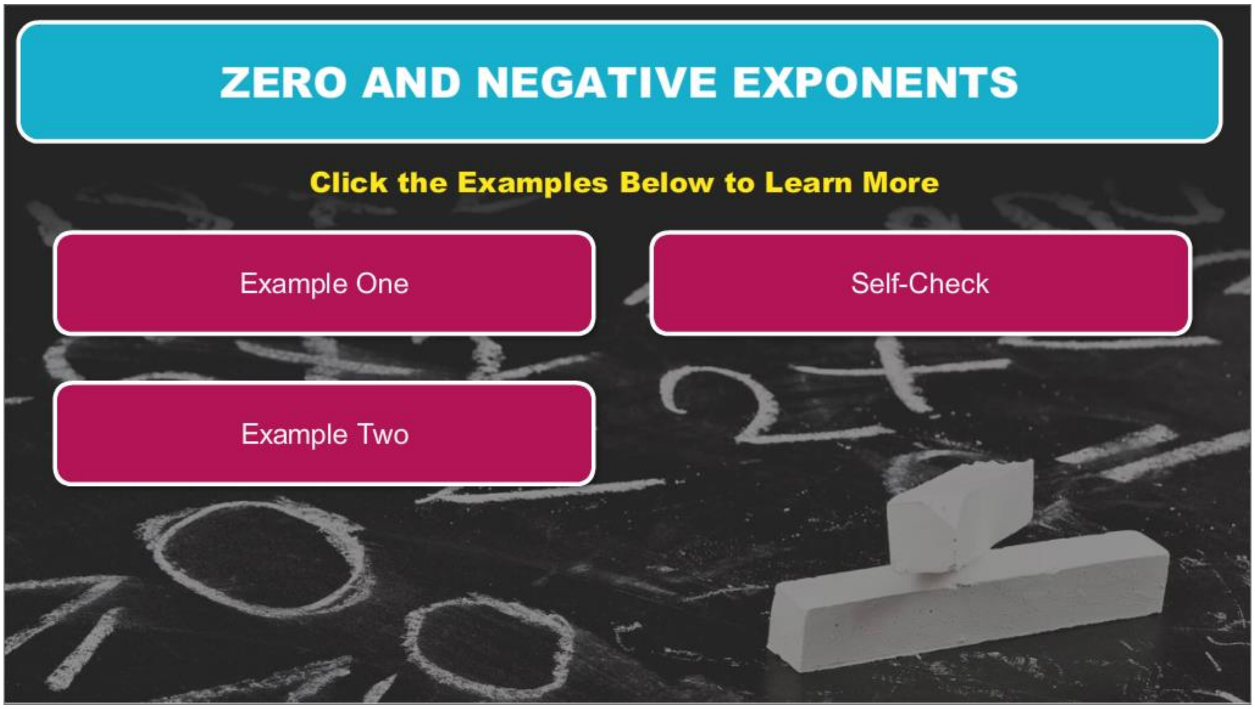
In the table, you also determined that $2^{-1} = \frac{1}{2}$, which is equivalent to $\frac{1}{2^1}$; and $2^{-2} = \frac{1}{4}$ which is equivalent to $\frac{1}{2^2}$.

Definition of a Negative Exponent



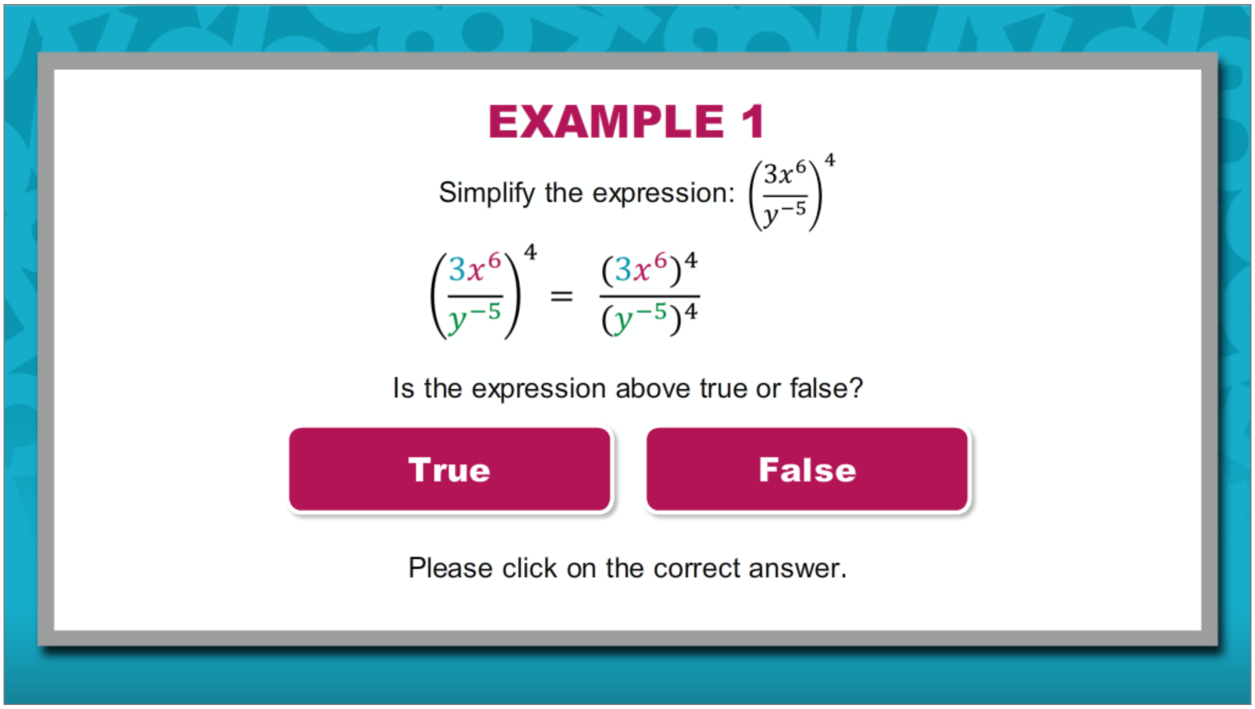
The definition of a negative exponent states that if $a \neq 0$, then $a^{-r} = \frac{1}{a^r}$, and $a^r = \frac{1}{a^{-r}}$.

Zero and Negative Exponents



Click the examples below the learn more.

Example 1



Simplify the following expression:

$$\left(\frac{3x^6}{y^{-5}}\right)^4$$

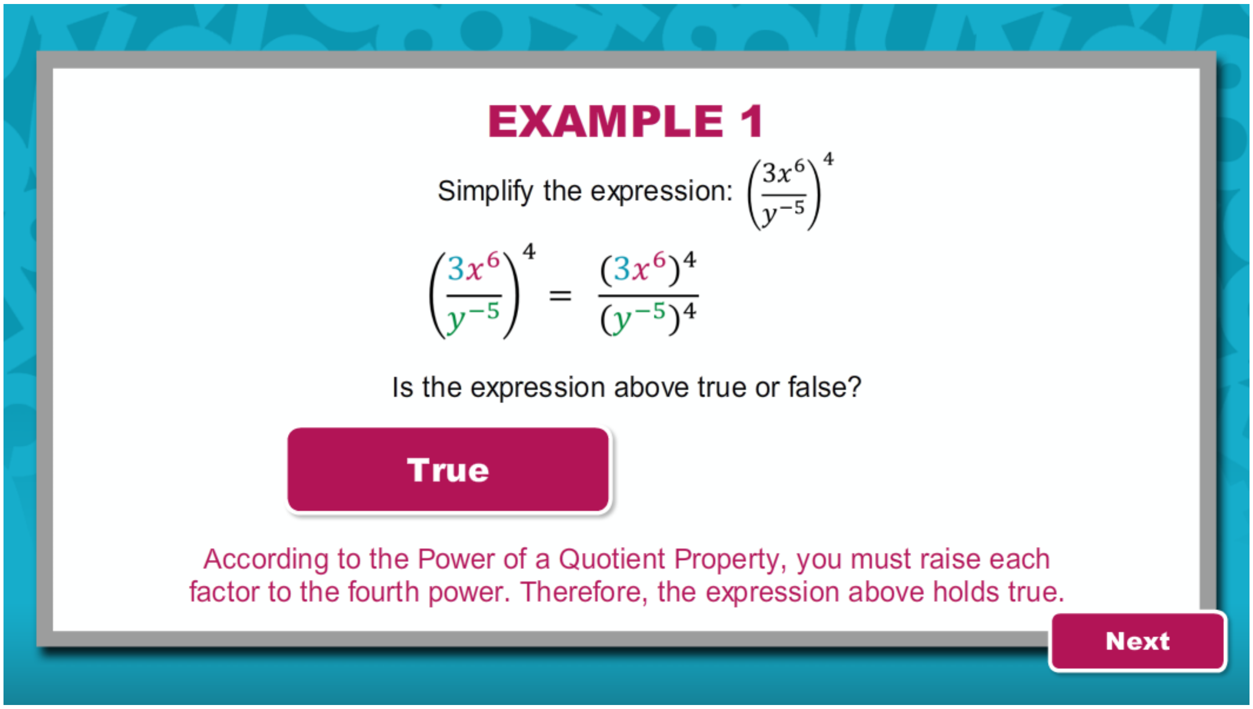
In this example, you must raise the quotient of $3x^6$ and y^{-5} to the fourth power. According to the Power of a Quotient Property, you must raise the numerator and denominator to the fourth power. Therefore,

A) True

B) False

Is the expression above true or false? Please click on the correct answer.

