Module 3: Adding and Subtracting Polynomials

Topic 3 Content: Factoring Trinomials With a Leading Coefficient Not Equal to One

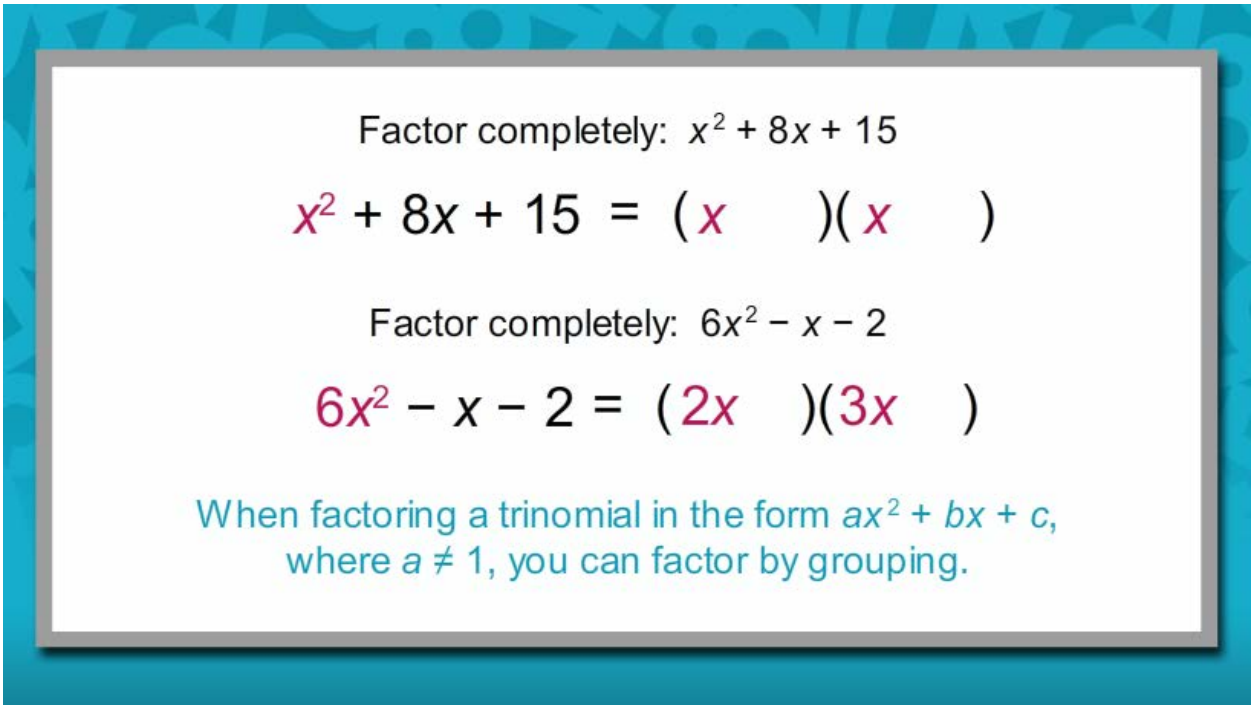
Introduction

Hello and welcome! I'm so glad to have you here for this lesson in Algebra I. In this lesson, you will extend your knowledge of factoring to include second-degree trinomials that have a leading coefficient not equal to one.

Module 3: Adding and Subtracting Polynomials

Topic 3 Content: Factoring Trinomials With a Leading Coefficient Not Equal to One

Anticipatory Set



Take for example the given trinomial.

$$x^2 + 8x + 15$$

You know that it can be represented as the product of two binomials. The first term of the trinomial is x^2, so the first term of each of its factors must be x because $x \cdot x = x^2$.

What if you were asked to factor this trinomial?

$$6x^2 - x - 2$$

The three terms share no common factor other than 1. So, factoring out a greatest common factor wouldn't change much at all.

The first term of this trinomial is $6x^2$.

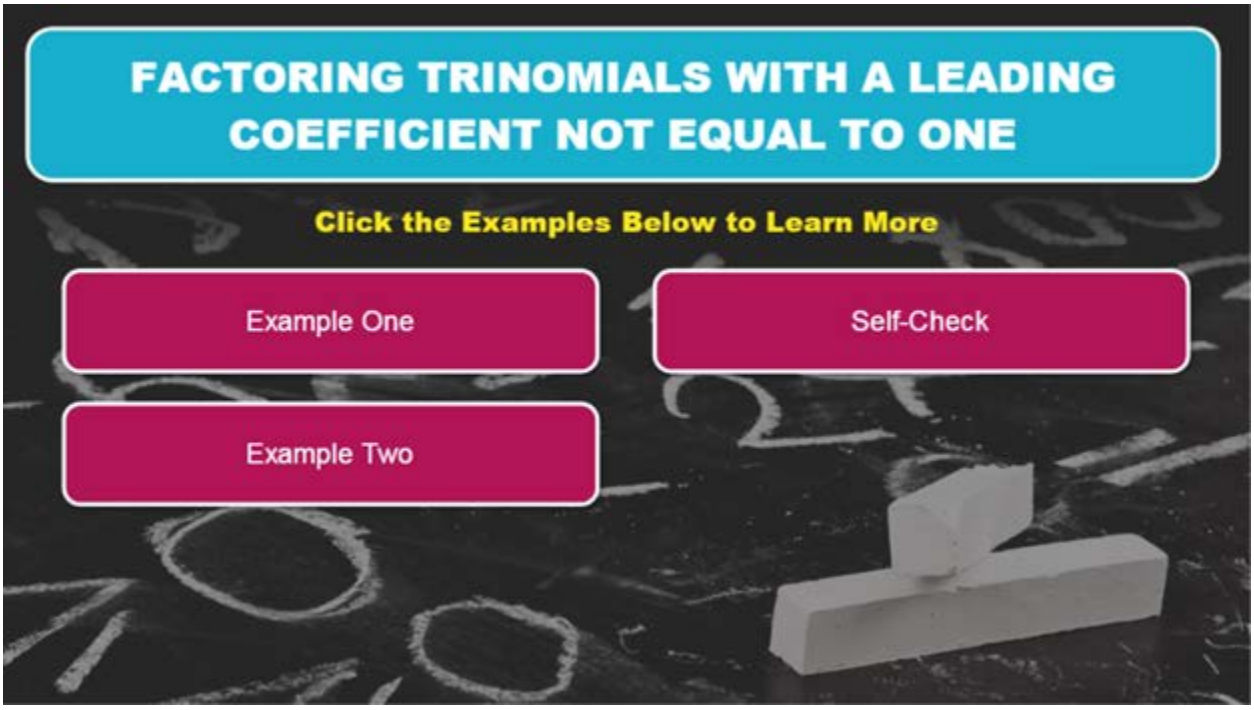
How could you determine what the first term of each of its binomial factors should be? Would they be x and $6x$ because $x \cdot 6x = 6x^2$? Or would they be $2x$ and $3x$ because $2x \cdot 3x = 6x^2$?

To factor a trinomial in the form $ax^2 + bx + c$, where $a \neq 1$, you could guess and check different pairs of factors. Or, you could factor by grouping.

Module 3: Adding and Subtracting Polynomials

Topic 3 Content: Factoring Trinomials With a Leading Coefficient Not Equal to One

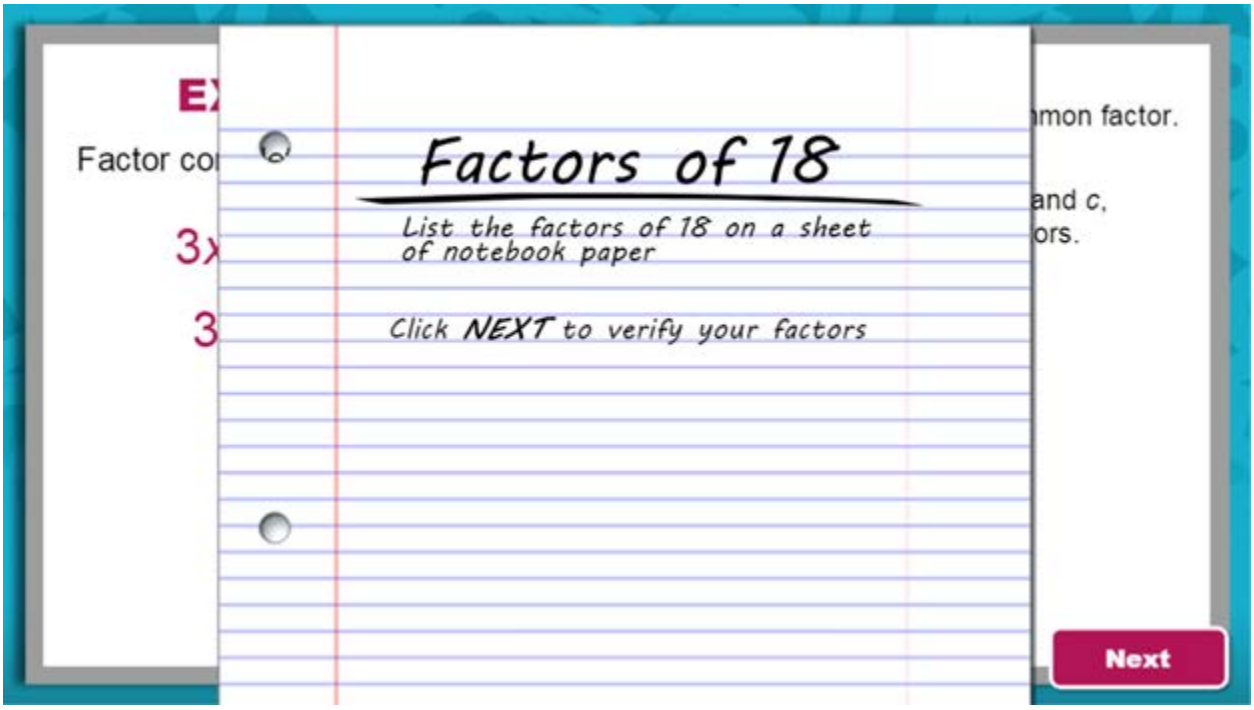
Factoring Trinomials With a Leading Coefficient Not Equal to One



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Example 1



Factor completely:

$$3x^2 + 11x + 6.$$

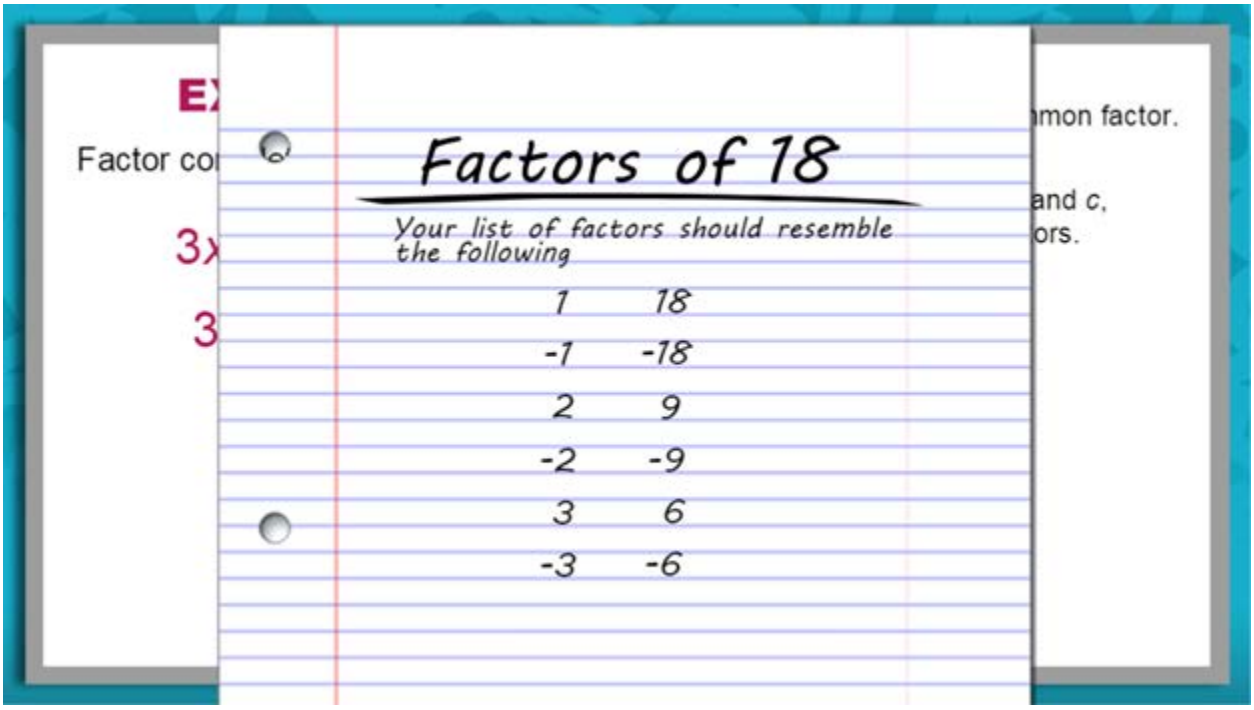
To factor by grouping, begin by factoring out the greatest common factor.

