Module 3: Adding and Subtracting Polynomials
Topic 3 Content: Factoring Trinomials With a Leading Coefficient of One

Introduction

Hello and welcome! In your earlier math studies, you learned how to factor integers. In this lesson, you will expand your knowledge of factoring to include second-degree trinomials that have a leading coefficient of one. Your knowledge of the distributive property will be very helpful to you during this lesson.

Anticipatory Set

Take for example, the product of the binomials $(x + 2)$ and $(x + 4)$. You have learned how to use the distributive property to evaluate a product like this one.

$$(x + 2)(x + 4) = x^2 + 4x + 2x + 8$$

After combining like terms, you find that the product is equal to $x^2 + 6x + 8$.

Now consider the terms of this trinomial.

The first term is the product of the first terms of the binomial factors.

$$x \cdot x = x^2$$

The middle term has a coefficient of 6, which is the sum of the second terms of the binomial factors.

$$2 + 4 = 6$$

The last term is the product of the second terms of the binomial factors.

$$2 \cdot 4 = 8$$

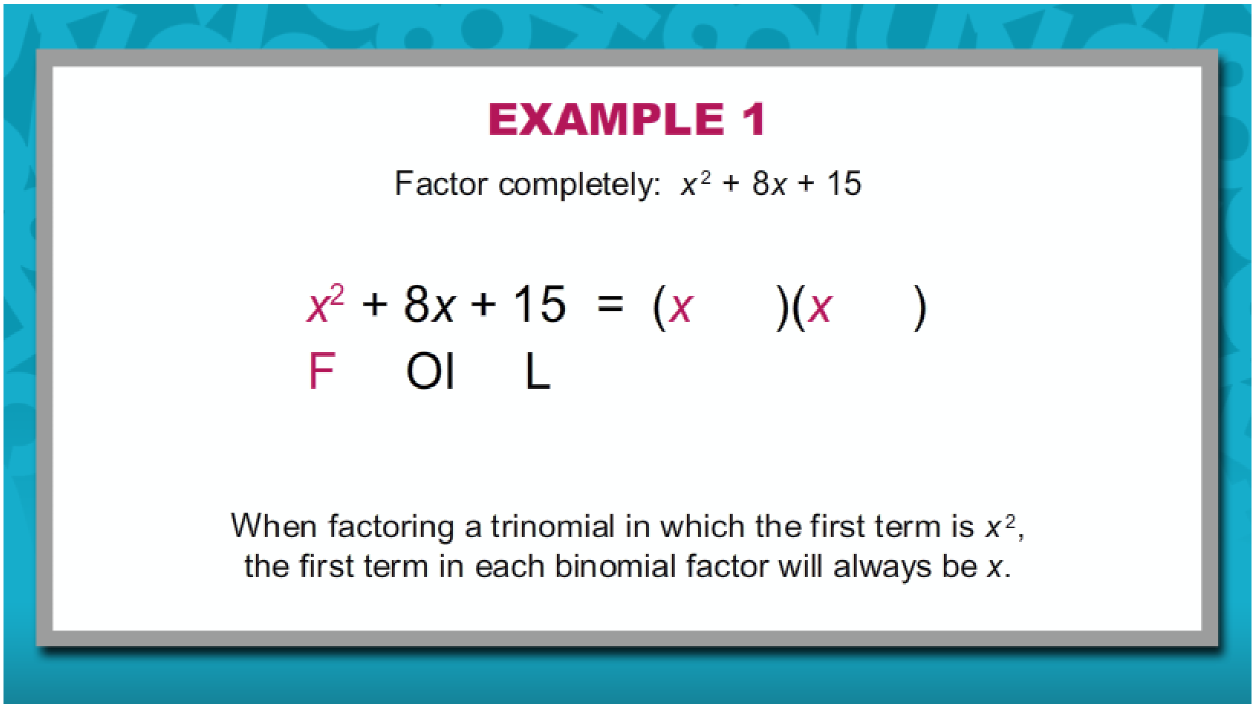
The trinomial $x^2 + 6x + 8$ is in the form $ax^2 + bx + c$, where the leading coefficient, a, is equal to 1. When factoring a trinomial in this form, the key is to determine two numbers that have a sum equal to the coefficient of the middle term and a product equal to the value of the last term.