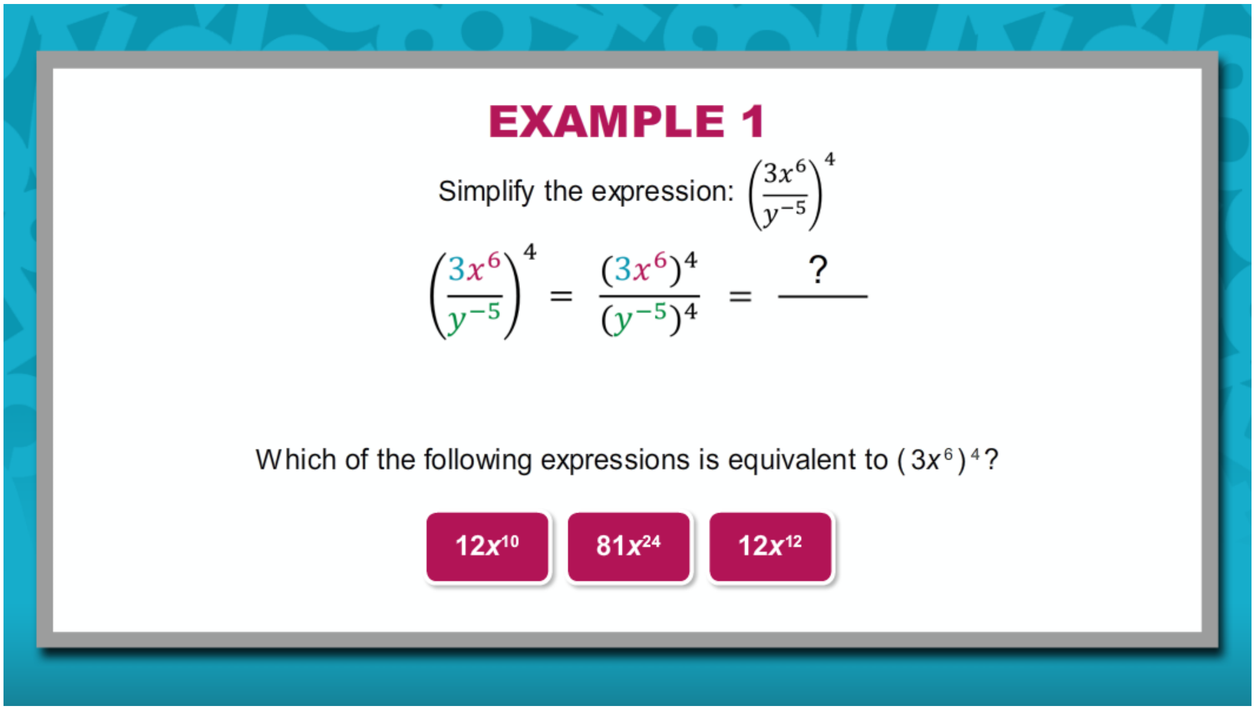
Example 1 (continued)



$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{\left(3x^6\right)^4}{(y^{-5})^4}$$

Feedback: According to the Power of a Quotient Property, you must raise each factor to the fourth power. Therefore, the expression above holds true.

Example 1 (continued)



To simplify the expression in the numerator, $(3x^6)^4$, you must apply the Power of a Product Property.

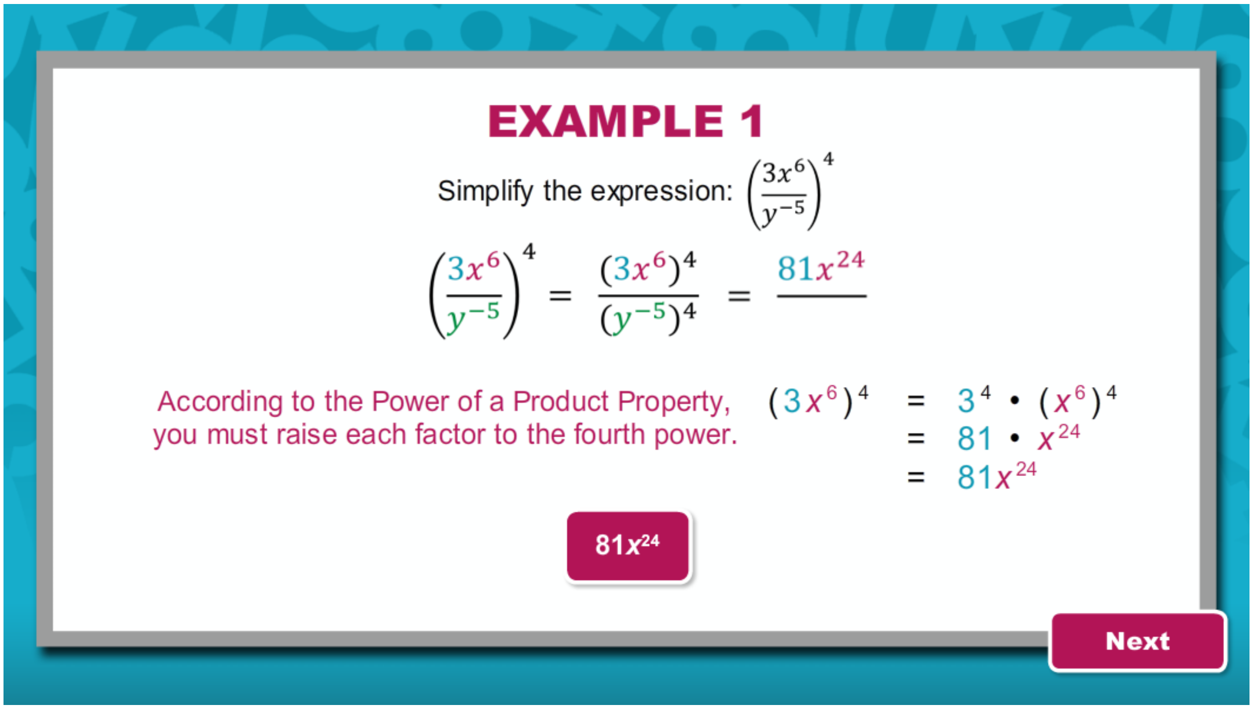
Which of the following expressions is equivalent to $(3x^6)^4$?

A) $12x^{10}$

B) $81x^{24}$

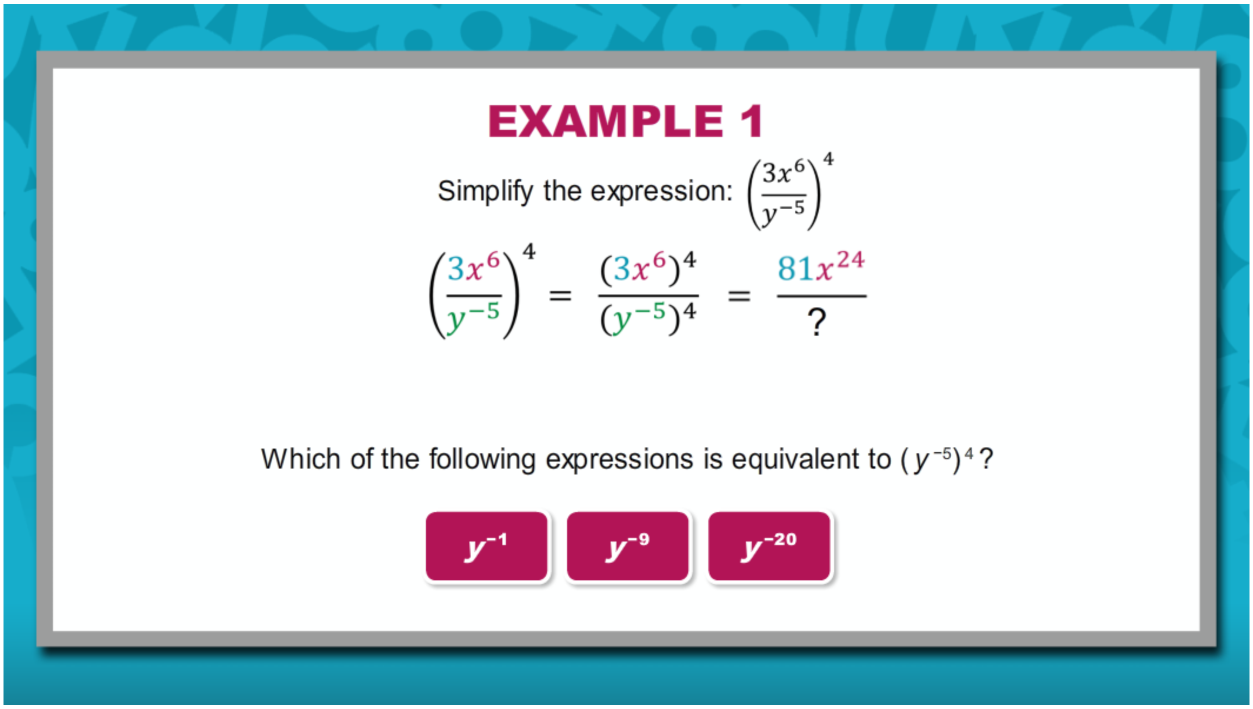
C) $12x^{12}$

Example 1 (continued)



Feedback: According to the Power of a Product Property, you must raise each factor to the fourth power.