The terms of this trinomial share no common factor other than 1, so you can move on to the next step of the process, which is to find the product of a and c.

$$3 \cdot 6 = 18$$

Now list the factors of 18.

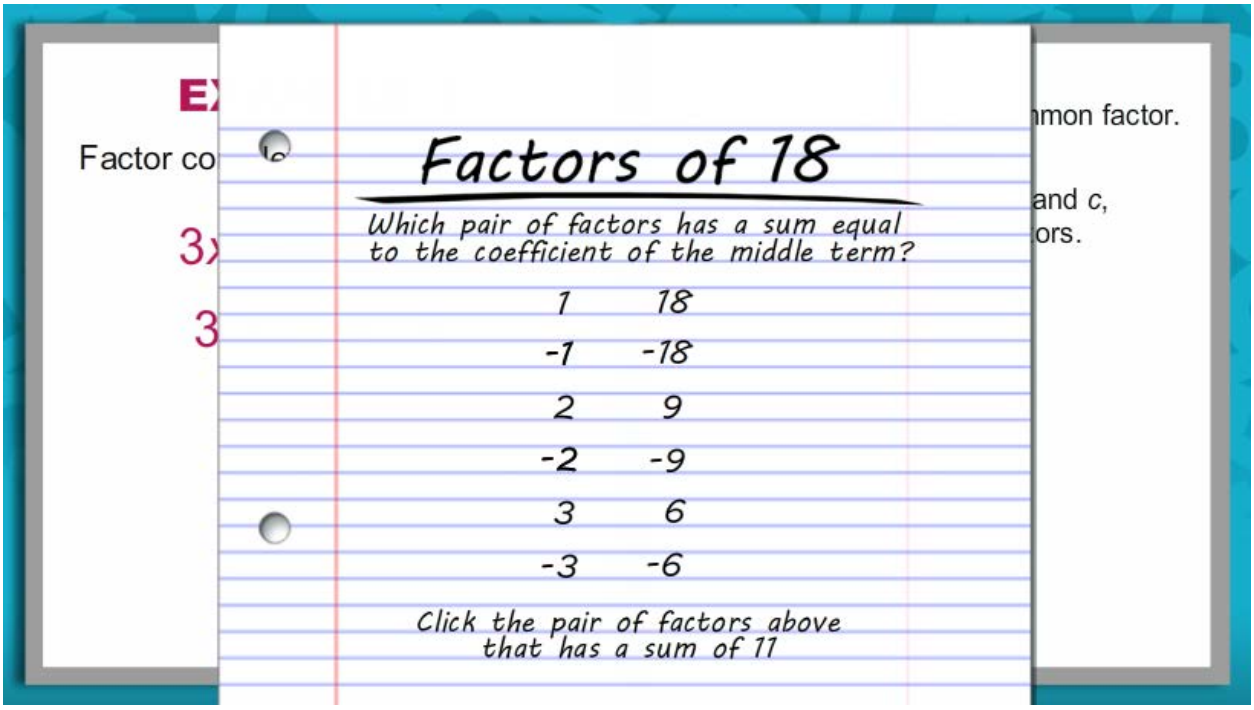
Example 1 (continued)



Your list of factors should resemble the following:

Factors of 18	
1	18
−1	−18
2	9
−2	−9
3	6
−3	−6

Example 1 (continued)

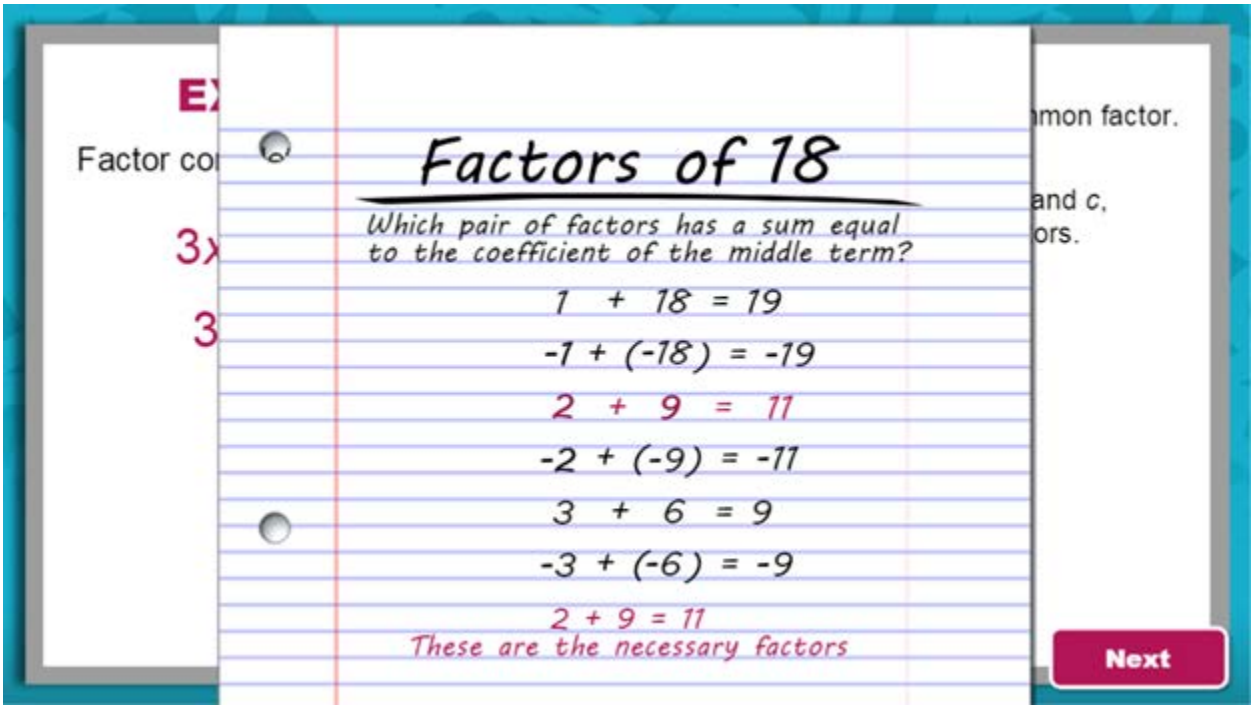


Which pair of factors has a sum equal to the coefficient of the middle term?

Factors of 18	
1	18
−1	−18
2	9
−2	−9
3	6
−3	−6

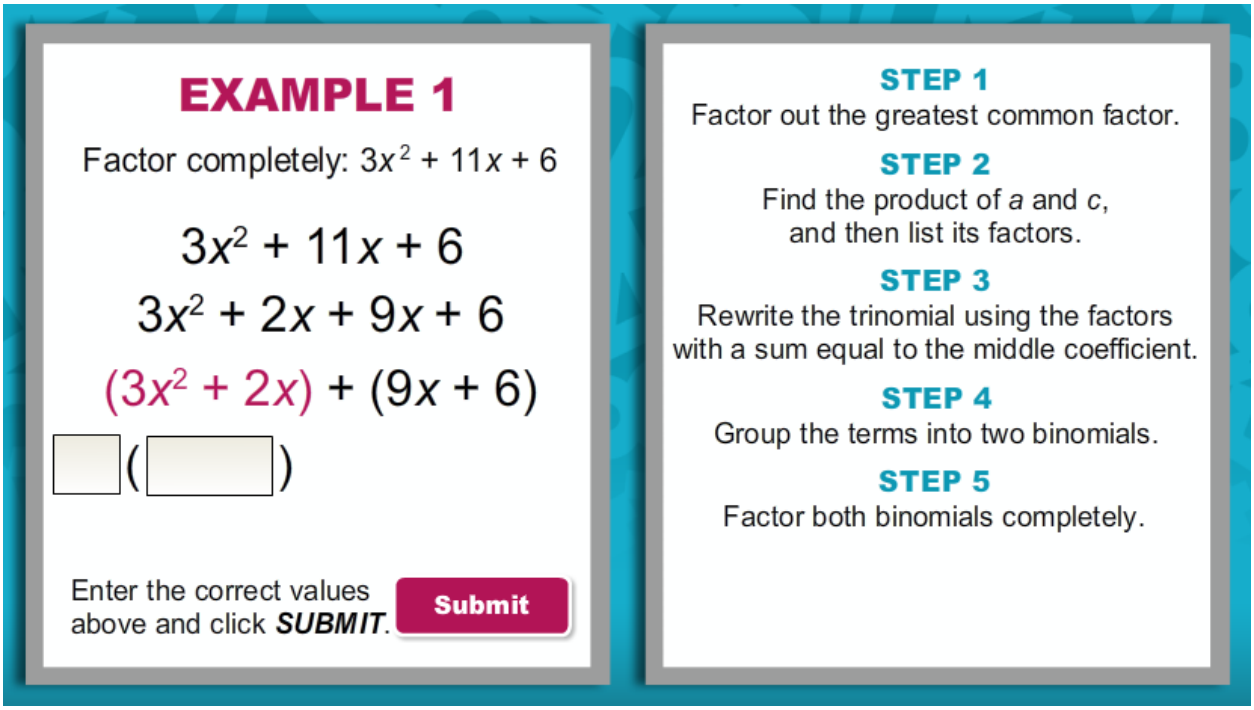
Click the pair of factors above that has a sum of 11.

Example 1 (continued)



The numbers 2 and 9 are the factors of 18 that have a sum of 11.

Example 1 (continued)



Now rewrite the trinomial. This time, use the sum of the chosen factors to represent the coefficient of the middle term.

$3x^2 + 11x + 6$
$3x^2 + 2x + 9x + 6$

Although the polynomial appears to have changed, you haven't changed its value at all because $2x + 9x = 11x$.

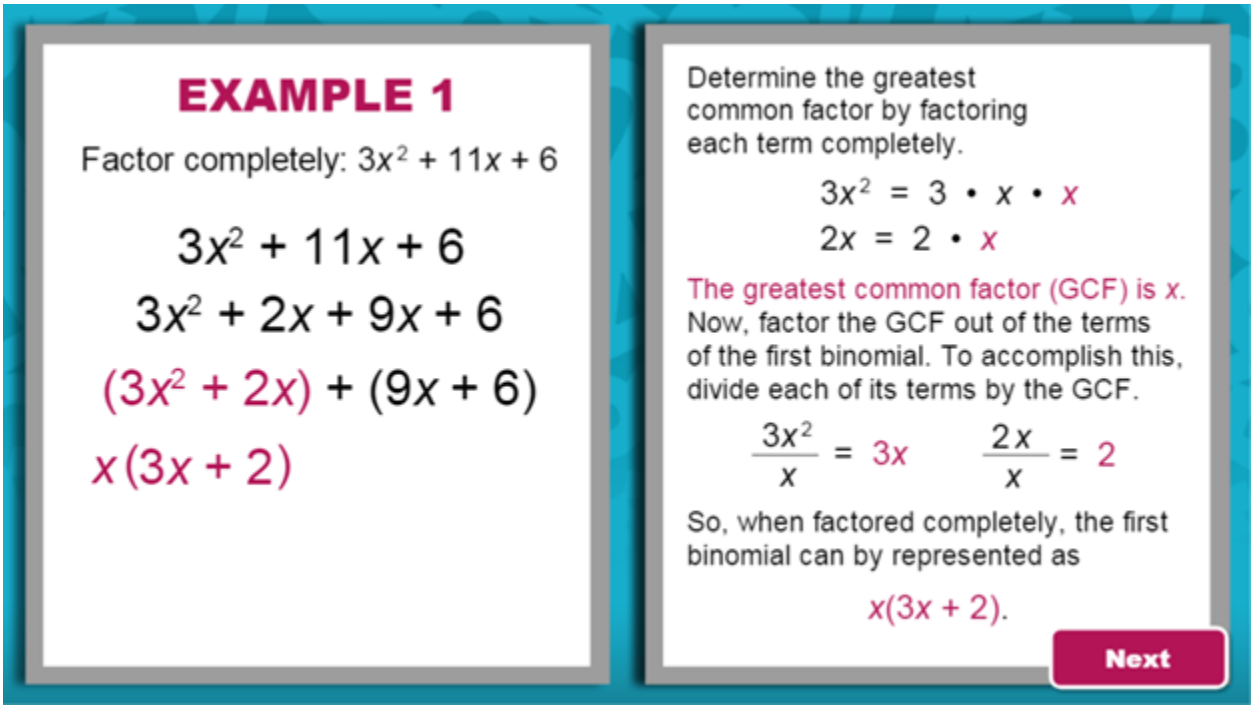
Next, group the terms into two binomials.

$(3x^2 + 2x) + (9x + 6)$

Now factor the first binomial completely.

$3x^2 + 2x$

Example 1 (continued)



Determine the greatest common factor by factoring each term completely.

$$3x^2 = 3 \cdot x \cdot x$$
$$2x = 2 \cdot x$$

The greatest common factor is x.