Factoring Trinomials With a Leading Coefficient of One

Click the examples below to learn more.

- Example One
- Example Two
- Example Three
- Self-Check

Example 1



Factor completely:

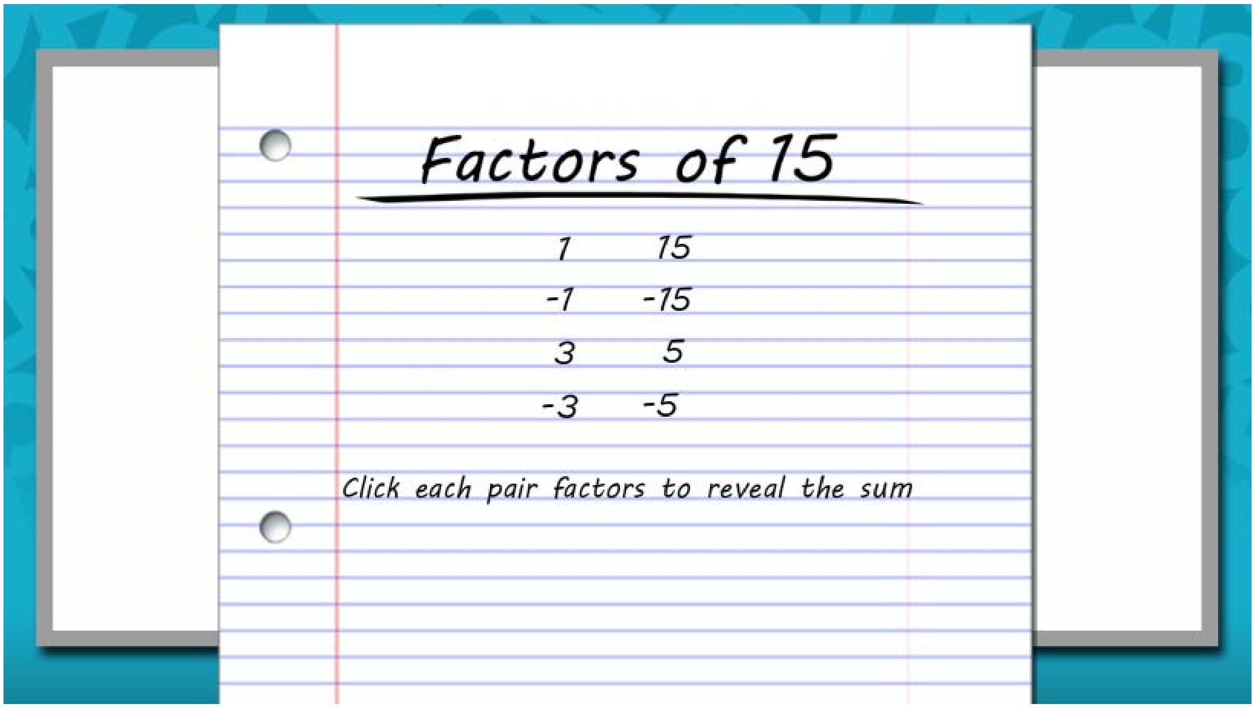
$$x^2 + 8x + 15.$$

The trinomial is in the form $ax^2 + bx + c$. So you know that it can be represented as the product of two binomials.

Recall that the first term of the trinomial is generated by multiplying the first terms of the binomial factors.

Because x^2 is the first term of the trinomial, the first term of each of its factors must be x. When factoring a trinomial in which the first term is x^2, the first term in each binomial factor will always be x.

Example 1 (continued)



Now it's time to determine the second term in each binomial factor. To accomplish this, you will need to consider the last term of the trinomial,15, and the coefficient of the middle term, 8.