Example 1 (continued)



To simplify the expression in the denominator, you must apply the Power of a Power Property.

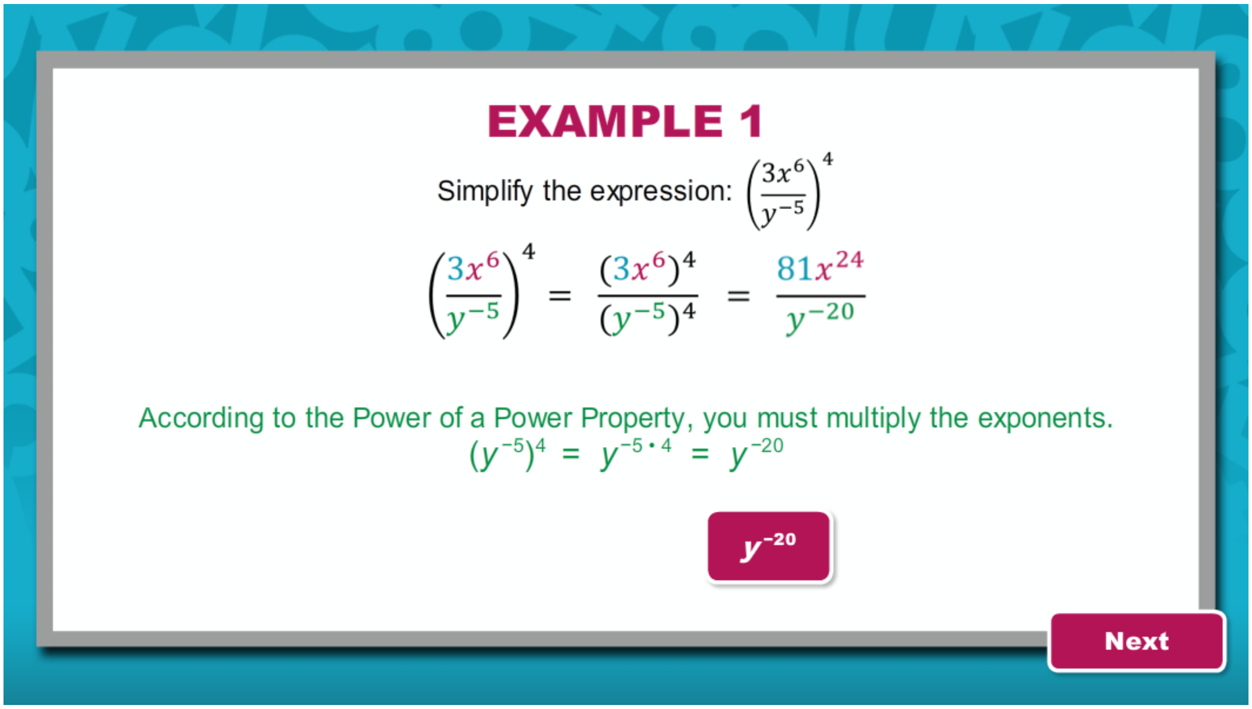
Which of the following expression is equivalent to $(y^{-5})^4$?

A) y^{-1}

B) y^{-9}

C) y^{-20}

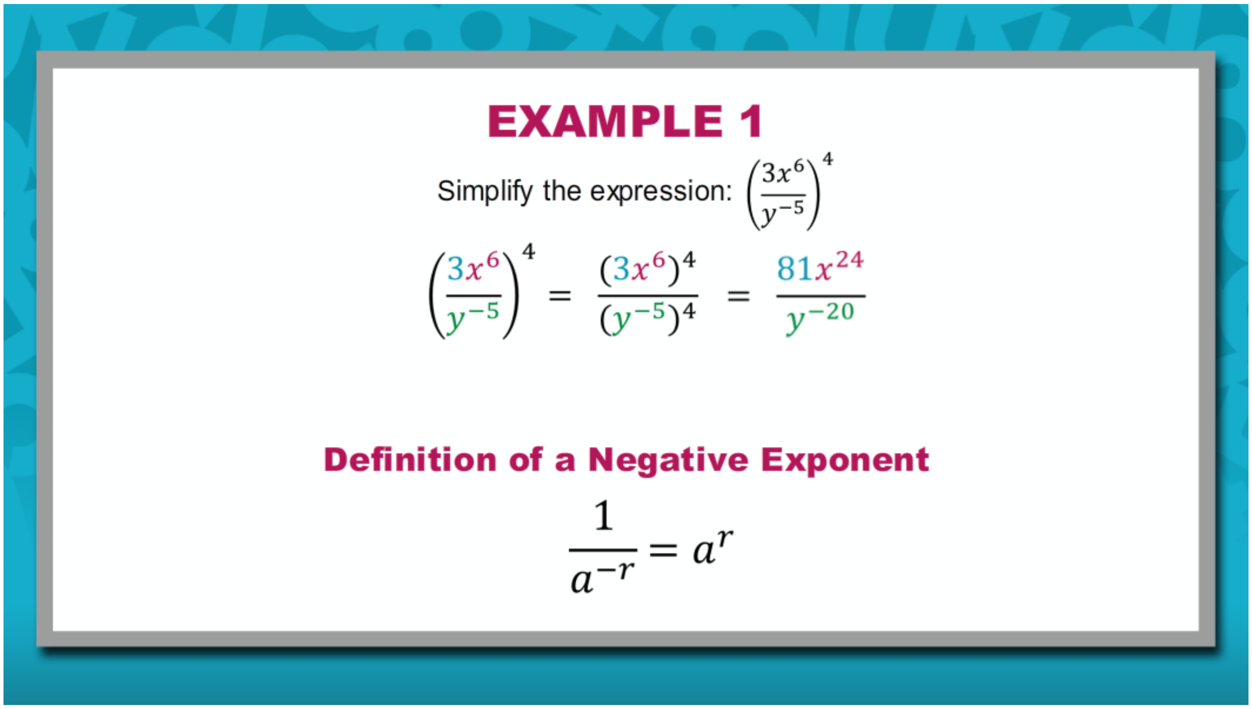
Example 1 (continued)



Feedback: According to the Power of a Power Property you must multiply the exponents.

$$(y^{-5})^4 = y^{-5 \cdot 4} = y^{-20}$$

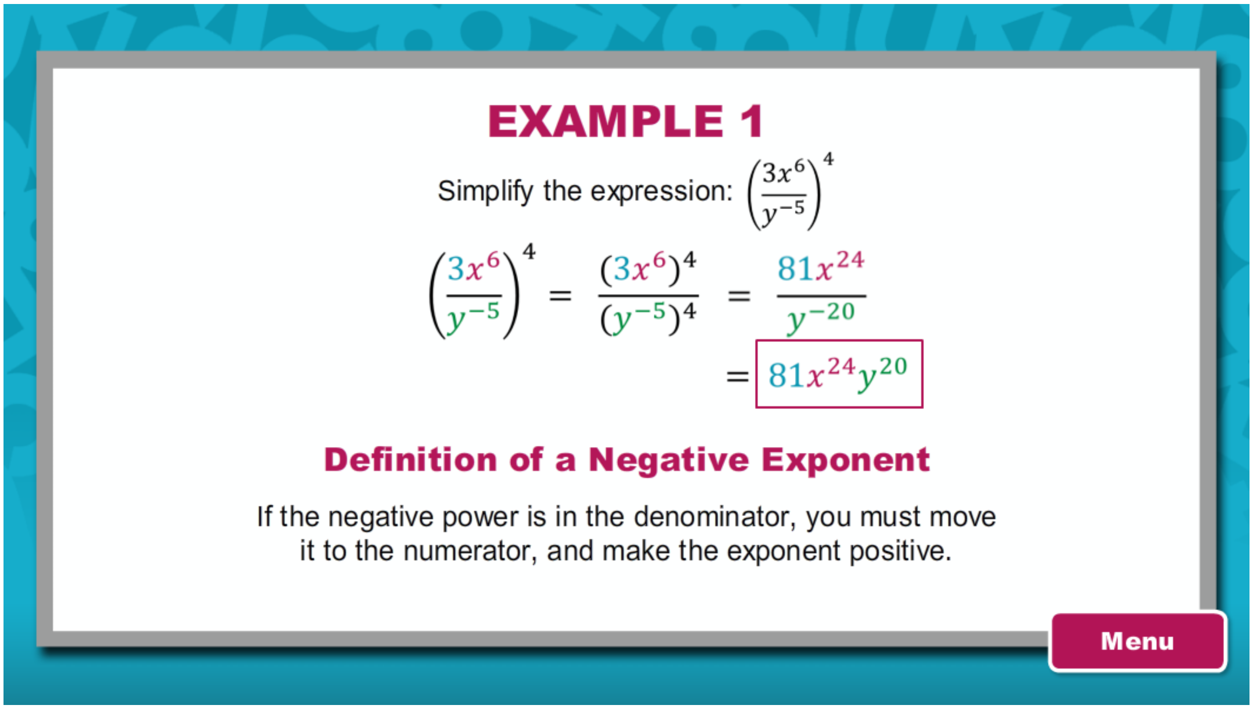
Example 1 (continued)



Now you must use the Definition of a Negative Exponent in order to eliminate the negative power in the denominator.