Now that you have identified the greatest common factor, factor it out of the terms of the first binomial. To accomplish this, divide each of its terms by the greatest common factor.

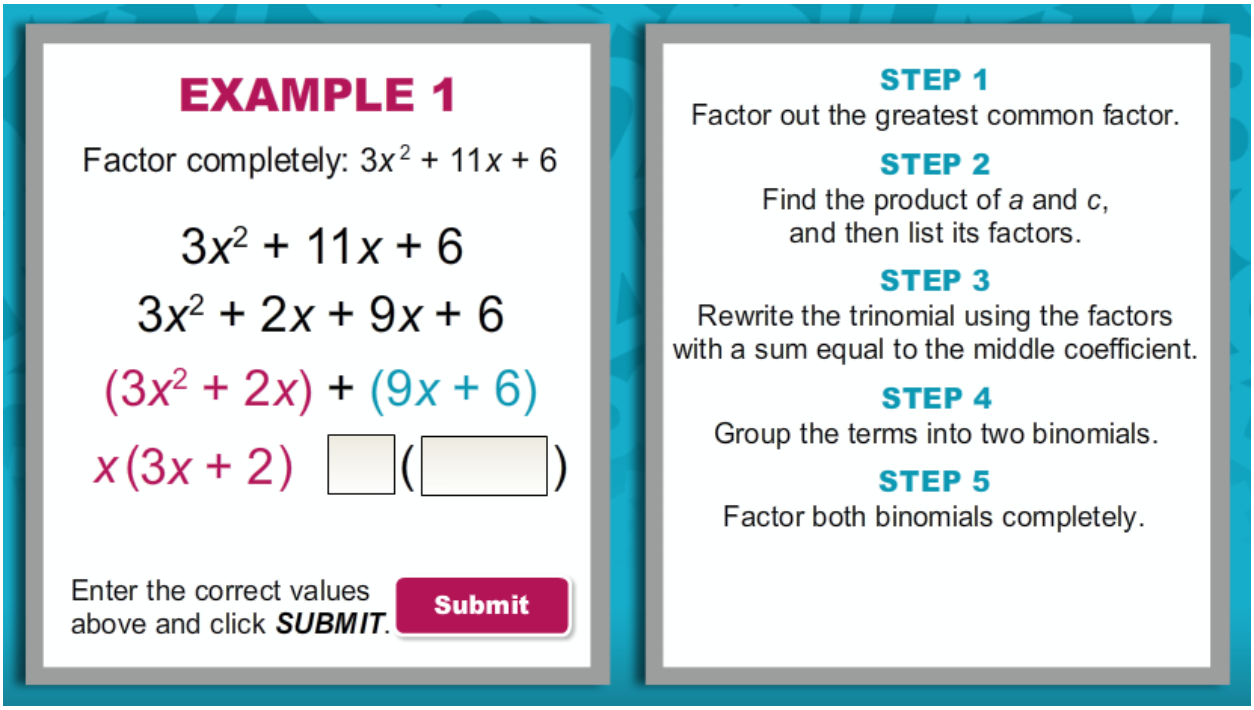
$$\frac{3x^2}{x} = 3x$$

$$\frac{2x}{x} = 2$$

So when factored completely, the first binomial can be represented as

$$x(3x + 2).$$

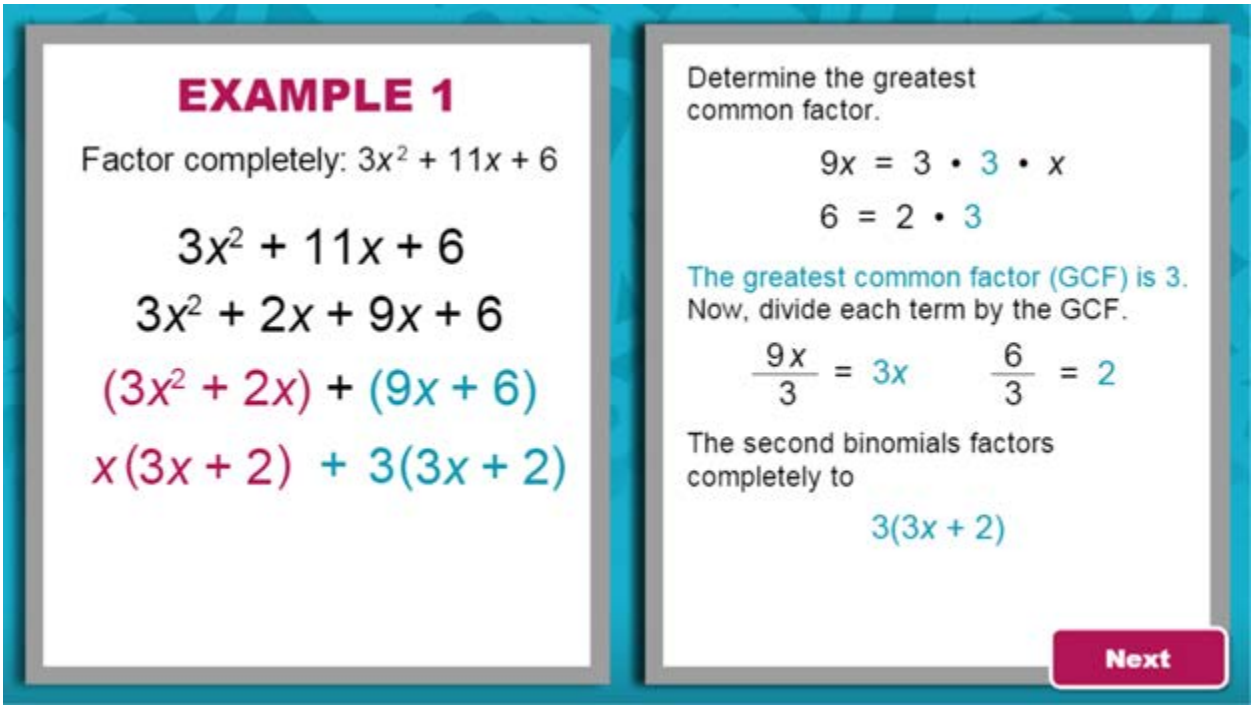
Example 1 (continued)



Now factor the second binomial completely.

$9x + 6$

Example 1 (continued)



Determine the greatest common factor.

$$9x = 3 \cdot 3 \cdot x$$
$$6 = 2 \cdot 3$$

The greatest common factor is 3.

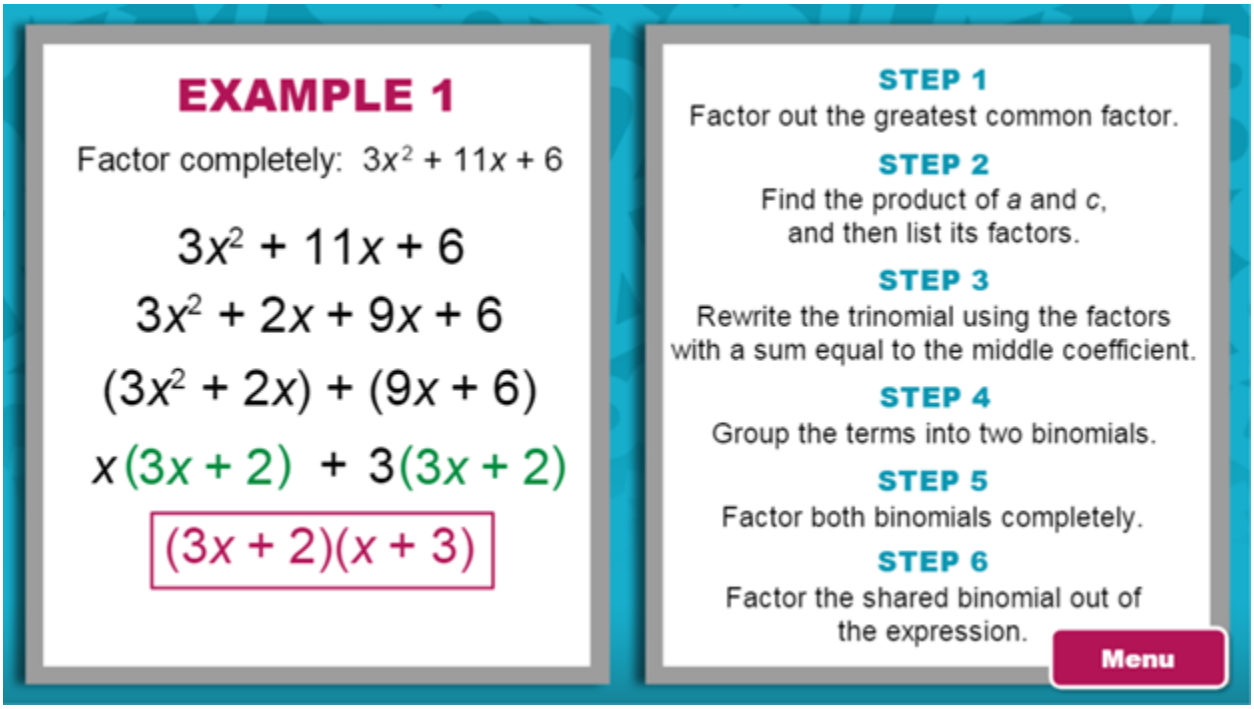
Now divide each term by the greatest common factor.

$$\frac{9x}{3} = 3x$$

$$\frac{6}{3} = 2$$

The second binomial factors completely to $3(3x + 2)$.

Example 1 (continued)



Notice that the groups each have a binomial factor of $(3x + 2)$. Factor it out of the expression.

$$x(3x + 2) + 3(3x + 2)$$
$$(3x + 2)(x + 3)$$

Finally, you can represent the expression as the product of the binomials

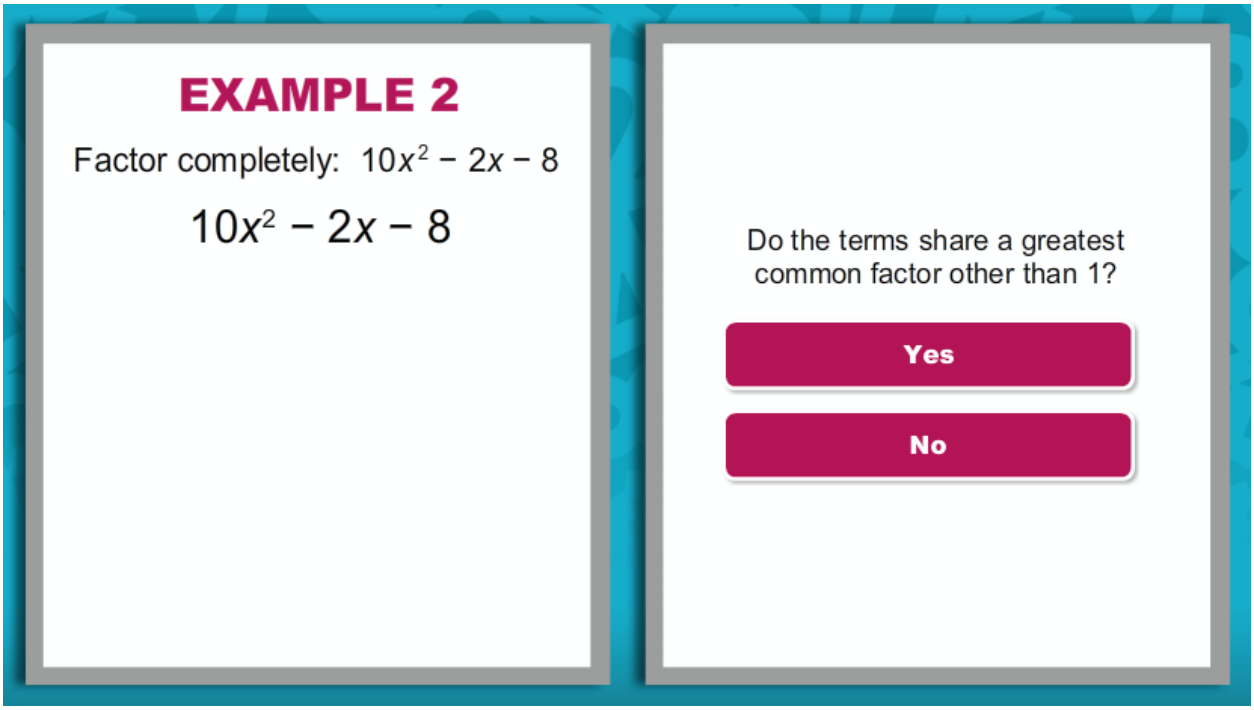
$$(3x + 2)$$

and

$$(x + 3).$$

You have factored the given trinomial completely.

