Introduction



Hello and welcome! I'm so glad to have you here for this lesson in Algebra I. In this lesson, you will extend your knowledge of factoring to include second-degree trinomials that have a leading coefficient not equal to one.



Anticipatory Set



Take for example the given trinomial.

 $x^2 + 8x + 15$

You know that it can be represented as the product of two binomials. The first term of the trinomial is x^2 , so the first term of each of its factors must be x because $x \cdot x = x^2$.

What if you were asked to factor this trinomial?

 $6x^2 - x - 2$

The three terms share no common factor other than 1. So, factoring out a greatest common factor wouldn't change much at all.

The first term of this trinomial is $6x^2$.

How could you determine what the first term of each of its binomial factors should be? Would they be *x* and 6*x* because $x \cdot 6x = 6x^2$? Or would they be 2*x* and 3*x* because $2x \cdot 3x = 6x^2$?

To factor a trinomial in the form $ax^2 + bx + c$, where $a \neq 1$, you could guess and check different pairs of factors. Or, you could factor by grouping.



Factoring Trinomials With a Leading Coefficient Not Equal to One



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check



Example 1

EX Factor cor 3) 3	0	Factors of 18 on a sheet of notebook paper Click NEXT to verify your factors	and c, ors.
	0		Next

Factor completely:

 $3x^2 + 11x + 6$.

To factor by grouping, begin by factoring out the greatest common factor.

The terms of this trinomial share no common factor other than 1, so you can move on to the next step of the process, which is to find the product of *a* and *c*.

 $3 \cdot 6 = 18$

Now list the factors of 18.



Example 1 (continued)

E Factor co 3) 3) 3) 4 5 5 7 5 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	and c, ors.
--	----------------

Your list of factors should resemble the following:

Facto	rs of 18
1	18
-1	-18
2	9
-2	-9
3	6
-3	-6



Example 1 (continued)

E Factor co 3) 3) 4) 5) 5) 6) 6) 6) 7) 7) 7) 7) 7) 7) 7) 7) 7) 7	nmon factor. and <i>c</i> , ors.
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Which pair of factors has a sum equal to the coefficient of the middle term?

Facto	rs of 18
1	18
-1	-18
2	9
-2	-9
3	6
-3	-6

Click the pair of factors above that has a sum of 11.



Example 1 (continued)



The numbers 2 and 9 are the factors of 18 that have a sum of 11.



Example 1 (continued)



Now rewrite the trinomial. This time, use the sum of the chosen factors to represent the coefficient of the middle term.

 $3x^2 + 11x + 6$ $3x^2 + 2x + 9x + 6$

Although the polynomial appears to have changed, you haven't changed its value at all because 2x + 9x = 11x.

Next, group the terms into two binomials.

 $(3x^2 + 2x) + (9x + 6)$

Now factor the first binomial completely.

 $3x^2 + 2x$



Example 1 (continued)

EXAMPLE 1
Factor completely:
$$3x^2 + 11x + 6$$

 $3x^2 + 11x + 6$
 $3x^2 + 2x + 9x + 6$
 $(3x^2 + 2x) + (9x + 6)$
 $x(3x + 2)$
Determine the greatest
common factor by factoring
each term completely.
 $3x^2 = 3 \cdot x \cdot x$
 $2x = 2 \cdot x$
The greatest common factor (GCF) is x.
Now, factor the GCF out of the terms of
the first binomial. To accomplish this,
divide each of its terms by the GCF.
 $\frac{3x^2}{x} = 3x$ $\frac{2x}{x} = 2$
So, when factored completely, the first
binomial can by represented as
 $x(3x + 2)$.
Next

Determine the greatest common factor by factoring each term completely.

$$3x^2 = 3 \cdot x \cdot x$$
$$2x = 2 \cdot x$$

The greatest common factor is *x*.

Now that you have identified the greatest common factor, factor it out of the terms of the first binomial. To accomplish this, divide each of its terms by the greatest common factor.

$$\frac{3x^2}{x} = 3x$$
$$\frac{2x}{x} = 2$$

So when factored completely, the first binomial can be represented as

x(3x + 2).



Example 1 (continued)

EXAMPLE 1 Factor completely: $3x^2 + 11x + 6$ $3x^2 + 11x + 6$ $3x^2 + 2x + 9x + 6$ $(3x^2 + 2x) + (9x + 6)$ x(3x + 2)	STEP 1 Factor out the greatest common factor. STEP 2 Find the product of <i>a</i> and <i>c</i> , and then list its factors. STEP 3 Rewrite the trinomial using the factors with a sum equal to the middle coefficient. STEP 4 Group the terms into two binomials. STEP 5 Factor both binomials completely.
Enter the correct values above and click <i>SUBMIT</i> .	