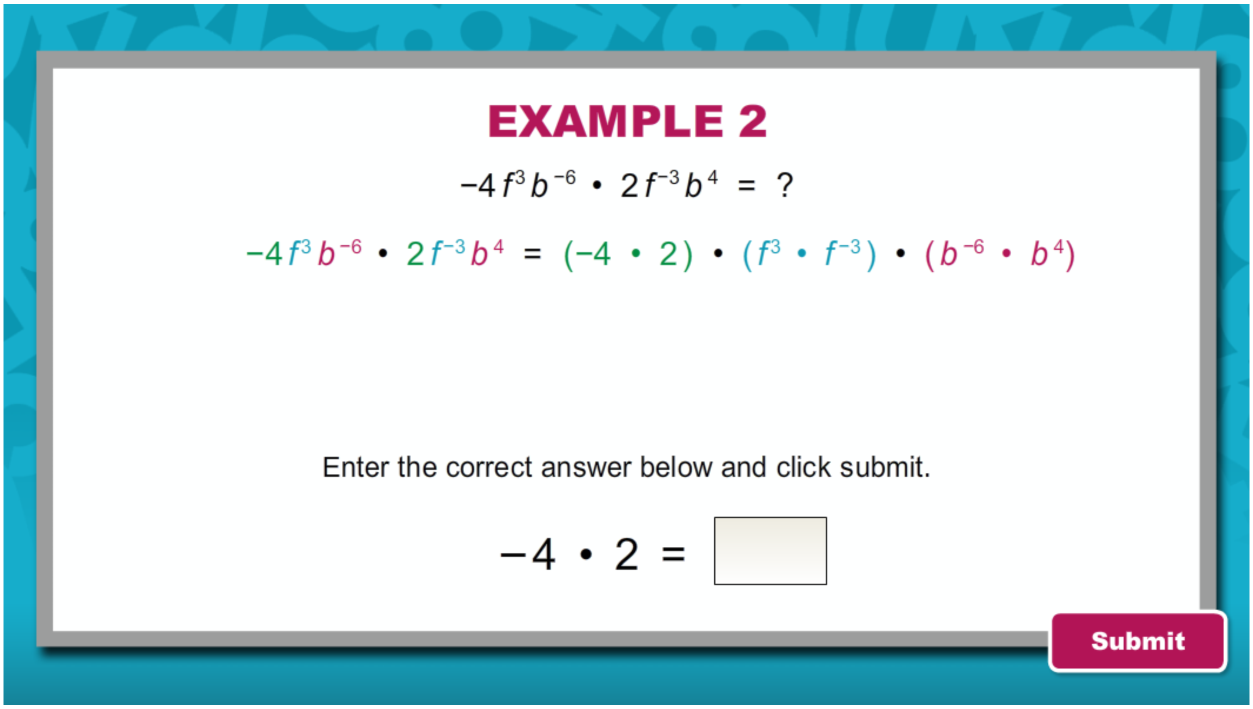
According to the Definition of a Negative Exponent, $\frac{1}{a^{-r}} = a^r$. So, $\frac{81x^{24}}{y^{-20}} = 81x^{24}y^{20}$.

Example 1 (continued)



One way to think about this is if the term that includes the negative power is in the denominator, you must move it to the numerator, and make the exponent positive.

Example 2

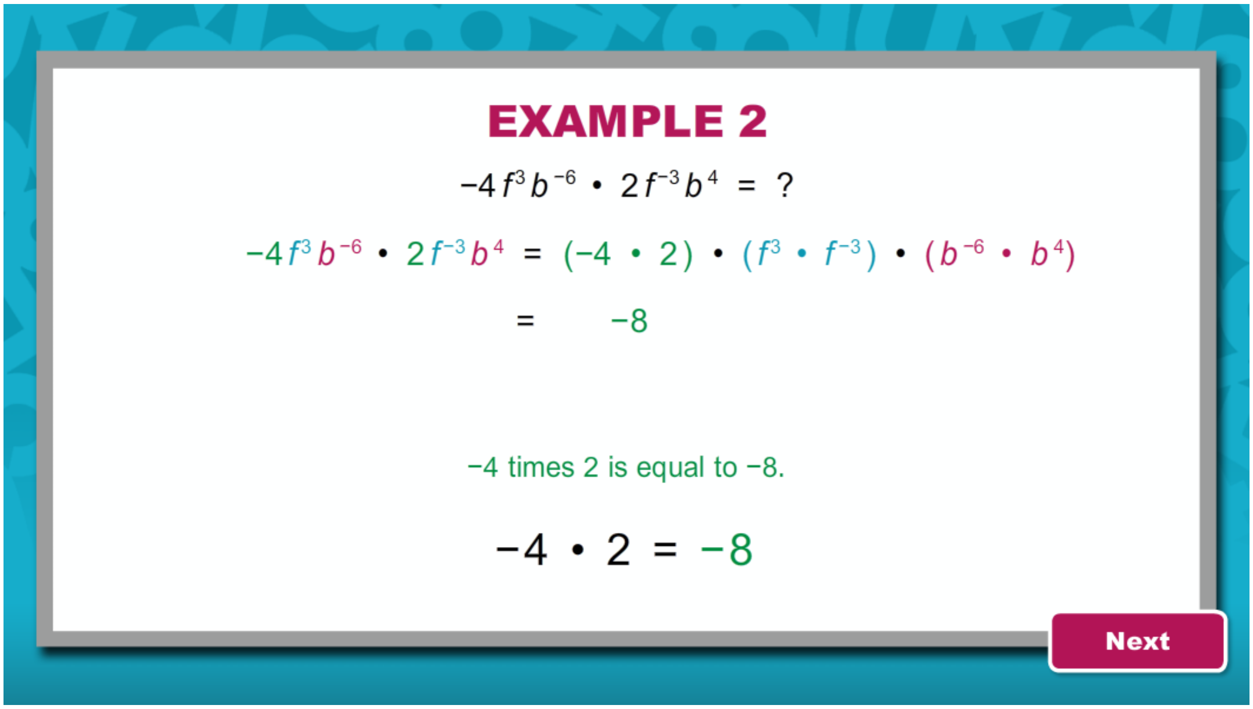


In this example, you must find the product of exponential expressions that include coefficients and more than one base. Recall that the Commutative Property of Multiplication states that you can multiply values in any order. Apply that property in this example by changing the order of multiplication, so that you multiply the coefficients first, then f^3 and f^{-3}, and finally b^{-6} and b^4.