Example 2

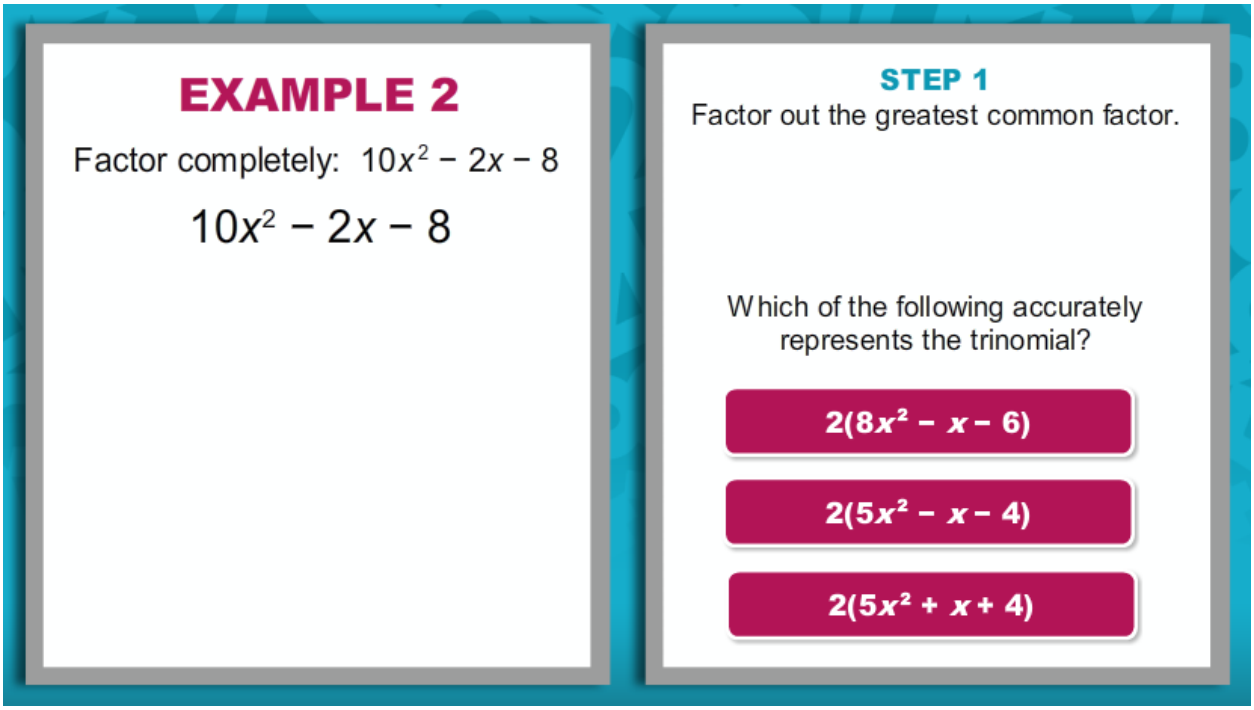


Factor completely:

$$10x^2 - 2x - 8.$$

Begin by considering the terms of the trinomial. Do the terms share a greatest common factor other than 1?

Example 2 (continued)



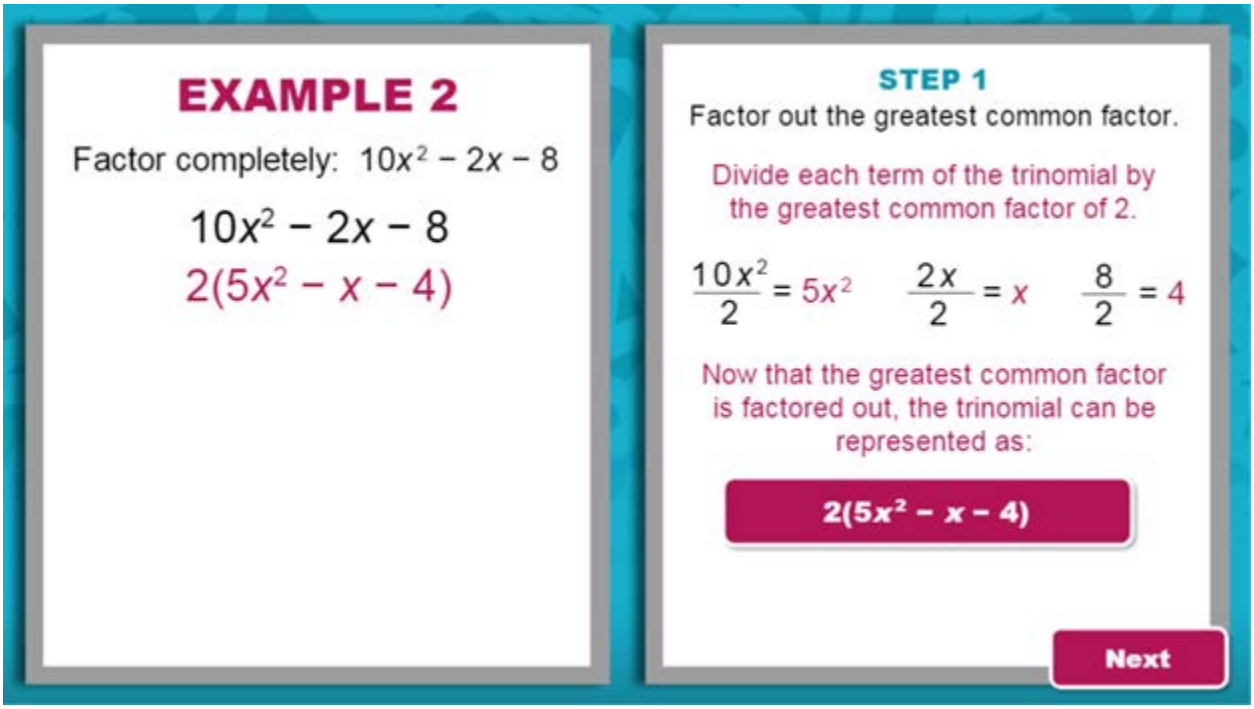
The answer is yes. The terms of the trinomial have a greatest common factor of 2.

Factor out the greatest common factor.

Which of the following accurately represents the trinomial?

A) $2(8x^2 - x - 6)$
B) $2(5x^2 - x - 4)$
C) $2(5x^2 + x + 4)$

Example 2 (continued)



To determine the answer, divide each term of the trinomial by the greatest common factor of 2.

$$10x^2 - 2x - 8$$

$$\frac{10x^2}{2} = 5x^2$$

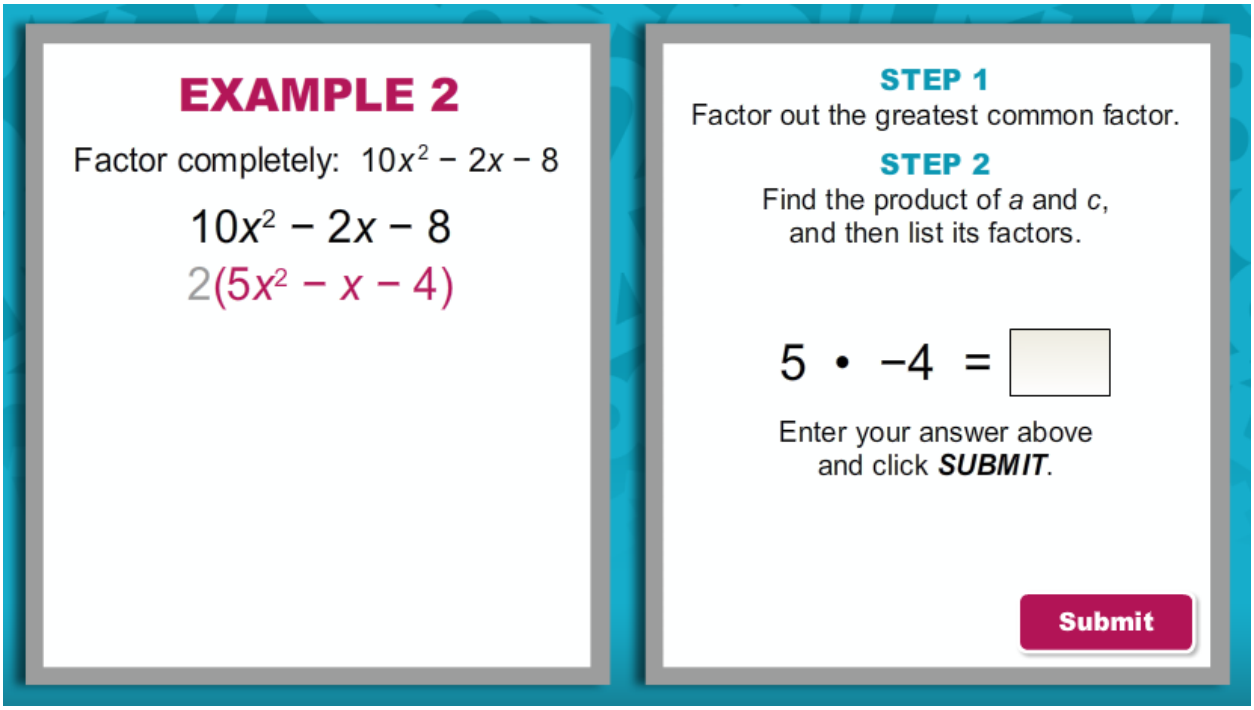
$$\frac{-2x}{2} = -x$$

$$\frac{-8}{2} = -4$$

Now that the greatest common factor is factored out, the trinomial can be represented as

$$2(5x^2 - x - 4).$$

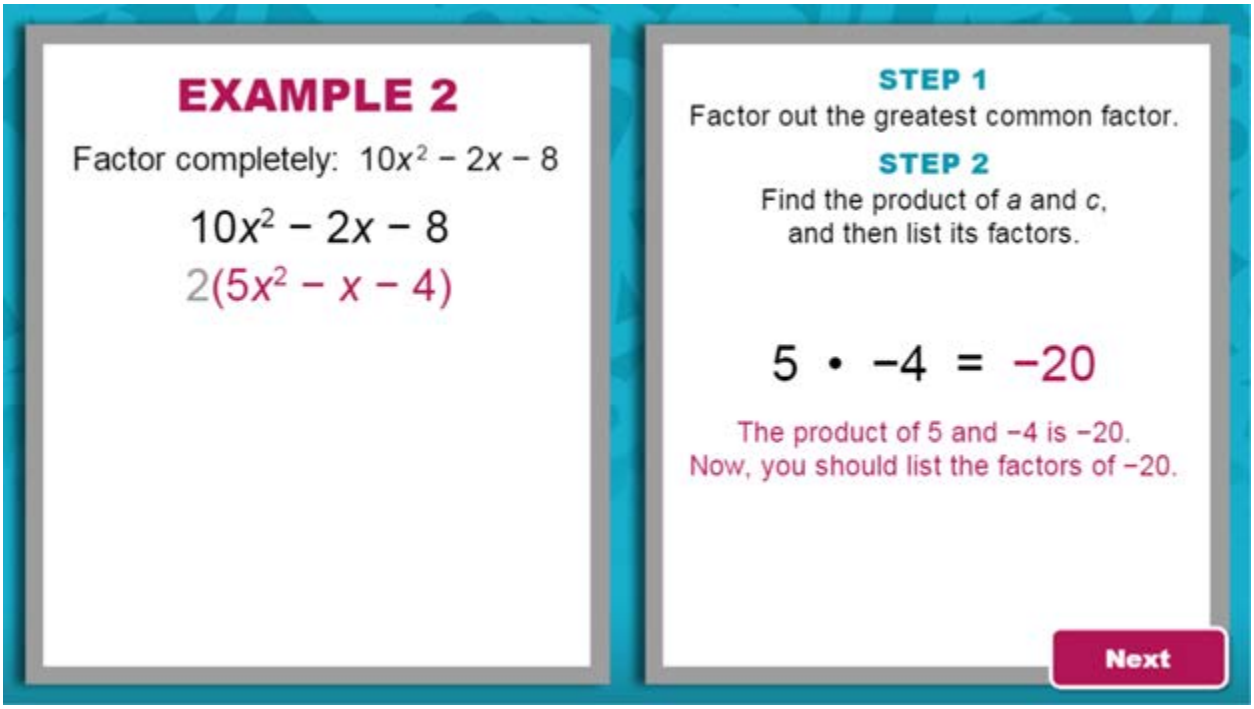
Example 2 (continued)



For a moment, set aside the greatest common factor of 2 and focus your attention on the trinomial $5x^2 - x - 4$. You can factor this trinomial by grouping. Begin by finding the product of a and c.

What is the product of 5 and -4?