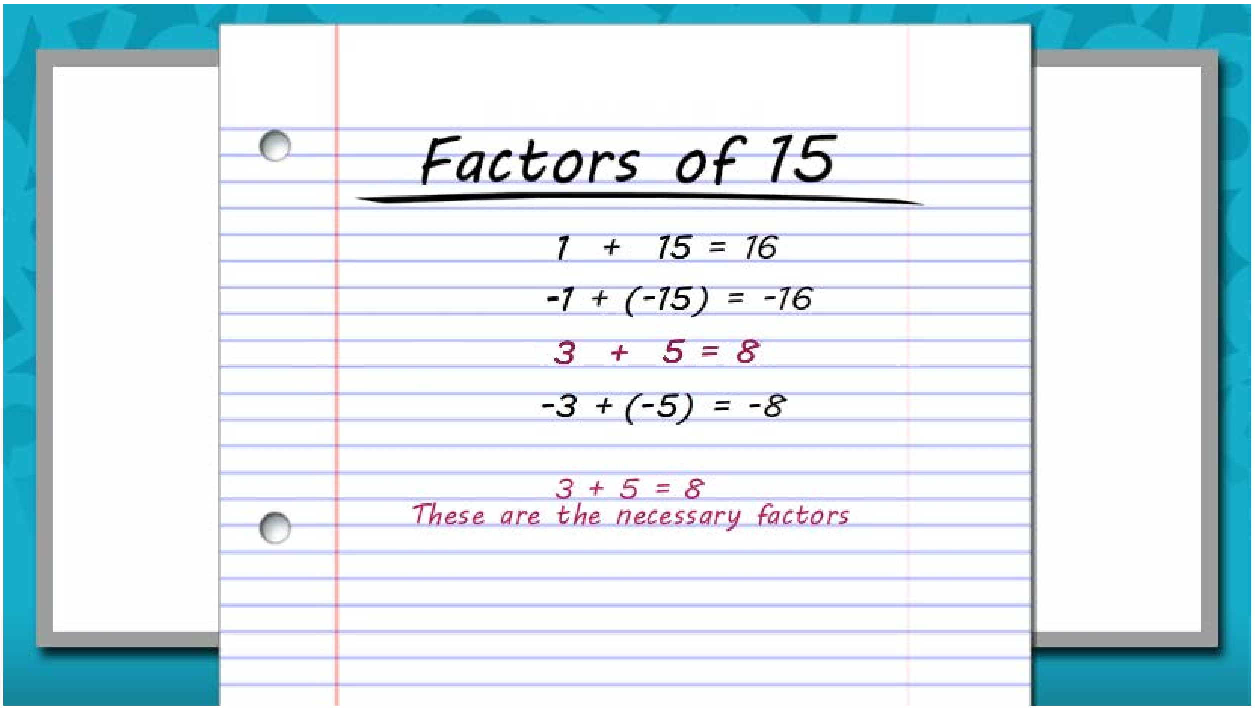
You must find the factors of 15, whose sum is equal to 8.

What pairs of numbers will multiply to equal 15?

$1 \cdot 15 = 15$
$-1 \cdot -15 = 15$
$3 \cdot 5 = 15$
$-3 \cdot -5 = 15$

Now, which pair of factors has a sum of 8?