$-4 \cdot 2 = ?$

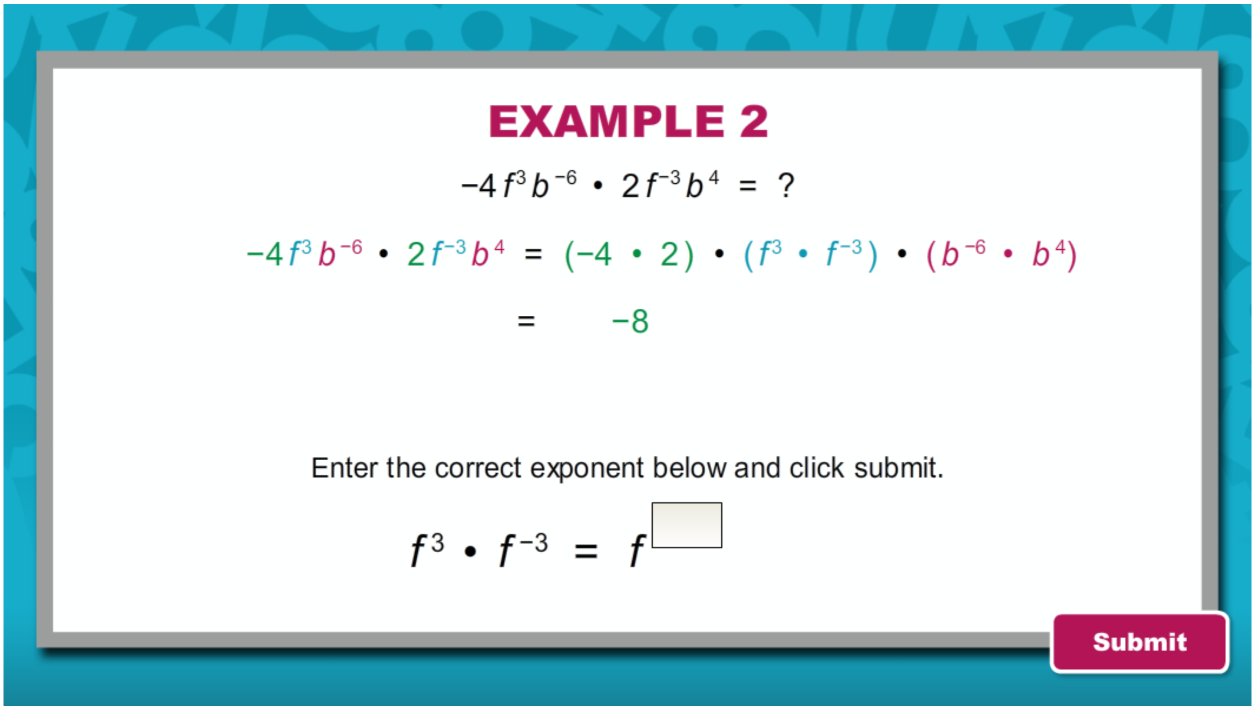
Enter the correct answer below and click submit.

Example 2 (continued)



Feedback: $-4 \cdot 2 = -8$

Example 2 (continued)

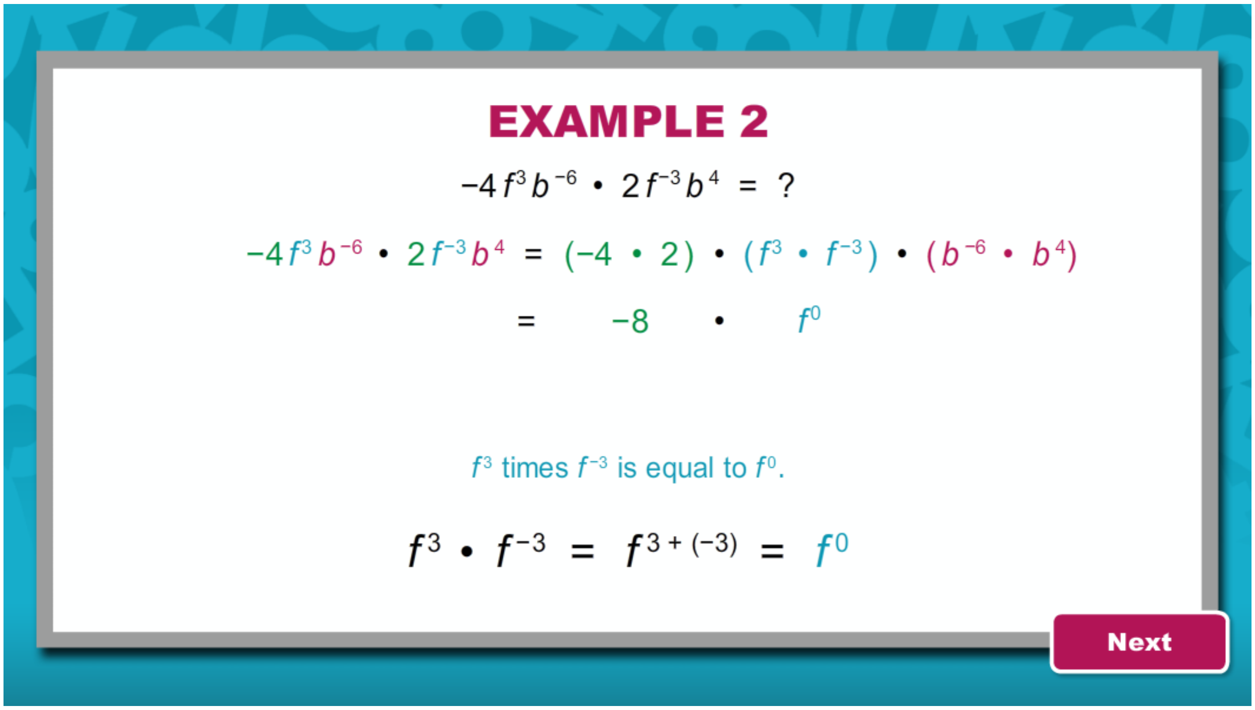


In order to multiply f^3 and f^{-3}, you will need to add the exponents.

$f^3 \cdot f^{-3} = f^?$

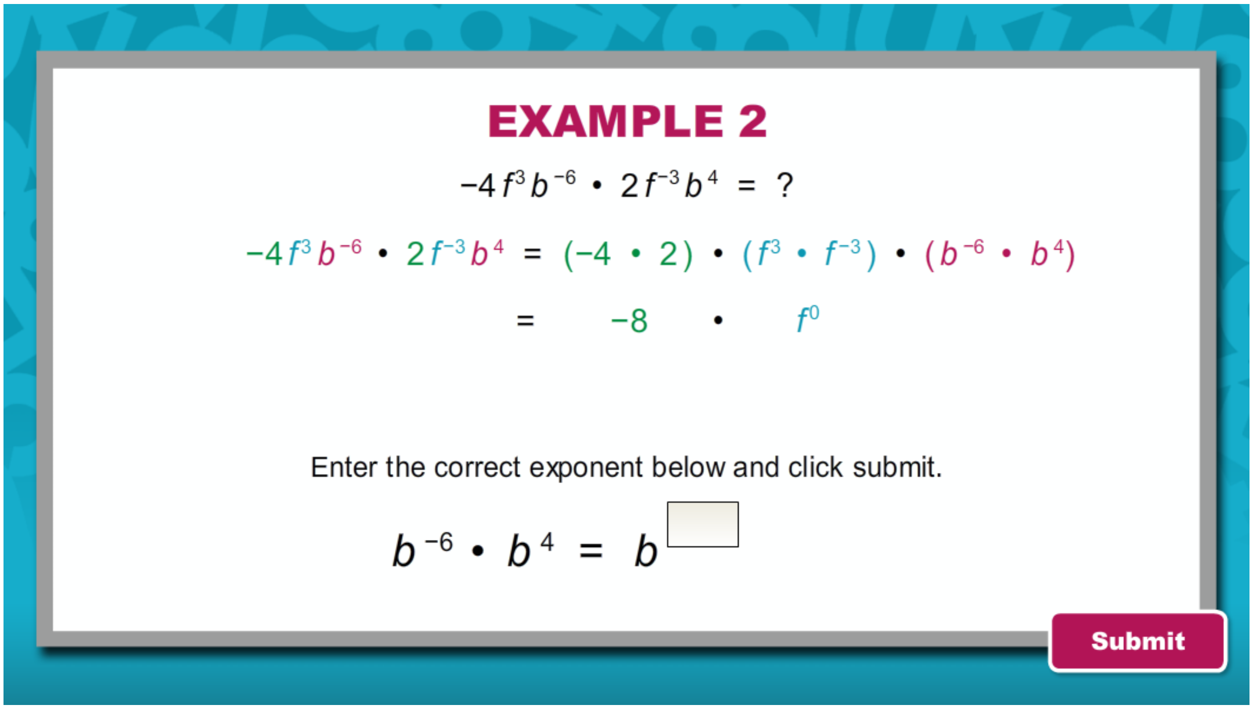
Enter the correct exponent below and click submit.