Example 2 (continued)

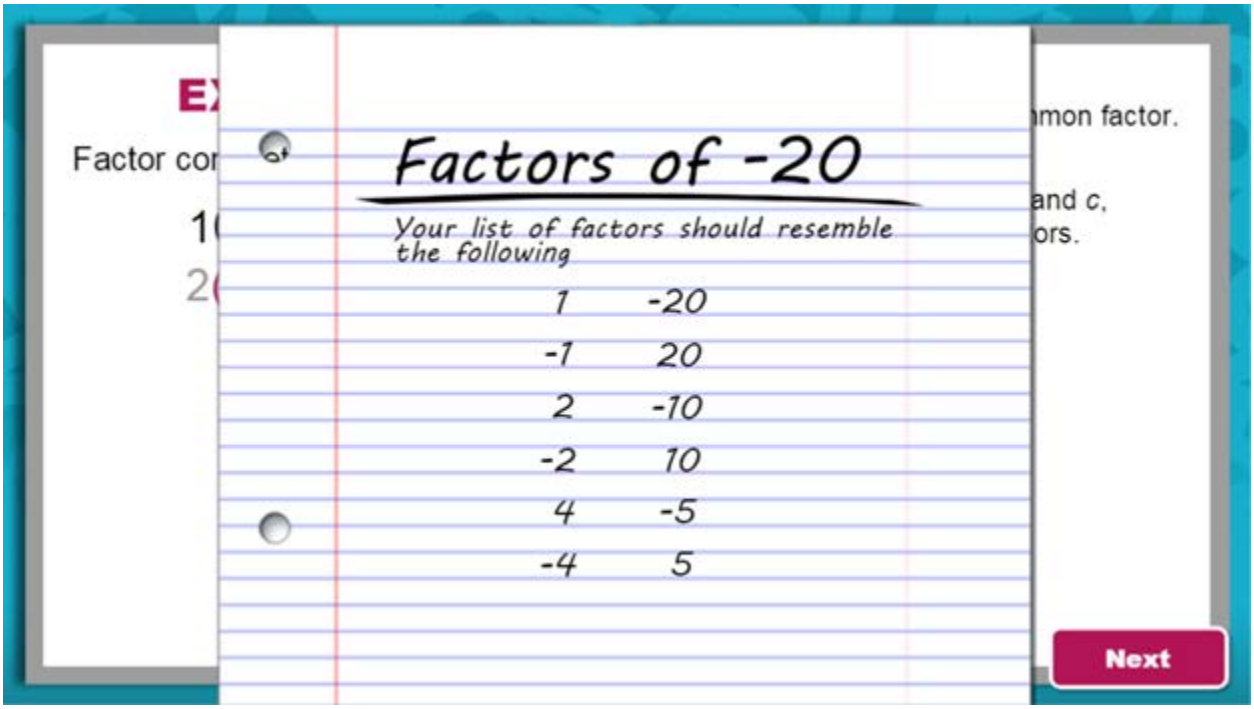


The correct answer is

$5 \cdot -4 = -20.$

Now list the factors of -20 on a sheet of notebook paper.

Example 2 (continued)

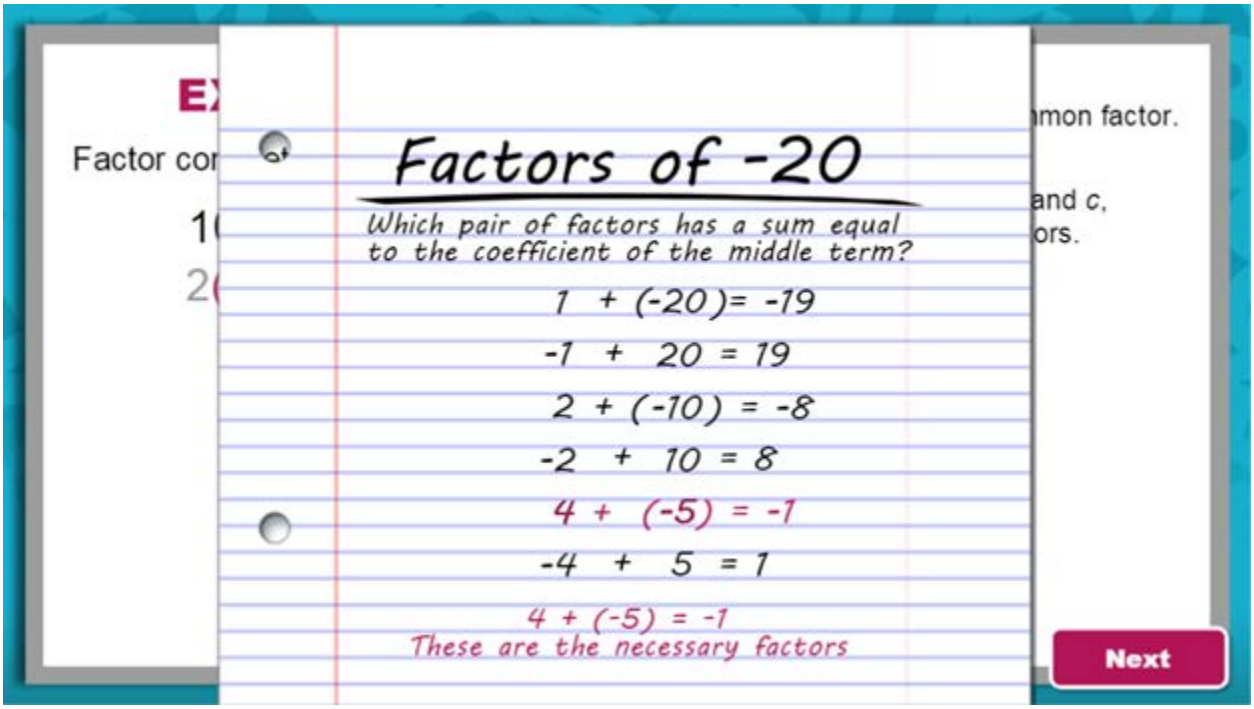


Your list of factors should resemble the following:

Factors of -20	
1	-20
-1	20
2	-10
-2	10
4	-5
-4	5

Identify the factors of -20 that have a sum of -1.

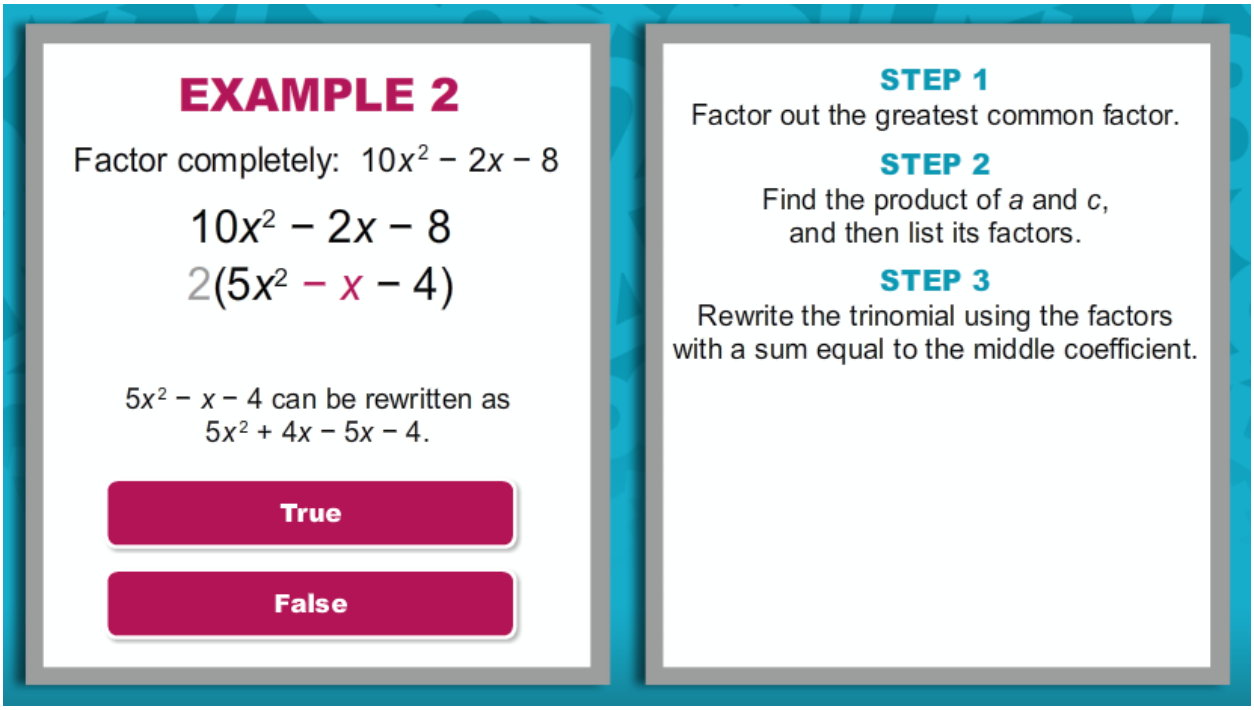
Example 2 (continued)



Which two numbers will multiply to equal -20 and add to equal -1?

The numbers 4 and -5 are the factors that have a sum of -1.

Example 2 (continued)

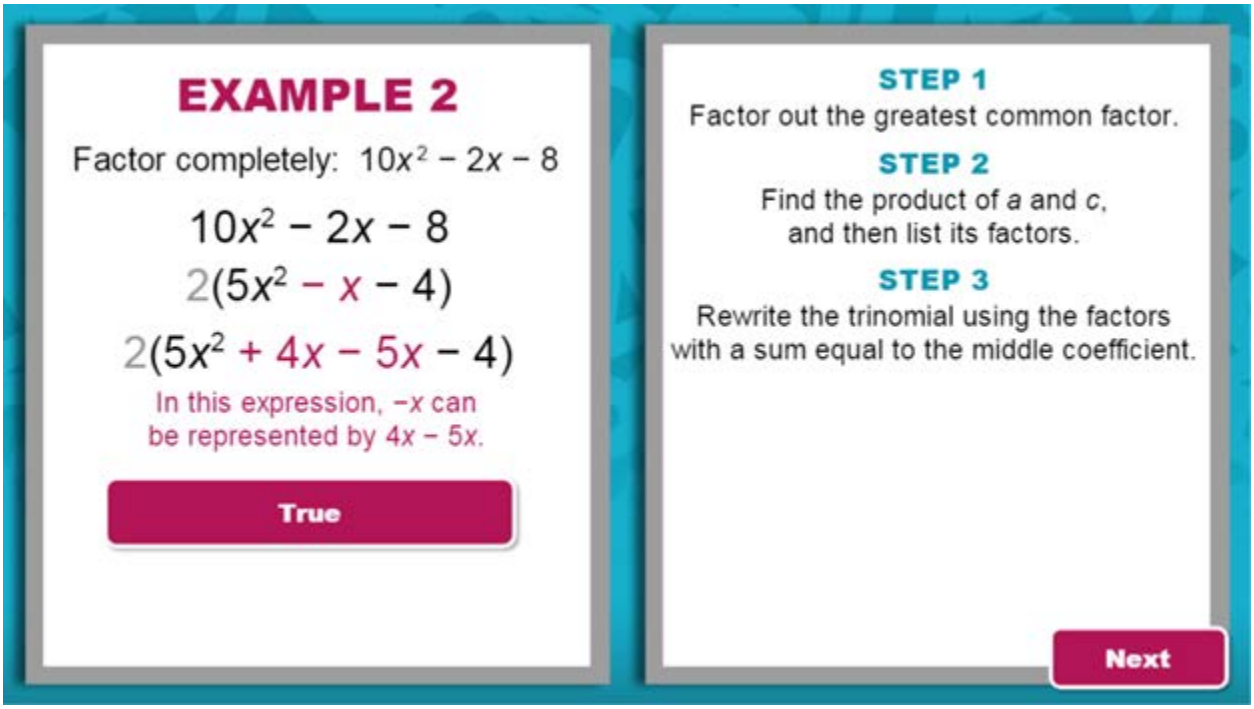


Now rewrite the trinomial using the sum of the chosen factors to represent the coefficient of the middle term.

The polynomial $5x^2 - x - 4$ can be rewritten as $5x^2 + 4x - 5x - 4$.