Now factor the second binomial completely.

9x + 6



Example 1 (continued)

EXAMPLE 1Factor completely:
$$3x^2 + 11x + 6$$
 $3x^2 + 11x + 6$ $3x^2 + 2x + 9x + 6$ $(3x^2 + 2x) + (9x + 6)$ $x(3x + 2) + 3(3x + 2)$ Determine the greatestcommon factor (GCF) is 3.Now, divide each term by the GCF. $\frac{9x}{3} = 3x$ $\frac{6}{3} = 2$ The second binomials factorscompletely to $3(3x + 2)$

Determine the greatest common factor.

$$9x = 3 \cdot 3 \cdot x$$

$$6 = 2 \cdot 3$$

The greatest common factor is 3.

Now divide each term by the greatest common factor.

$$\frac{9x}{3} = 3x$$
$$\frac{6}{3} = 2$$

The second binomial factors completely to 3(3x + 2).



Example 1 (continued)



Notice that the groups each have a binomial factor of (3x + 2). Factor it out of the expression.

x(3x+2) + 3(3x+2)(3x+2)(x+3)

Finally, you can represent the expression as the product of the binomials

(3x + 2)

and

(x + 3).

You have factored the given trinomial completely.



Example 2



Factor completely:

 $10x^2 - 2x - 8$.

Begin by considering the terms of the trinomial. Do the terms share a greatest common factor other than 1?



Example 2 (continued)



The answer is yes. The terms of the trinomial have a greatest common factor of 2.

Factor out the greatest common factor.

Which of the following accurately represents the trinomial?

A) $2(8x^2 - x - 6)$ B) $2(5x^2 - x - 4)$ C) $2(5x^2 + x + 4)$



Example 2 (continued)



To determine the answer, divide each term of the trinomial by the greatest common factor of 2.

$$10x^2 - 2x - 8$$

$$\frac{10x^2}{2} = 5x^2$$
$$\frac{-2x}{2} = -x$$
$$\frac{-8}{2} = -4$$

Now that the greatest common factor is factored out, the trinomial can be represented as

$$2(5x^2 - x - 4).$$



Example 2 (continued)



For a moment, set aside the greatest common factor of 2 and focus your attention on the trinomial $5x^2 - x - 4$. You can factor this trinomial by grouping. Begin by finding the product of *a* and *c*.

What is the product of 5 and -4?



Example 2 (continued)



The correct answer is

$$5 \cdot -4 = -20.$$

Now list the factors of -20 on a sheet of notebook paper.



Example 2 (continued)

E) Factor cor	Factors of -20	imon factor.
1	Your list of factors should resemble the following	ors.
2	1 -20	
	-1 20	
	2 -10	
	-2 10	
0	4 -5	
	-4 5	
		Next

Your list of factors should resemble the following:

Factors	s of -20
1	-20
-1	20
2	-10
-2	10
4	-5
-4	5

Identify the factors of -20 that have a sum of -1.



Example 2 (continued)

E			
Factor cor	Ģ	Factors of -20	imon factor.
1		Which pair of factors has a sum equal to the coefficient of the middle term?	and c, ors.
21		1 + (-20)= -19	
		-1 + 20 = 19	-
		2 + (-10) = -8	
		-2 + 10 = 8	_
	0	-4 + (-5) = -7	_
		4 + (-5) = -1	
		These are the necessary factors	Next

Which two numbers will multiply to equal -20 and add to equal -1?

The numbers 4 and -5 are the factors that have a sum of -1.



Example 2 (continued)



Now rewrite the trinomial using the sum of the chosen factors to represent the coefficient of the middle term.

The polynomial $5x^2 - x - 4$ can be rewritten as $5x^2 + 4x - 5x - 4$.

A) True B) False



Example 2 (continued)



In this expression, -x is represented by 4x - 5x.



Example 2 (continued)



Next, group the terms into two binomials.

$$(5x^2 + 4x) + (-5x - 4)$$

Now factor the first binomial completely.

 $(5x^2 + 4x)$



Example 2 (continued)

EXAMPLE 2
Factor completely:
$$10x^2 - 2x - 8$$

 $10x^2 - 2x - 8$
 $2(5x^2 - x - 4)$
 $2(5x^2 + 4x - 5x - 4)$
 $2((5x^2 + 4x) + (-5x - 4))$
 $2(x(5x + 4))$ (Meriation of the first binomial.
 $\frac{5x^2}{x} = 5x$ $\frac{4x}{x} = 4$
When factored completely, the first binomial can by represented as
 $x(5x + 4)$

First, find the greatest common factor of the terms of the first binomial.

$$5x^2 = 5 \cdot x \cdot x$$
$$4x = 2 \cdot 2 \cdot x$$

The greatest common factor is *x*.