Example 2 (continued)



Feedback: $f^3 \cdot f^{-3} = f^{3+(-3)} = f^0$

Example 2 (continued)

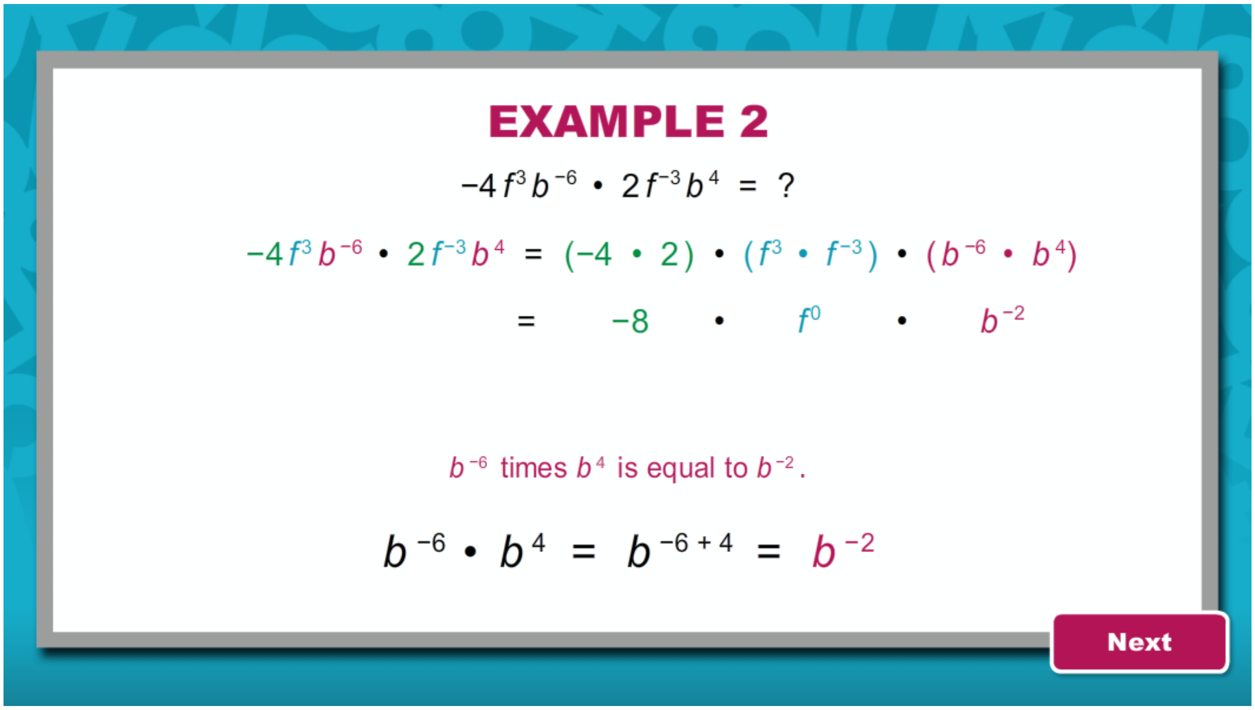


In order to multiply b^{-6} and b^4, you will need to add the exponents.

$b^{-6} \cdot b^4 = b^?$

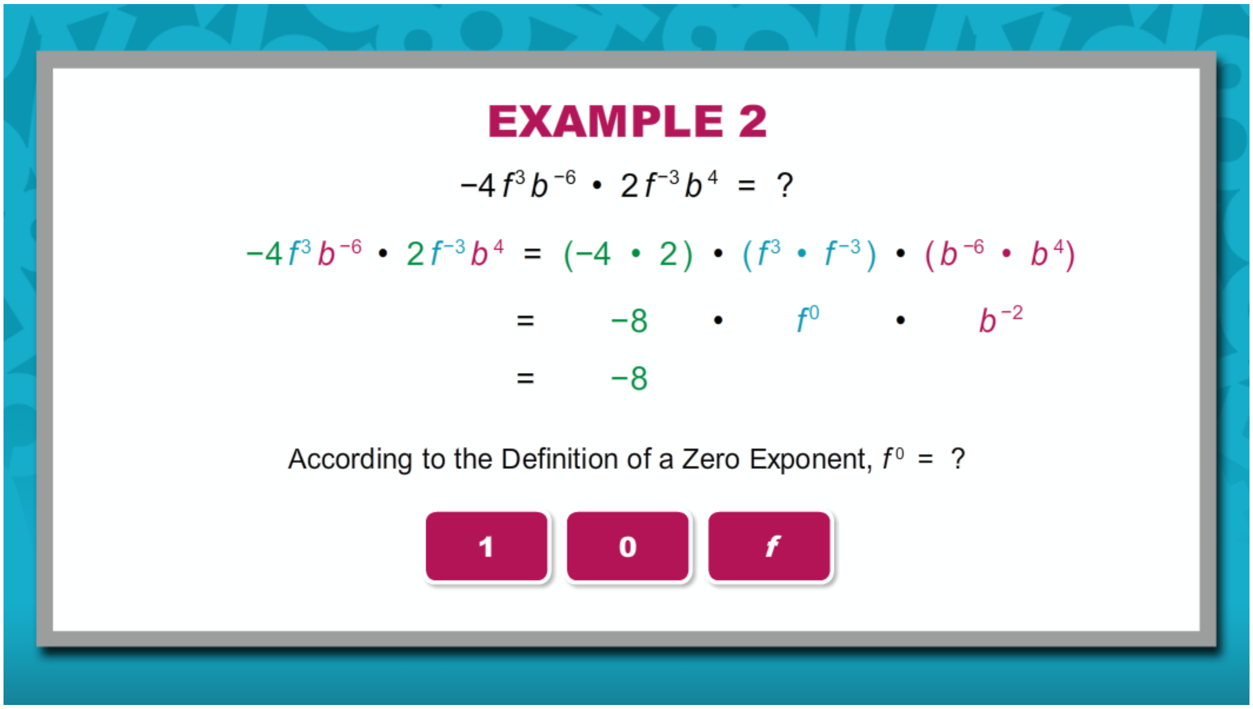
Enter the correct exponent below and click submit.

Example 2 (continued)



Feedback: $b^{-6} \cdot b^4 = b^{-6+4} = b^{-2}$

Example 2 (continued)



Continue simplifying the exponential expressions.

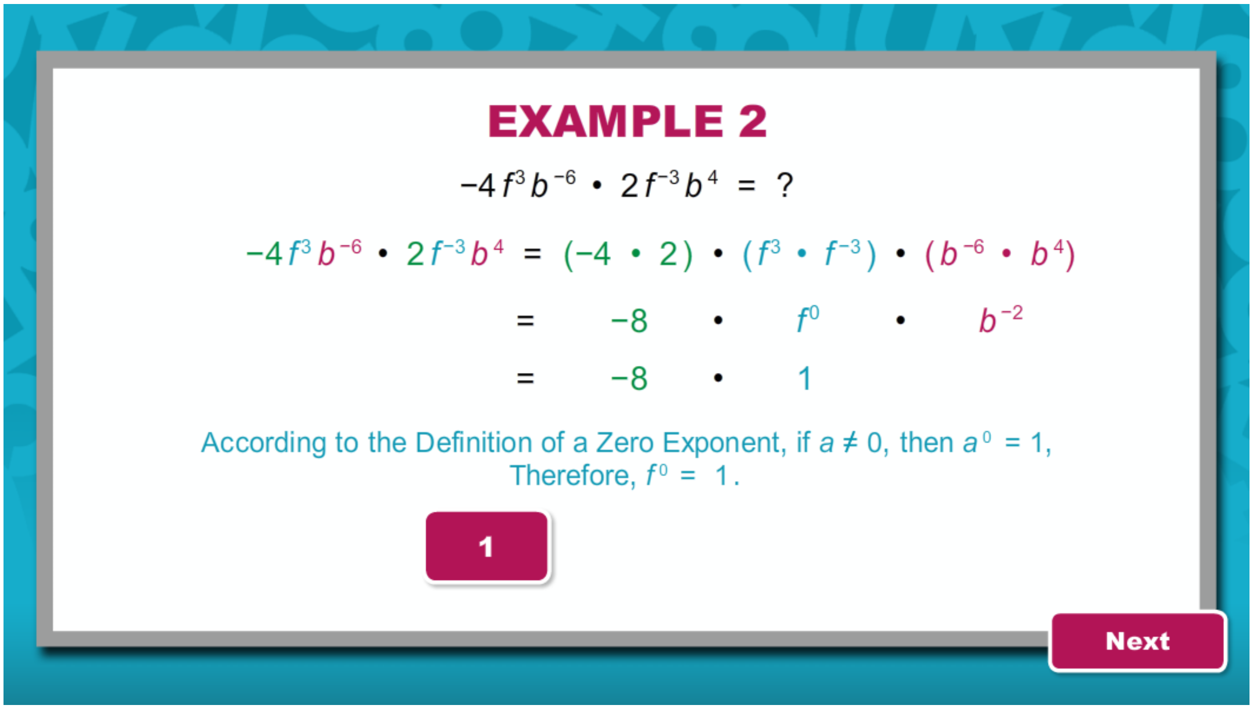
According to the Definition of a Zero Exponent, $f^0 = ?$