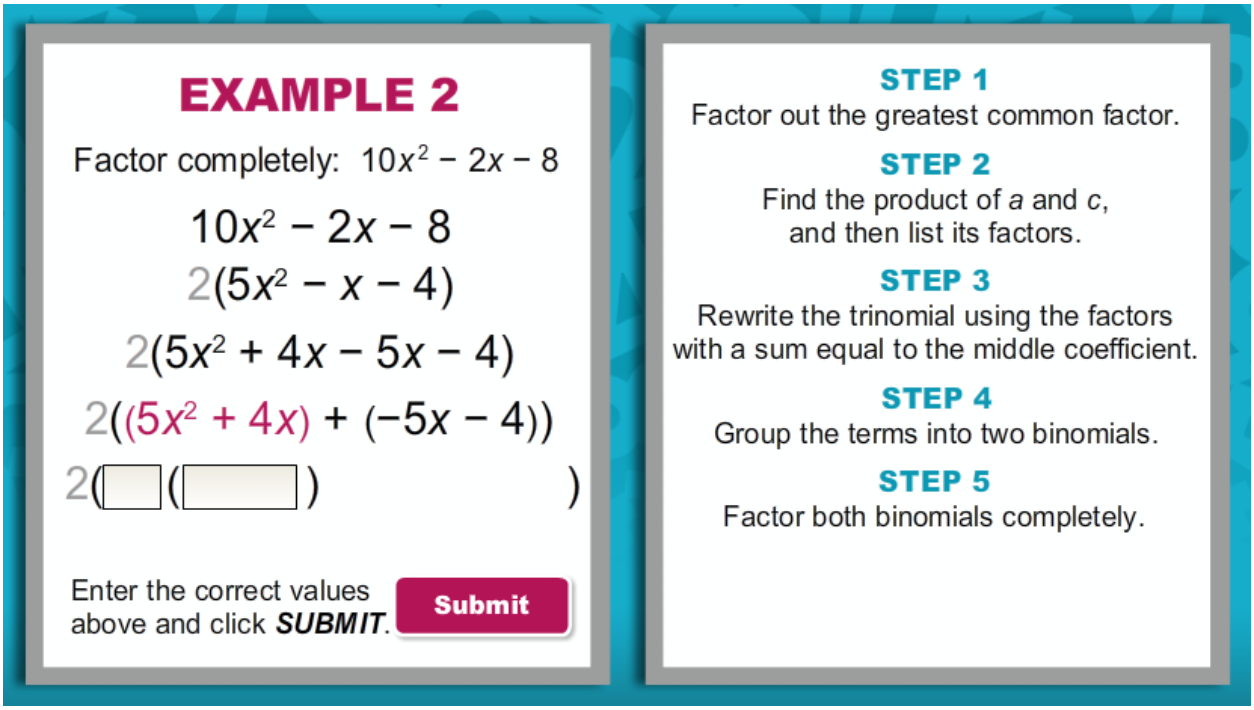
A) True
B) False

Example 2 (continued)



In this expression, $-x$ is represented by $4x - 5x$.

Example 2 (continued)



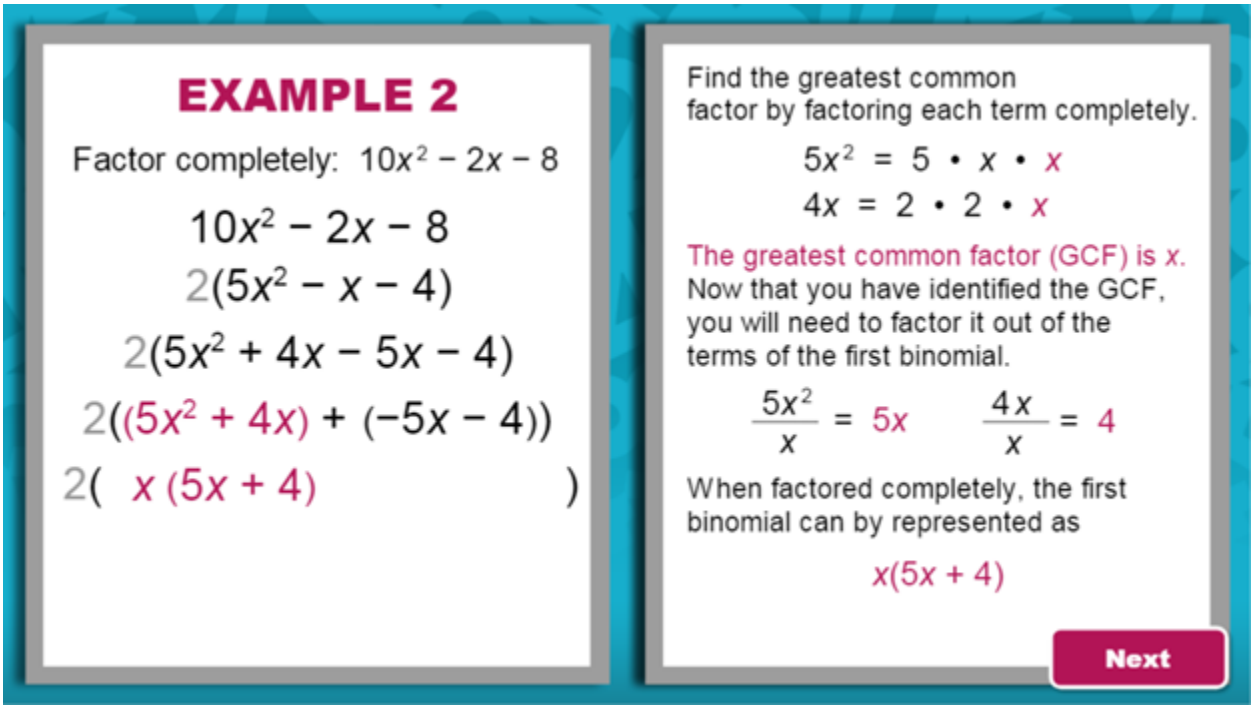
Next, group the terms into two binomials.

$$(5x^2 + 4x) + (-5x - 4)$$

Now factor the first binomial completely.

$$(5x^2 + 4x)$$

Example 2 (continued)



First, find the greatest common factor of the terms of the first binomial.

$$5x^2 = 5 \cdot x \cdot x$$
$$4x = 2 \cdot 2 \cdot x$$

The greatest common factor is x.

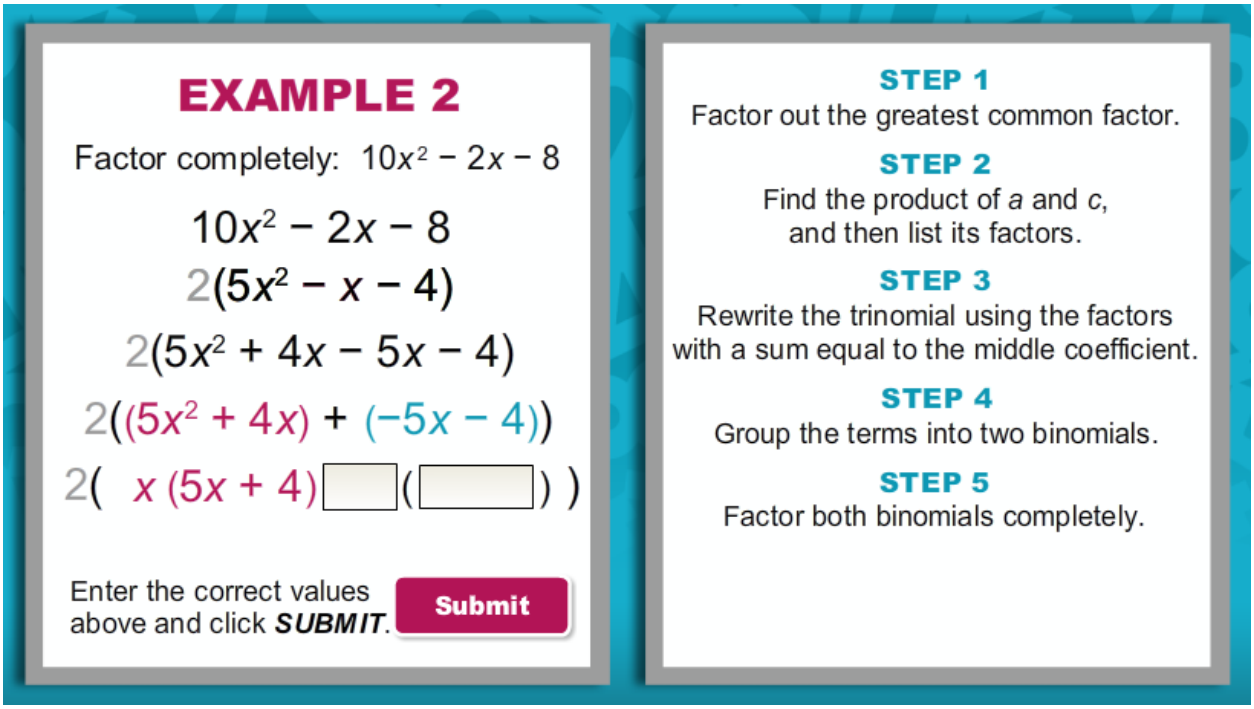
Now that you have identified the greatest common factor, you will need to factor it out of the terms of the first binomial.

$$\frac{5x^2}{x} = 5x$$

$$\frac{4x}{x} = 4$$

So when factored completely, the first binomial can be represented as $x(5x + 4)$.

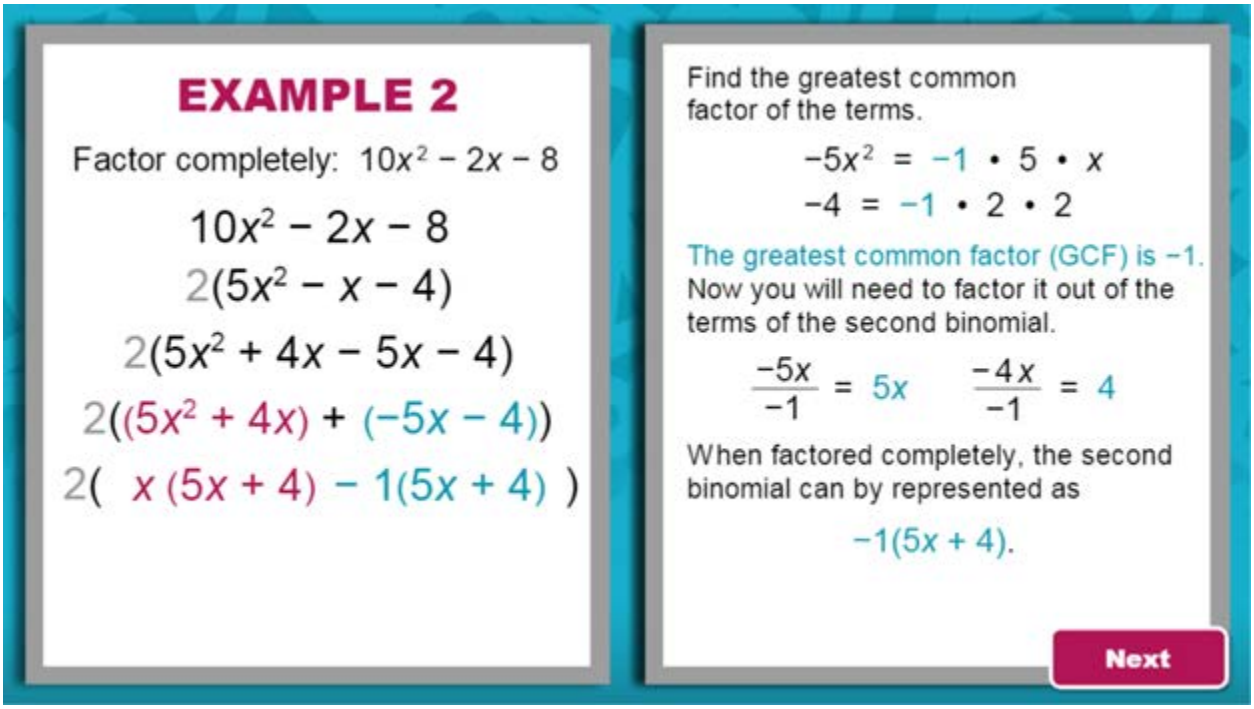
Example 2 (continued)



Now factor the second binomial completely.

$(-5x - 4)$

Example 2 (continued)



Find the greatest common factor of the terms of the first binomial.

$$-5x = -1 \cdot 5 \cdot x$$
$$-4 = -1 \cdot 2 \cdot 2$$

The greatest common factor is -1.

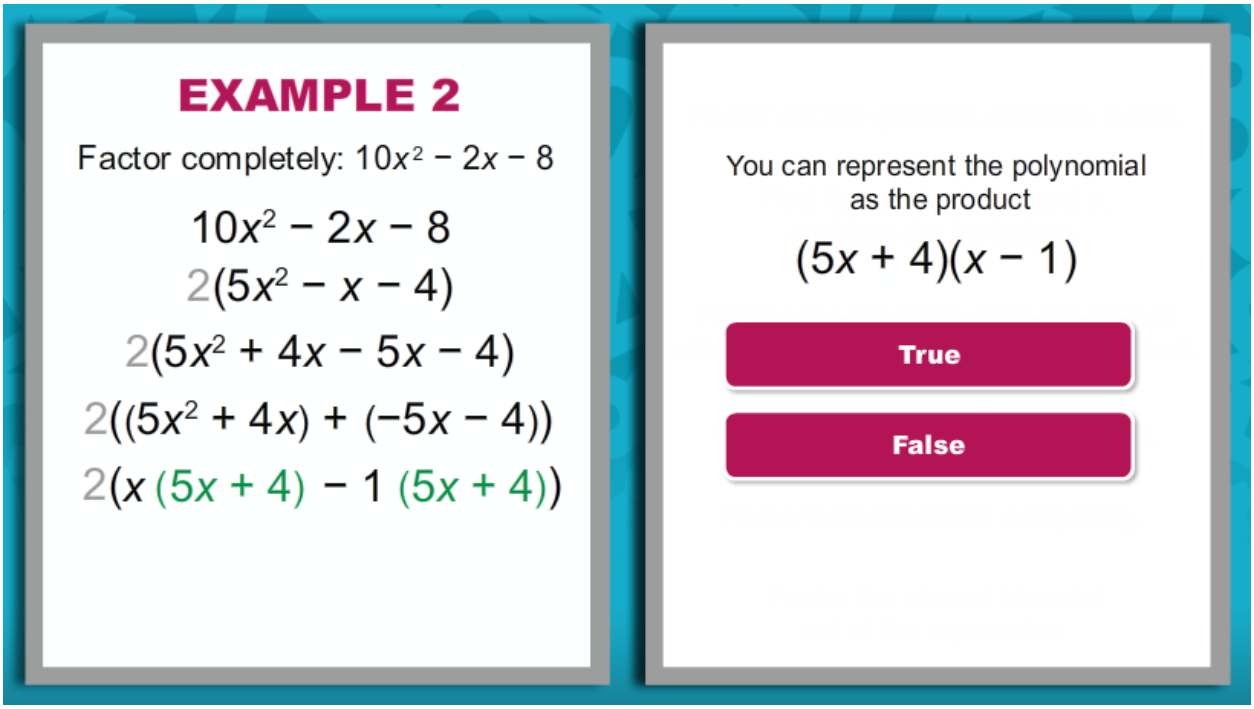
Now you will need to factor it out of the terms of the second binomial.

$$\frac{-5x}{-1} = 5x$$

$$\frac{-4}{-1} = 4$$

So when factored completely, the second binomial can be represented as $-1(5x + 4)$.