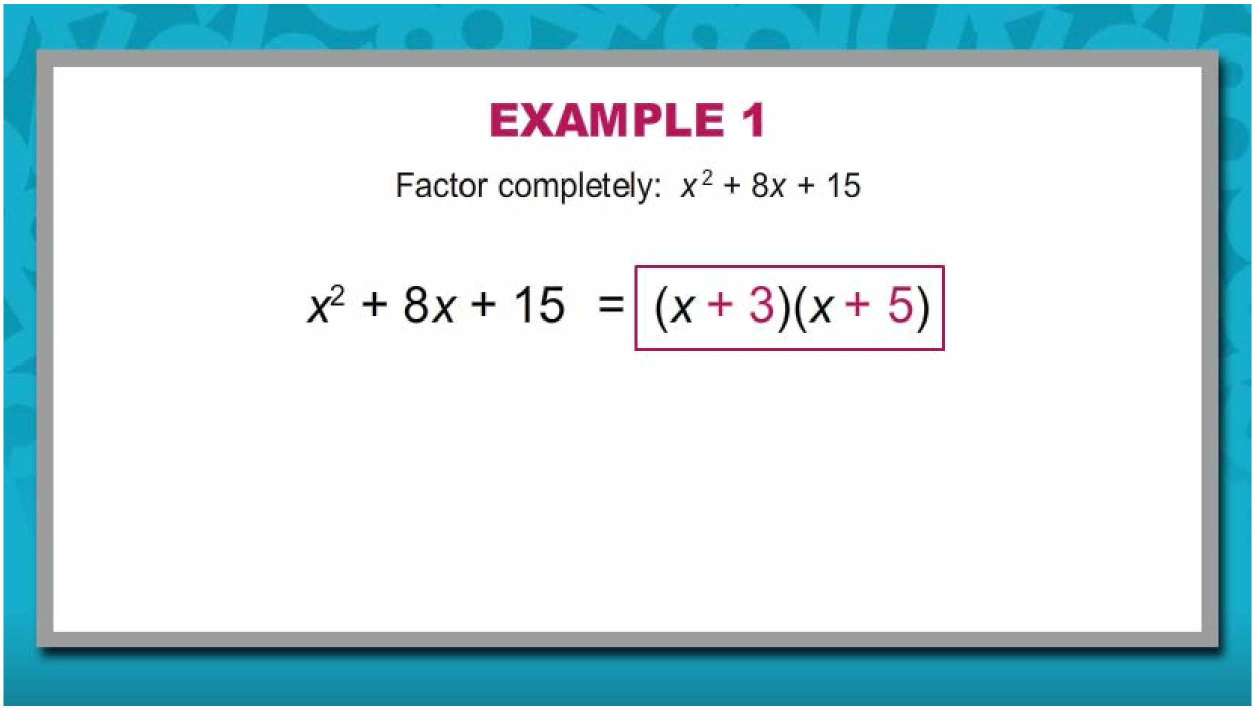
Example 1 (continued)



The necessary factors are 3 and 5.

$$3 + 5 = 8$$

Example 1 (continued)



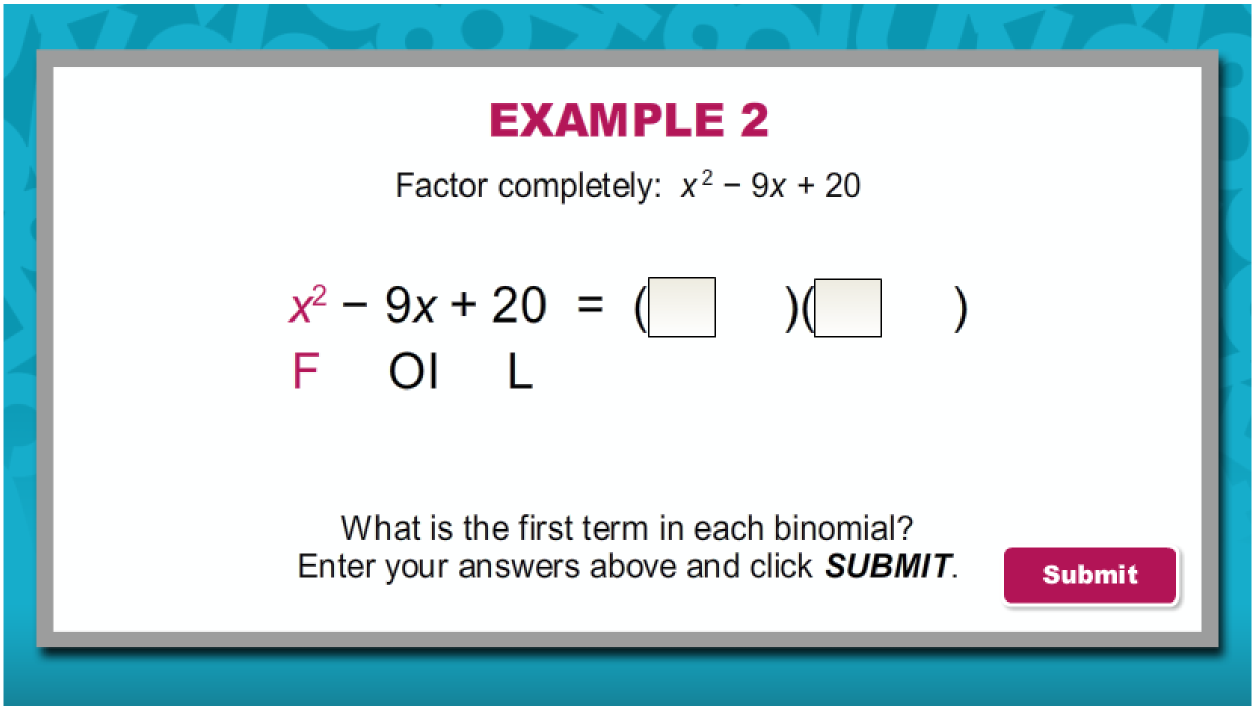
Now that you have identified the factors of 15 that have a sum of 8, you can insert them into the parentheses.