A) 1

B) 0

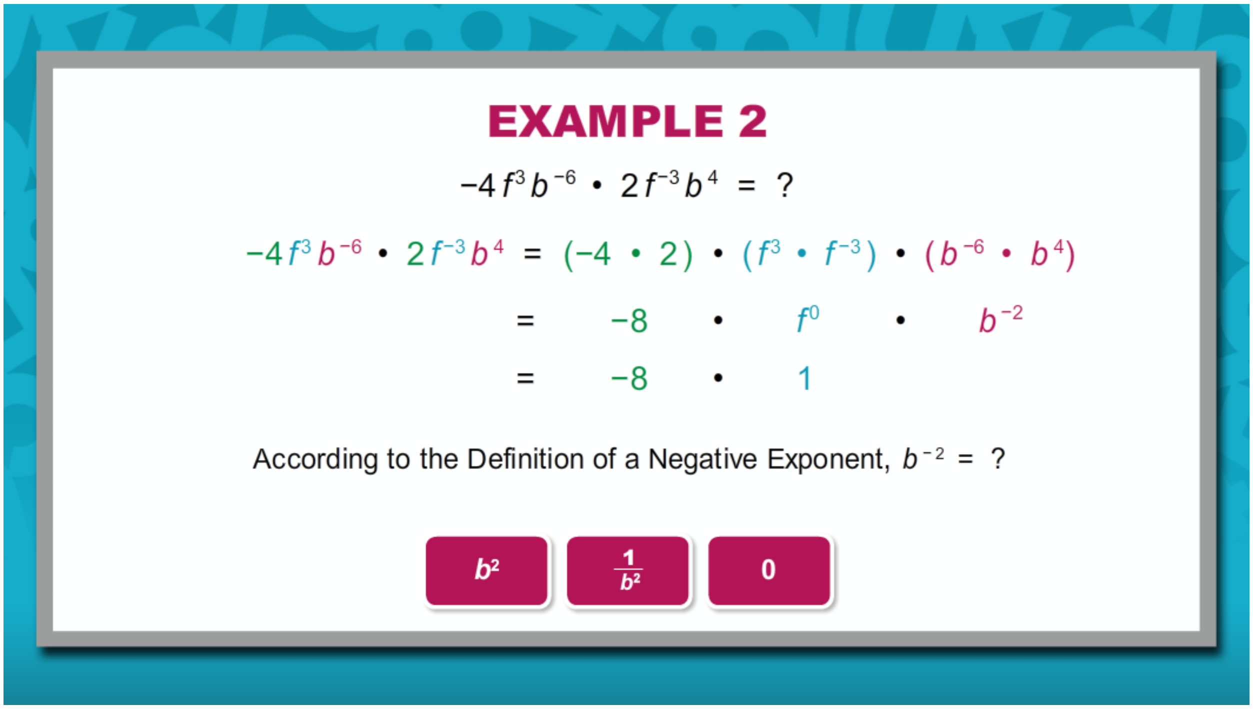
C) f

Example 2 (continued)



Feedback: According to the Definition of Zero Exponents, if $a \neq 0$, then $a^0 = 1$. Therefore, $f^0 = 1$.

Example 2 (continued)



You must use the Definition of a Negative Exponent in order to simplify b^{-2}. Recall that according to the Definition of Negative Exponents, $a^{-r} = \frac{1}{a^r}$.

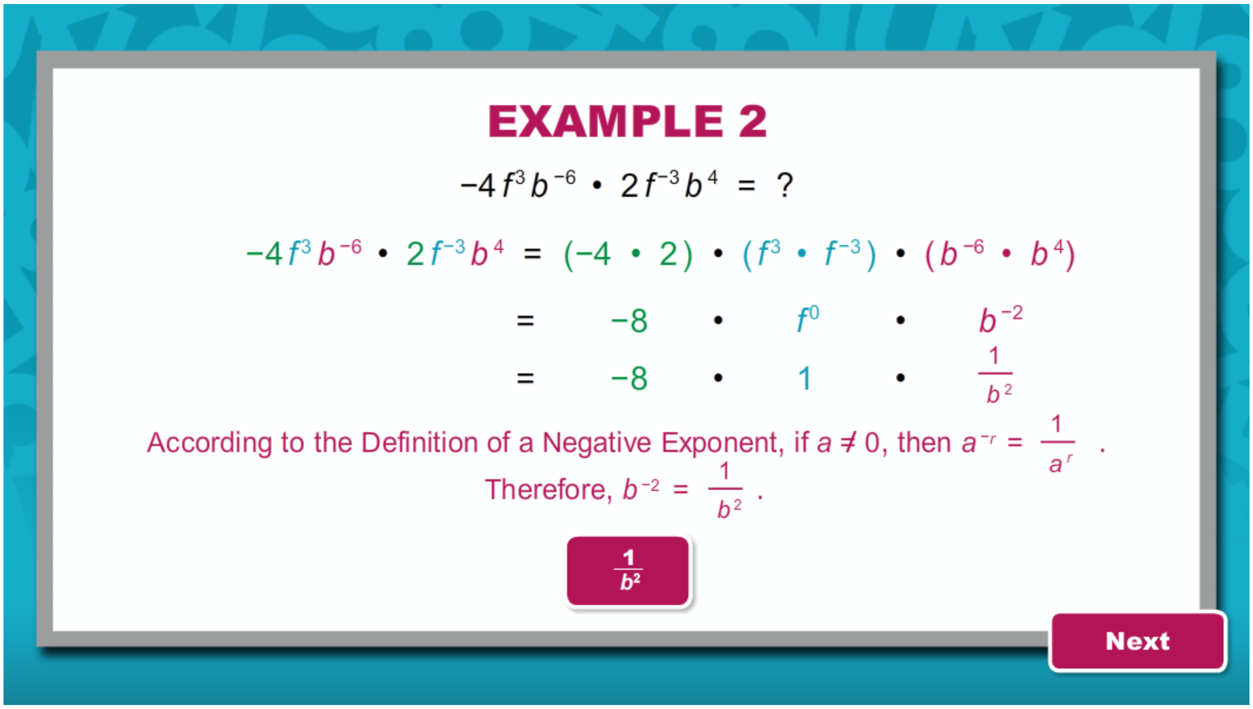
So, $b^{-2} = ?$.

A) b^2

B) $\frac{1}{b^2}$

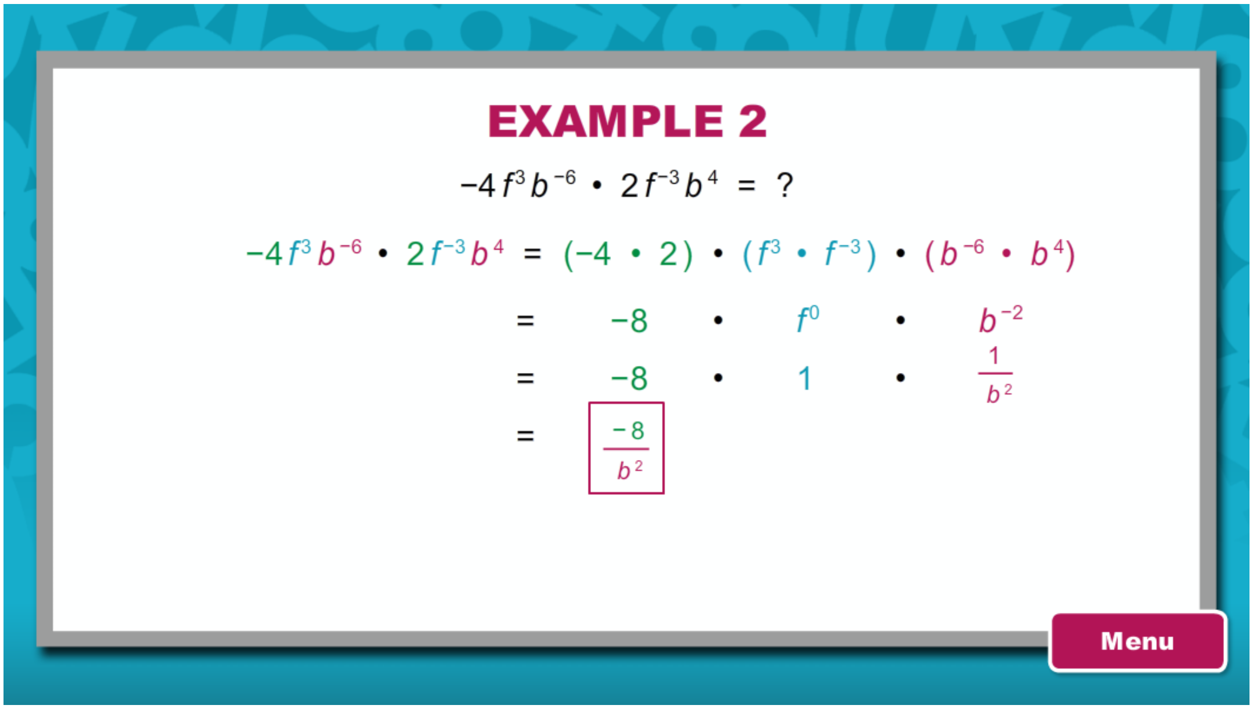
C) 0

Example 2 (continued)



Feedback: According to the Definition of a Negative Exponent, if $a \neq 0$, then $a^{-r} = \frac{1}{a^r}$

Example 2 (continued)