Example 2 (continued)

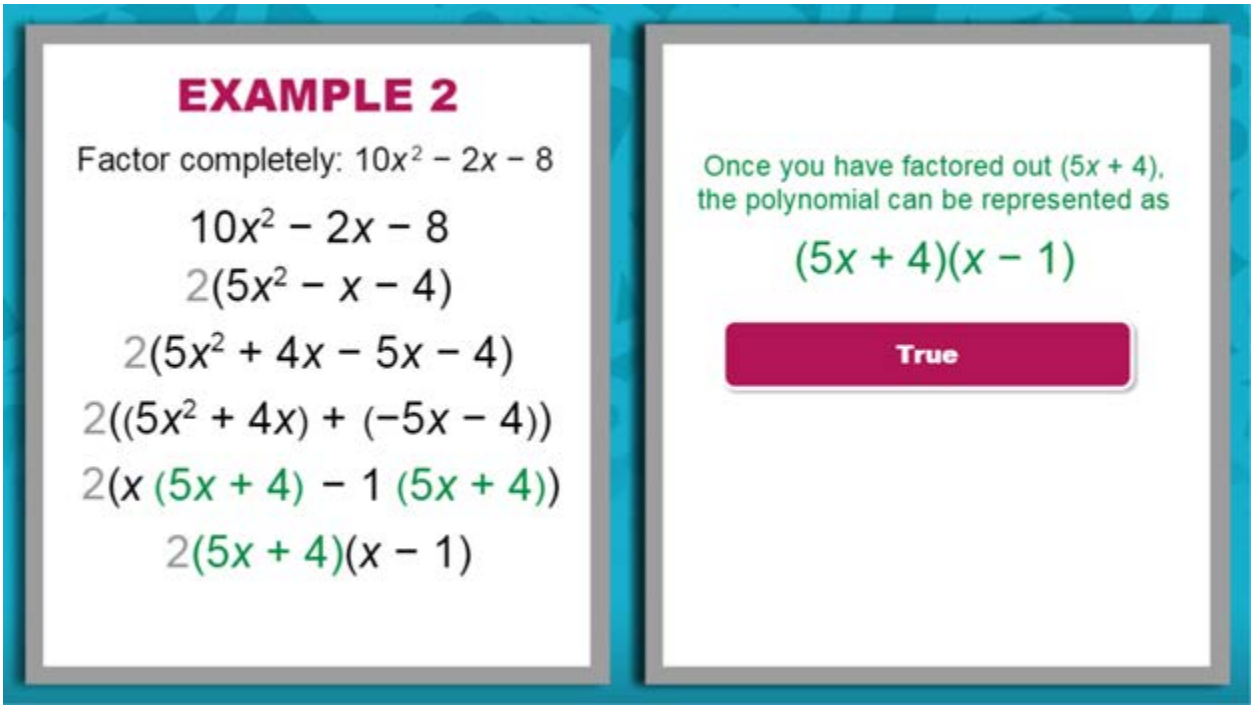


Notice that the groups of the polynomial each have a common factor, the binomial $5x + 4$.

You can represent the polynomial as the product $(5x + 4)(x - 1)$.

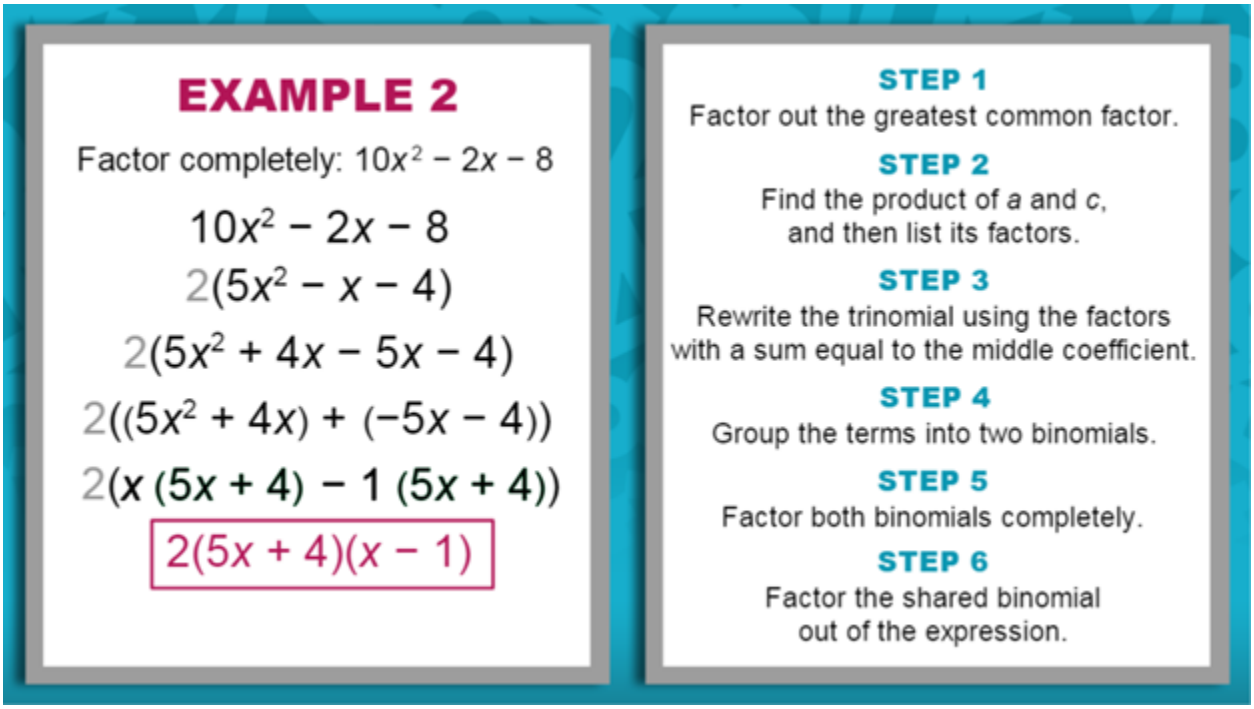
A) True
B) False

Example 2 (continued)



Once you have factored out $(5x + 4)$, the polynomial can be represented as $(5x + 4)(x - 1)$.

Example 2 (continued)

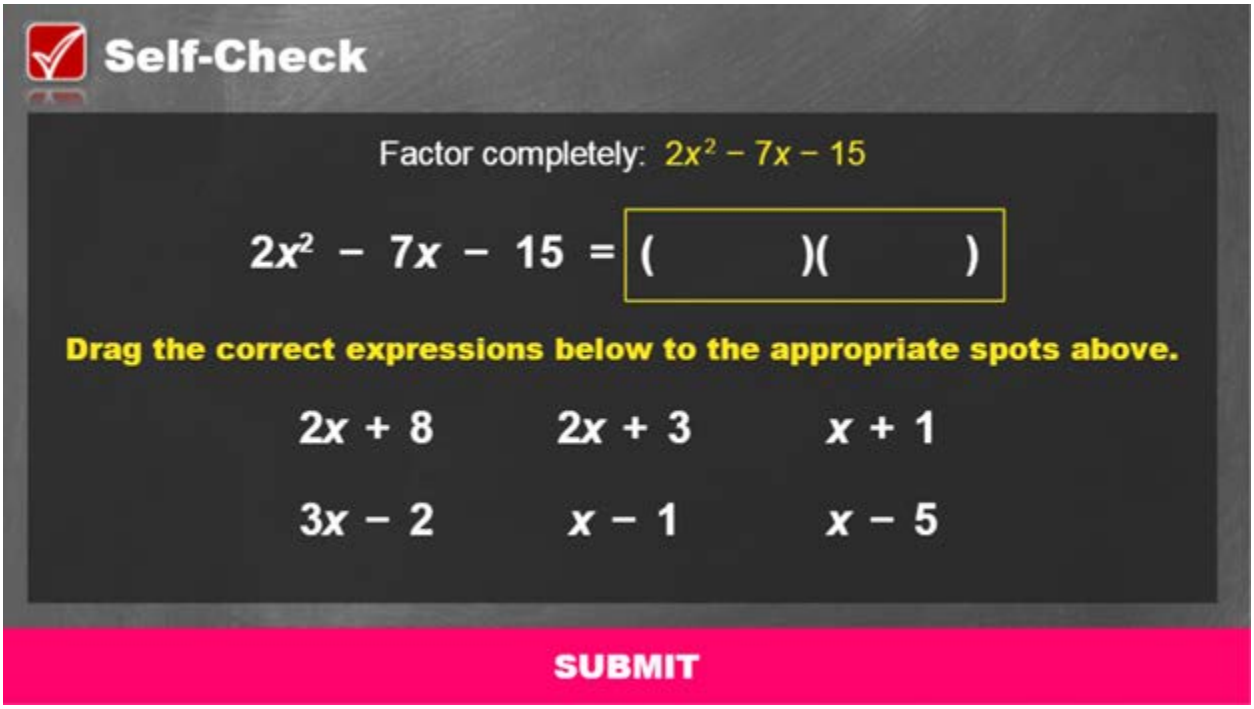


Once you have factored out $(5x + 4)$, the polynomial can be represented as $(5x + 4)(x - 1)$.

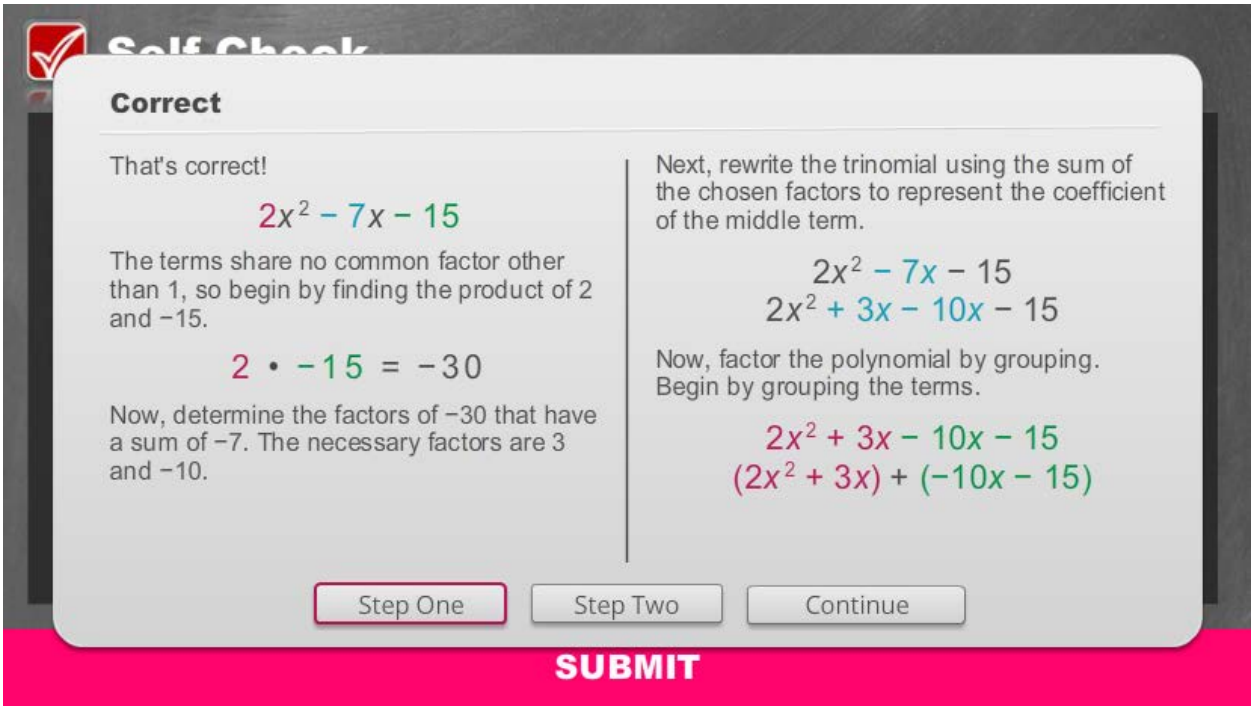
Now recall the greatest common factor of 2 that was initially factored out of the given trinomial. When factored completely, the trinomial $10x^2 - 2x - 8$ can be represented as