Now that you have identified the greatest common factor, you will need to factor it out of the terms of the first binomial.

$$\frac{5x^2}{x} = 5x$$
$$\frac{4x}{x} = 4$$

So when factored completely, the first binomial can be represented as x(5x + 4).



Example 2 (continued)



Now factor the second binomial completely.

$$(-5x - 4)$$



Example 2 (continued)

EXAMPLE 2
Factor completely:
$$10x^2 - 2x - 8$$

 $10x^2 - 2x - 8$
 $2(5x^2 - x - 4)$
 $2(5x^2 + 4x - 5x - 4)$
 $2((5x^2 + 4x) + (-5x - 4))$
 $2(x(5x + 4) - 1(5x + 4))$

Find the greatest common factor of the terms of the first binomial.

$$-5x = -1 \cdot 5 \cdot x$$
$$-4 = -1 \cdot 2 \cdot 2$$

The greatest common factor is -1.

Now you will need to factor it out of the terms of the second binomial.

$$\frac{-5x}{-1} = 5x$$
$$\frac{-4}{-1} = 4$$

So when factored completely, the second binomial can be represented as -1(5x + 4).



Example 2 (continued)



Notice that the groups of the polynomial each have a common factor, the binomial 5x + 4.

You can represent the polynomial as the product (5x + 4)(x - 1).

A) True B) False



Example 2 (continued)



Once you have factored out (5x + 4), the polynomial can be represented as (5x + 4)(x - 1).



Example 2 (continued)



Once you have factored out (5x + 4), the polynomial can be represented as (5x + 4)(x - 1).

Now recall the greatest common factor of 2 that was initially factored out of the given trinomial. When factored completely, the trinomial $10x^2 - 2x - 8$ can be represented as

$$2(5x+4)(x-1)$$
.



Self-Check 1





Self-Check 1: Answer Step One

That's correct! $2x^2 - 7x - 15$	Next, rewrite the trinomial using the sum of the chosen factors to represent the coefficien of the middle term.
The terms share no common factor other than 1, so begin by finding the product of 2 and -15.	$2x^2 - 7x - 15$ $2x^2 + 3x - 10x - 15$
$2 \cdot -15 = -30$	Now, factor the polynomial by grouping. Begin by grouping the terms
Now, determine the factors of -30 that have a sum of -7. The necessary factors are 3 and -10.	$2x^{2} + 3x - 10x - 15$ $(2x^{2} + 3x) + (-10x - 15)$
Ctan One	Tue



Self-Check 1: Answer Step Two

1 -	alf Chaak
Co	orrect
Fa	actor each group completely.
	(2x2 + 3x) + (-10x - 15)x(2x + 3) - 5(2x + 3)
Fa	actor out the common binomial expression.
	$ \begin{array}{c} x(2x+3) - 5(2x+3) \\ (2x+3)(x-5) \end{array} $
WI	hen factored completely, $2x^2 - 7x - 15$ can be represented as $(2x + 3)(x - 5)$
	Step One Step Two Continue
	SUBMIT



Self-Check 2





Self-Check 2: Answer Step One

$24x^{2} + 18x + 3$ $3(8x^{2} + 6x + 1)$ Now, focus your attention on factoring the trinomial completely. $8x^{2} + 6x + 1$ Find the product of 8 and 1. $8 \cdot 1 = 8$	Next rewrite the trinomial using the sum of the chosen factors to represent the coefficient of the middle term. $8x^{2} + 6x + 1$ $8x^{2} + 2x + 4x + 1$
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Self-Check 2: Answer Step Two

Correct Now, factor the polynomial by grouping. Begin by grouping the terms. $8x^2 + 2x + 4x + 1$	Factor out the common binomial expression. 2x(4x + 1) + 1(4x + 1) $(4x + 1)(2x + 1)$
$(8x^{2} + 2x) + (4x + 1)$ Factor each group completely. $(8x^{2} + 2x) + (4x + 1)$ $2x(4x + 1) + 1(4x + 1)$	Recall the greatest common factor of 3 that was initially factored out of the trinomial. When factored completely, the trinomial $24x^2 + 18x + 3$ can be represented as 3(4x + 1)(2x + 1)
Step One Step Two Continue SUBMIT	



Conclusion



Congratulations! You have reached the conclusion of this lesson, where you learned how to factor second-degree trinomials by grouping.