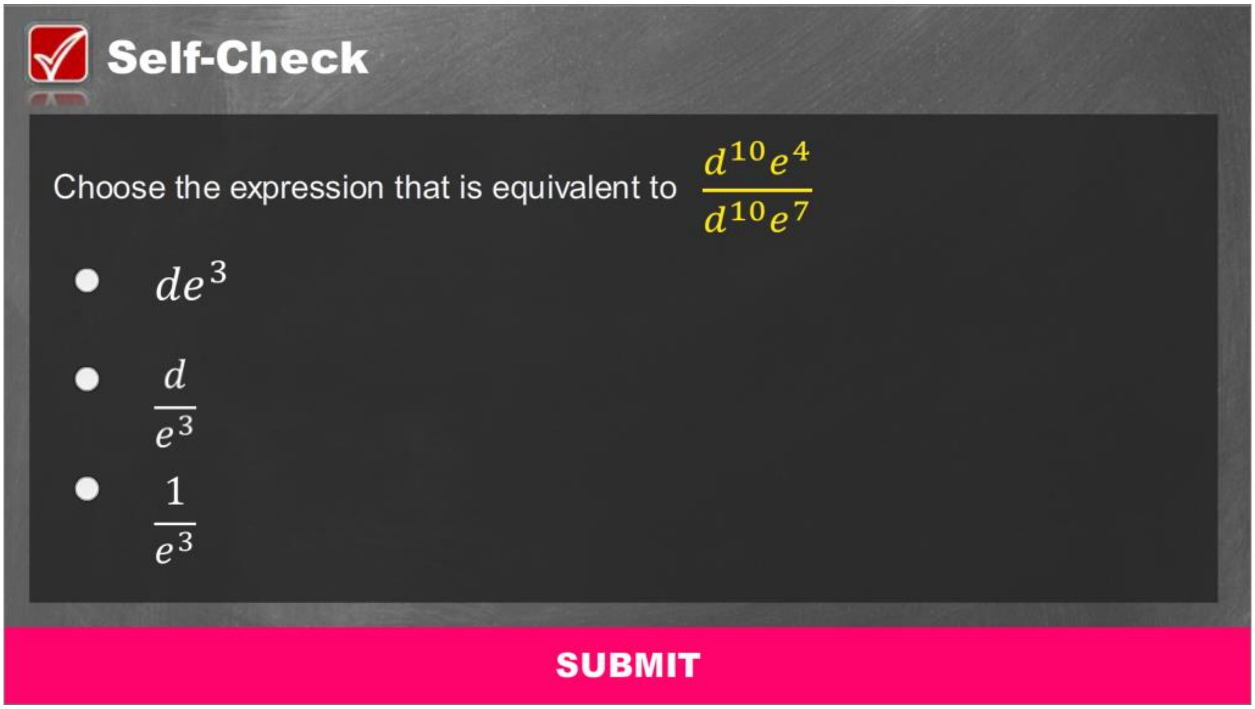


Now simplify the product.

$$-8 \cdot 1 \cdot \frac{1}{b^2} = \frac{-8}{b^2}$$

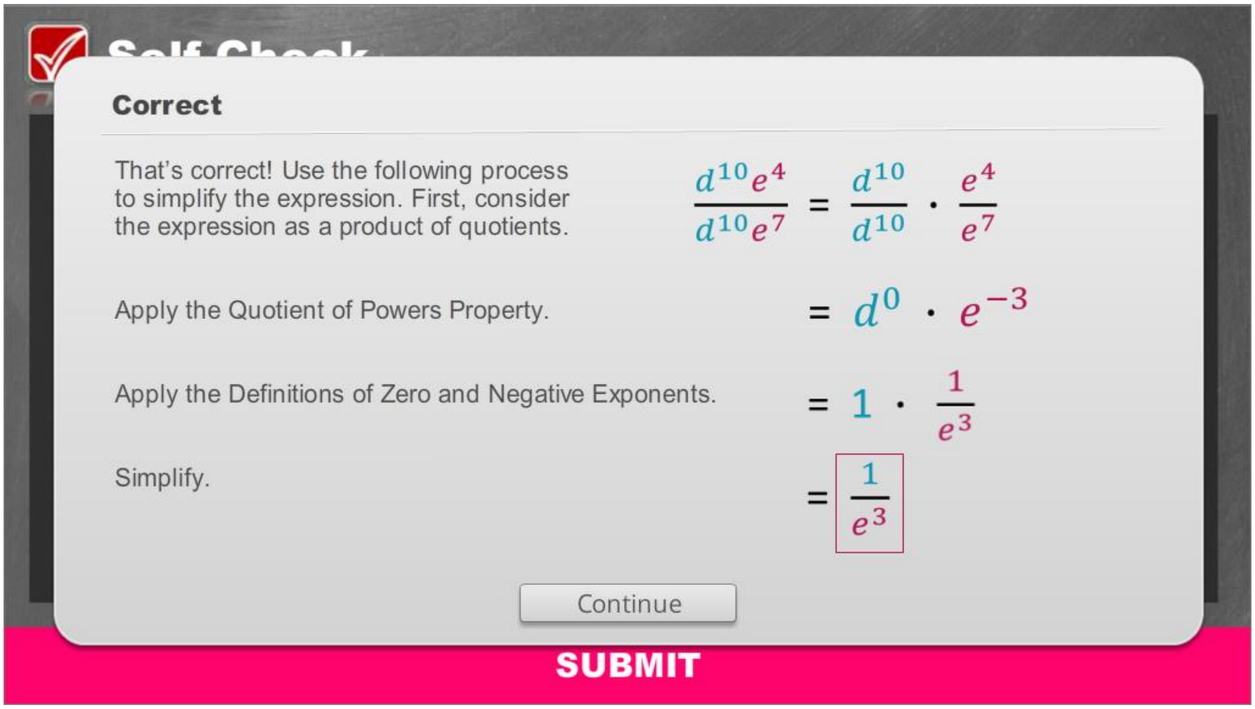
Your work is complete.

Self-Check



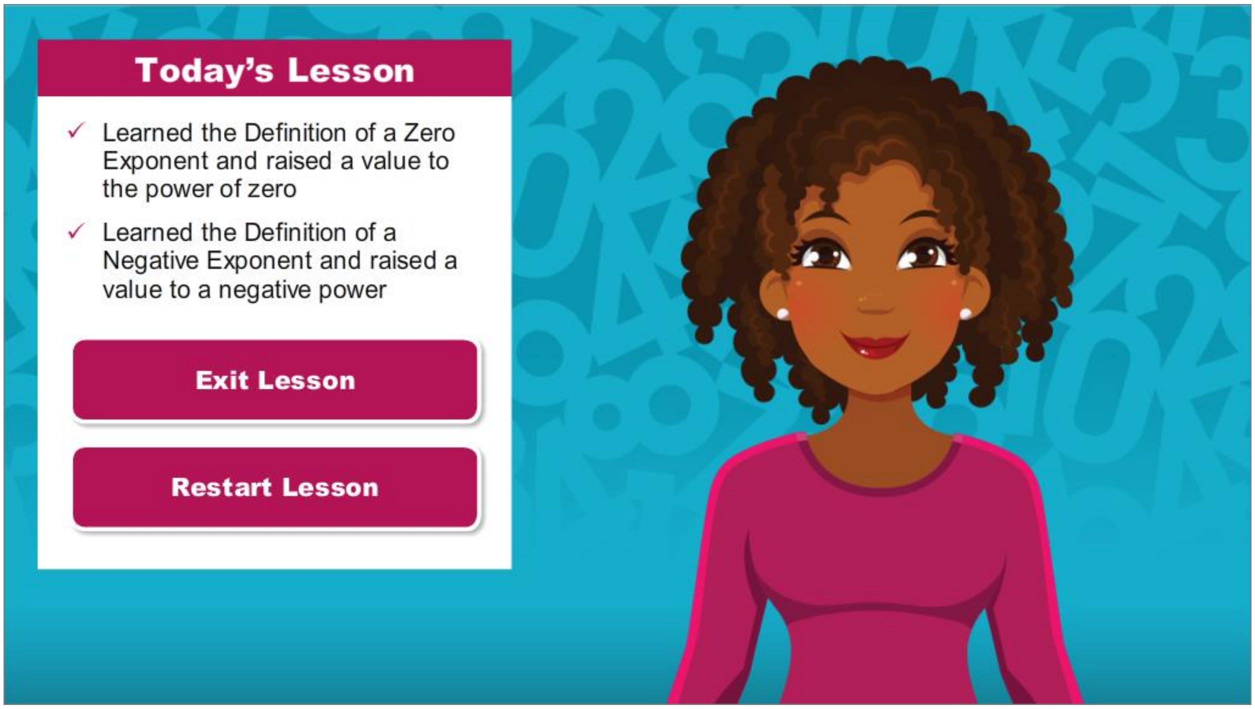
Solve the problem in the image above to check your understanding of the content.

Self-Check: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



Congratulations! You have reached the conclusion of this lesson in Algebra I. In this lesson, you applied your knowledge of exponents to discover definitions that allow you to raise a value to the power of zero or to a negative power.