$$2(5x + 4)(x - 1).$$

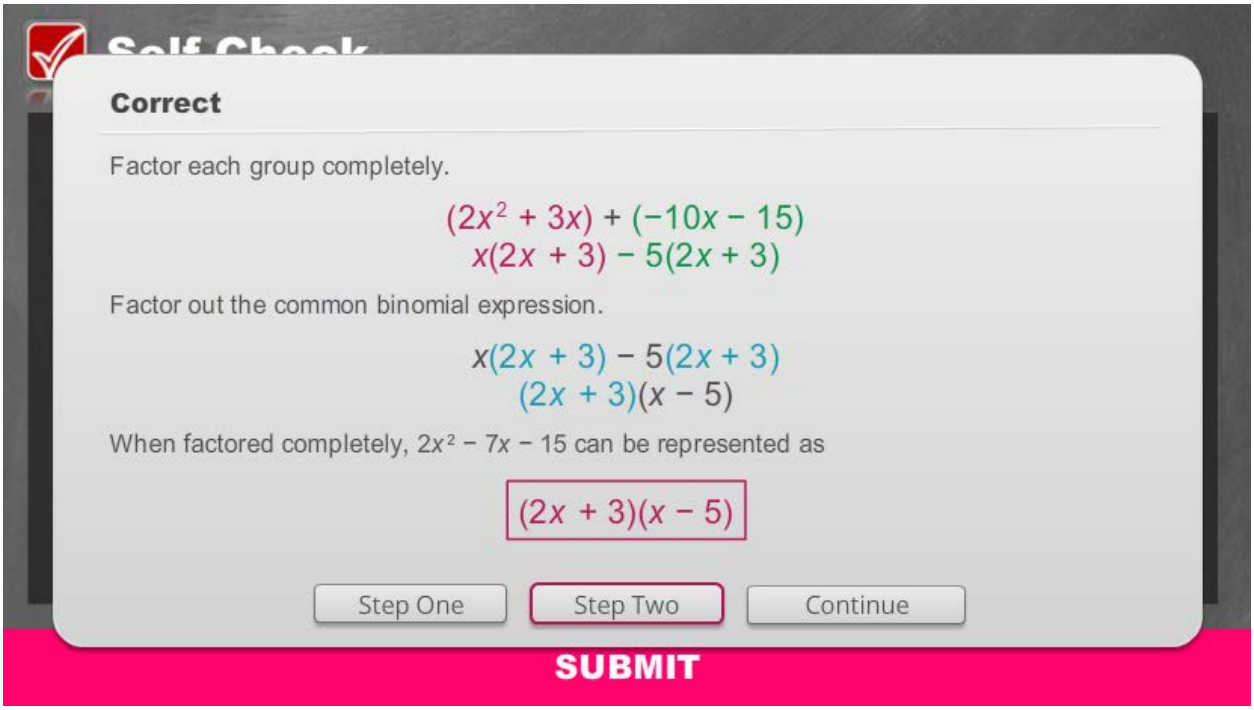
Self-Check 1


Self-Check
Factor completely: $2x^2 - 7x - 15$
$2x^2 - 7x - 15 = (\quad)(\quad)$
Drag the correct expressions below to the appropriate spots above.
$2x + 8$
$2x + 3$
$x + 1$
$3x - 2$
$x - 1$
$x - 5$
SUBMIT

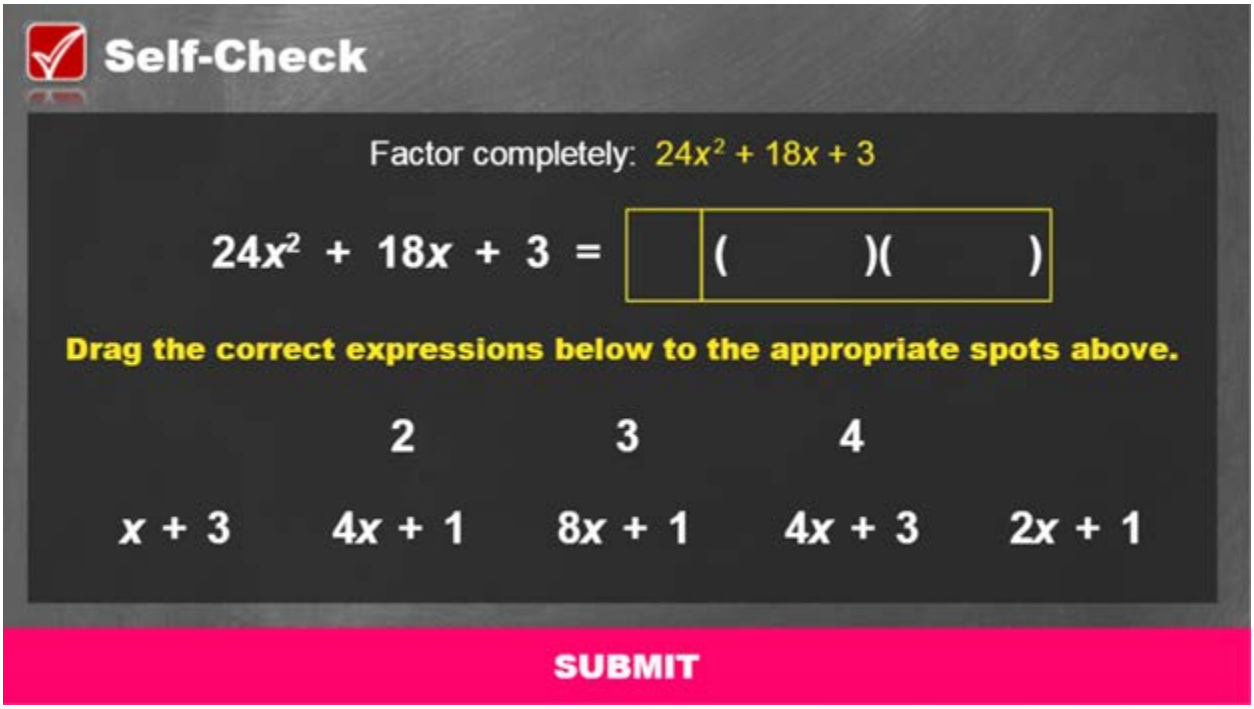
Self-Check 1: Answer Step One



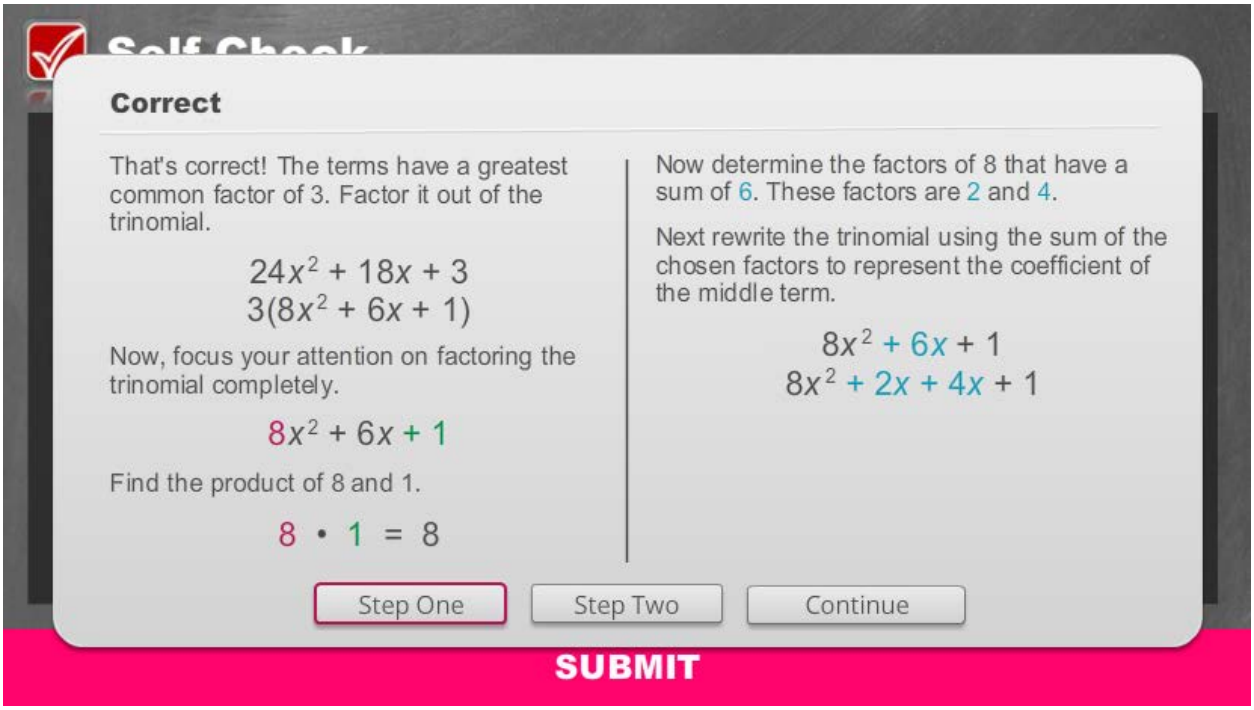
Self-Check 1: Answer Step Two



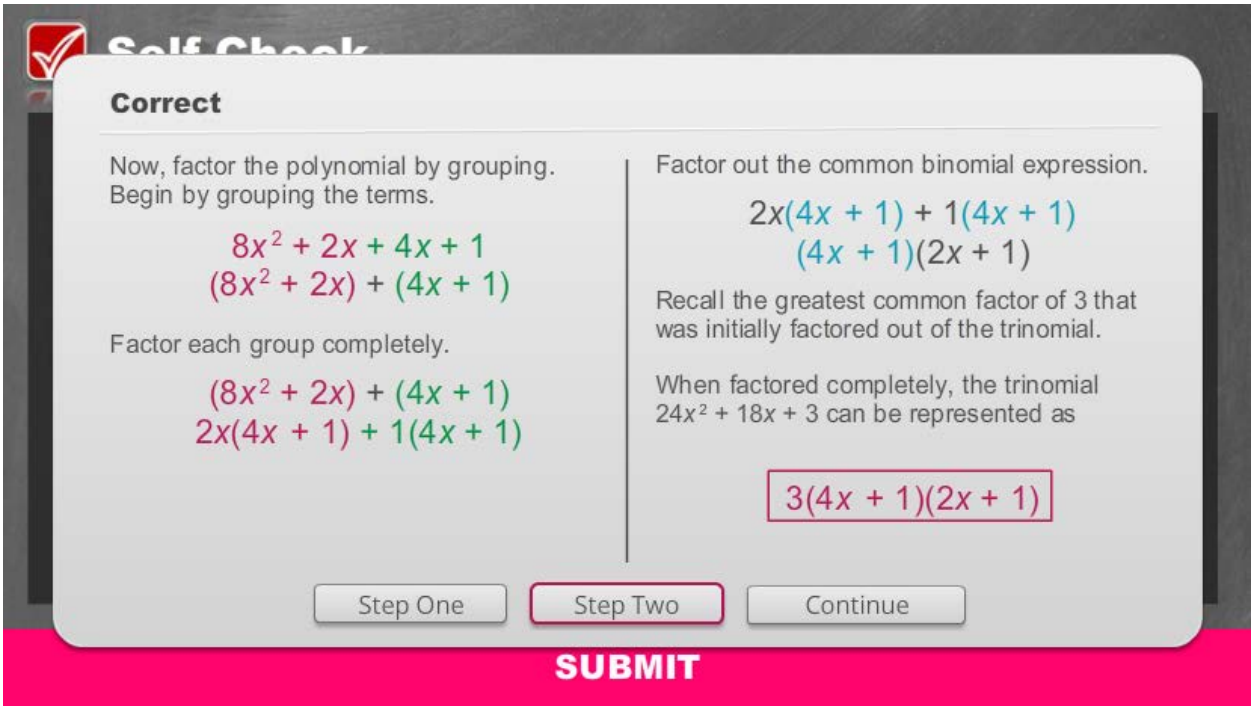
Self-Check 2


Self-Check
Factor completely: $24x^2 + 18x + 3$
$24x^2 + 18x + 3 =$ ( )( )
Drag the correct expressions below to the appropriate spots above.
2
3
4
$x + 3$
$4x + 1$
$8x + 1$
$4x + 3$
$2x + 1$
SUBMIT

Self-Check 2: Answer Step One



Self-Check 2: Answer Step Two



Conclusion

Congratulations! You have reached the conclusion of this lesson, where you learned how to factor second-degree trinomials by grouping.