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

Your work is complete. The trinomial factors completely to the binomials $(x + 3)(x + 5)$. It is important to note that you can also represent the binomial factors in reverse.

$$x^2 + 8x + 15 = (x + 5)(x + 3)$$

Example 2



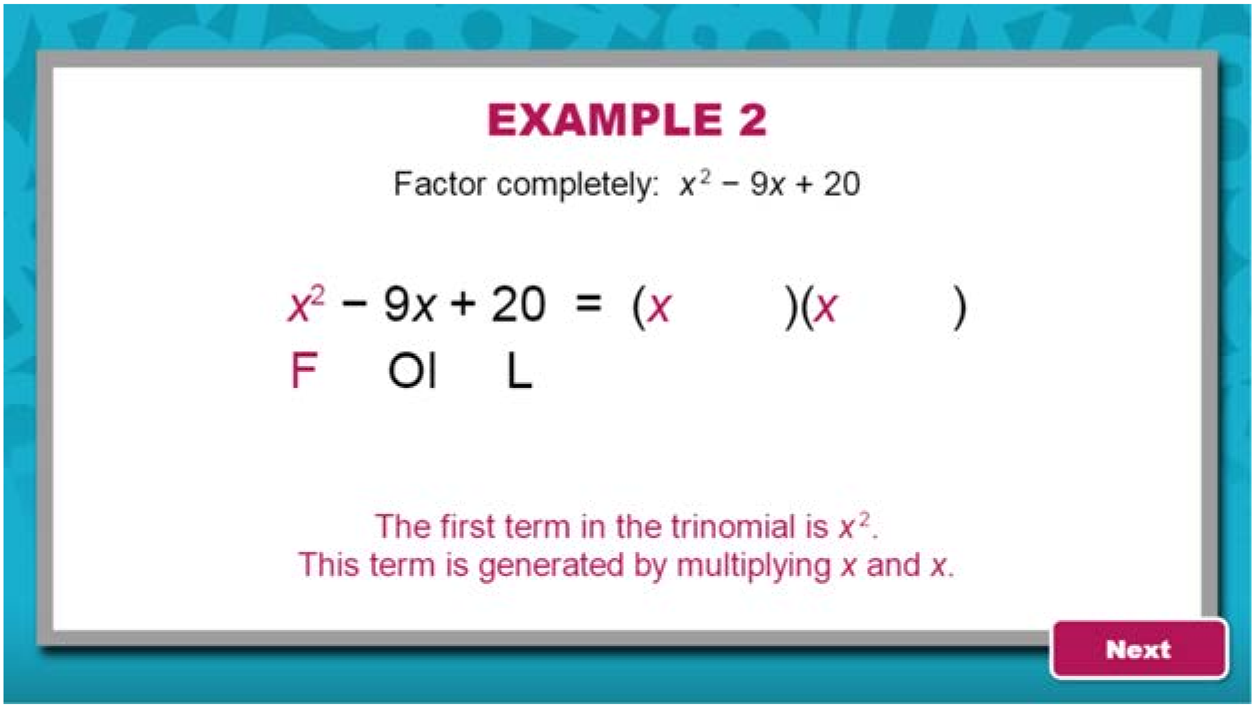
Factor completely: $x^2 - 9x + 20$.

The trinomial can be represented as the product of two binomials.

Recall that the first term of the trinomial is generated by multiplying the first terms of the binomial factors.

What is the first term in each binomial?

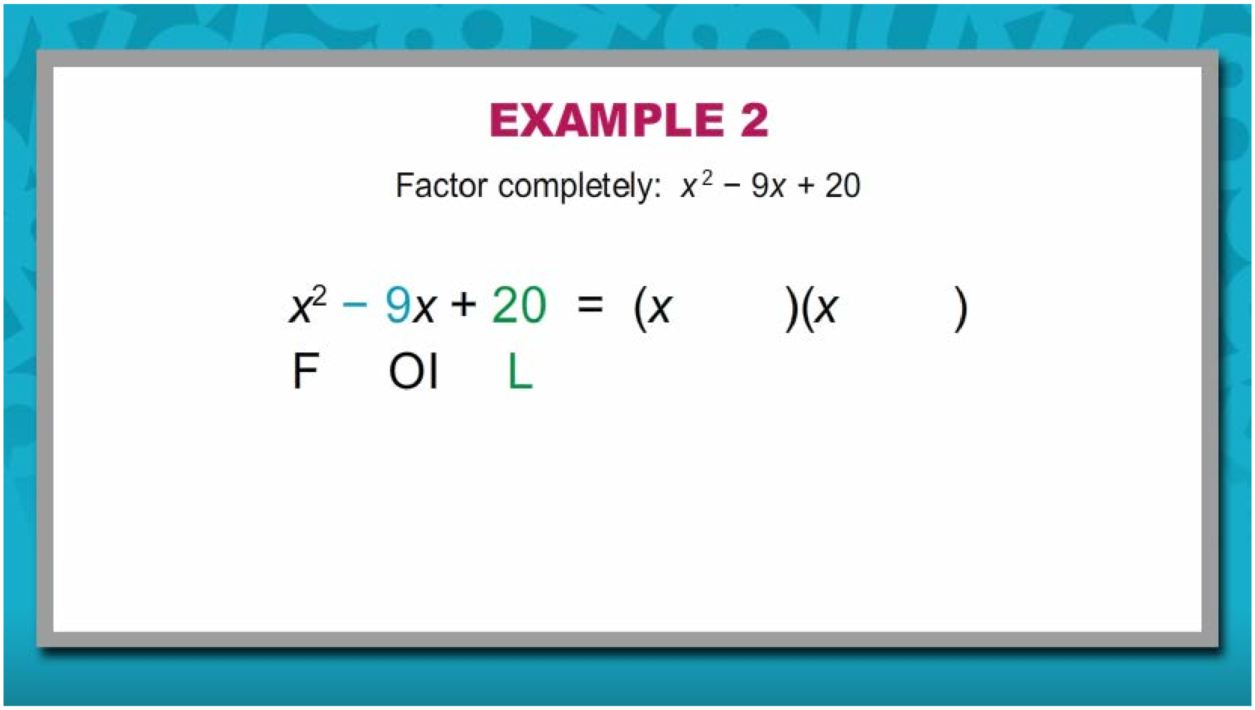
Example 2 (continued)



The first term in the trinomial is x^2. This term is generated by multiplying x and x.

$$x^2 - 9x + 20 = (x \quad)\,(x \quad)$$

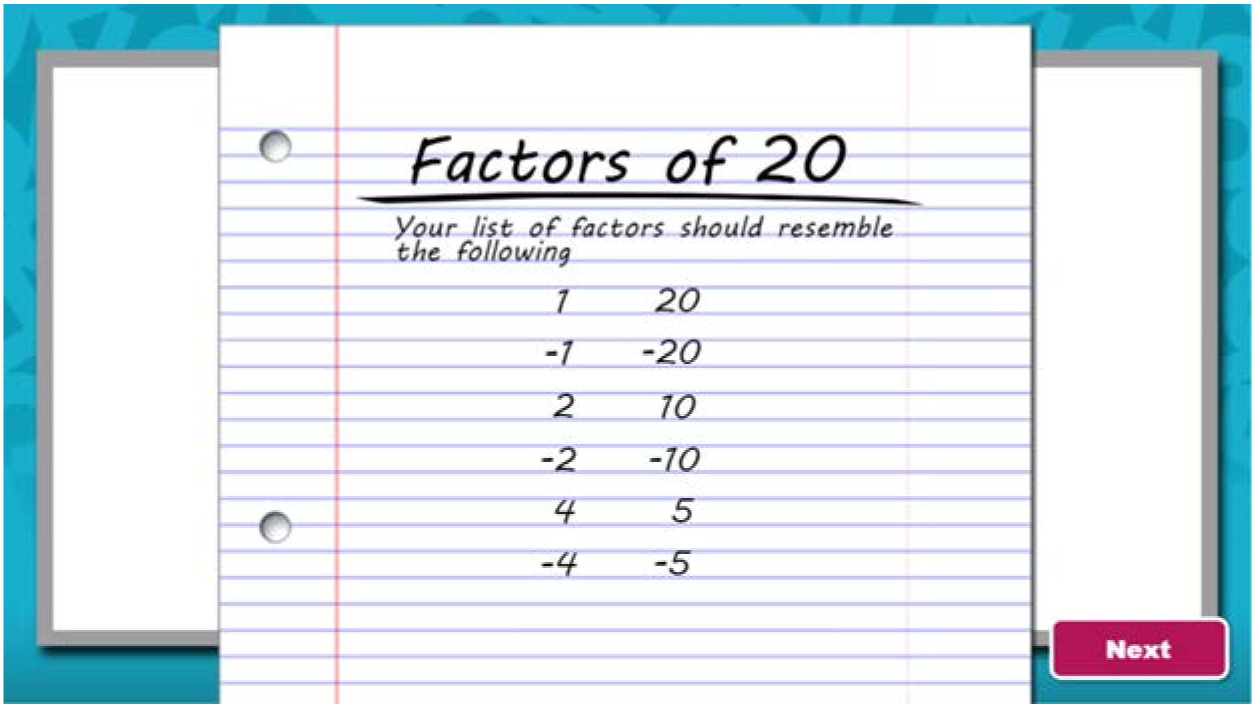
Example 2 (continued)



To determine the second term of each binomial, you must find the factors of 20 that have a sum of −9. In other words, you must find two numbers that multiply to equal 20 and add to equal −9.

Begin by listing the factors of 20.

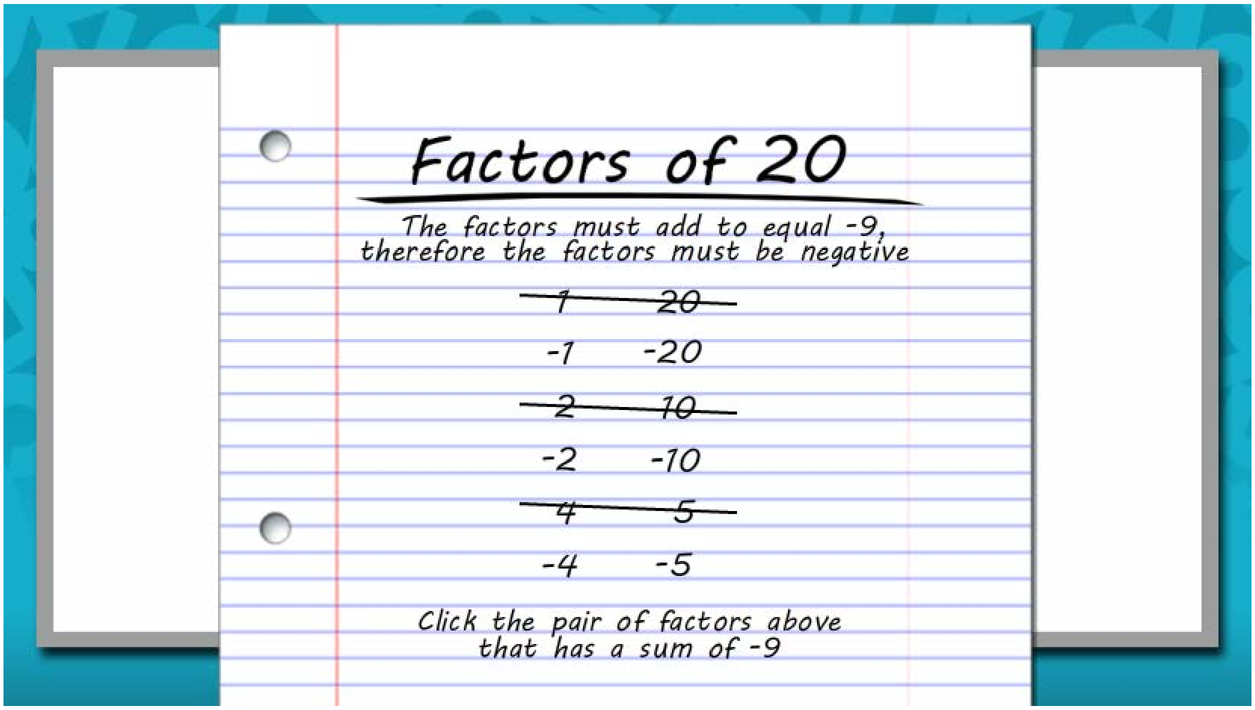
Example 2 (continued)



Your list of factors should resemble the following:

Factors of 20	
1	20
-1	-20
2	10
-2	-10
4	5
-4	-5

Example 2 (continued)

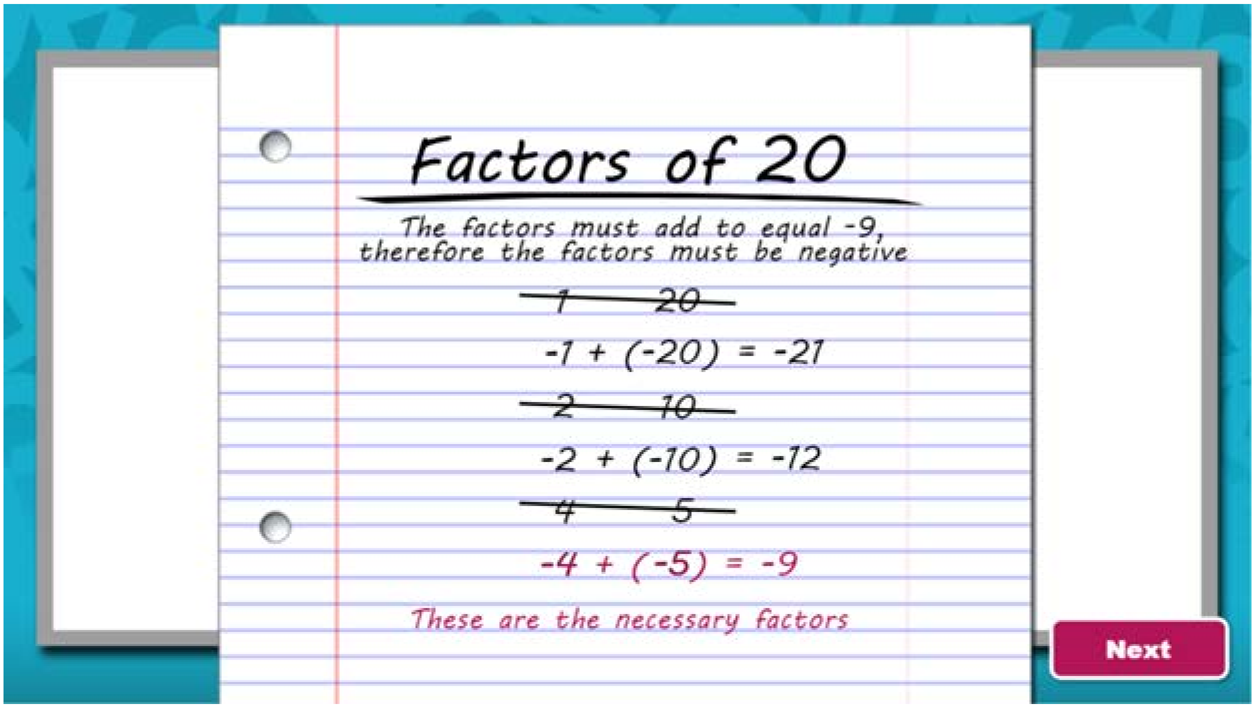


Because the necessary factors of 20 must also add to equal −9, this means that the factors must be negative. You can focus your attention on the negative factors of 20.

Factors of 20	
1	20
−1	−20
2	10
−2	−10
4	5
−4	−5

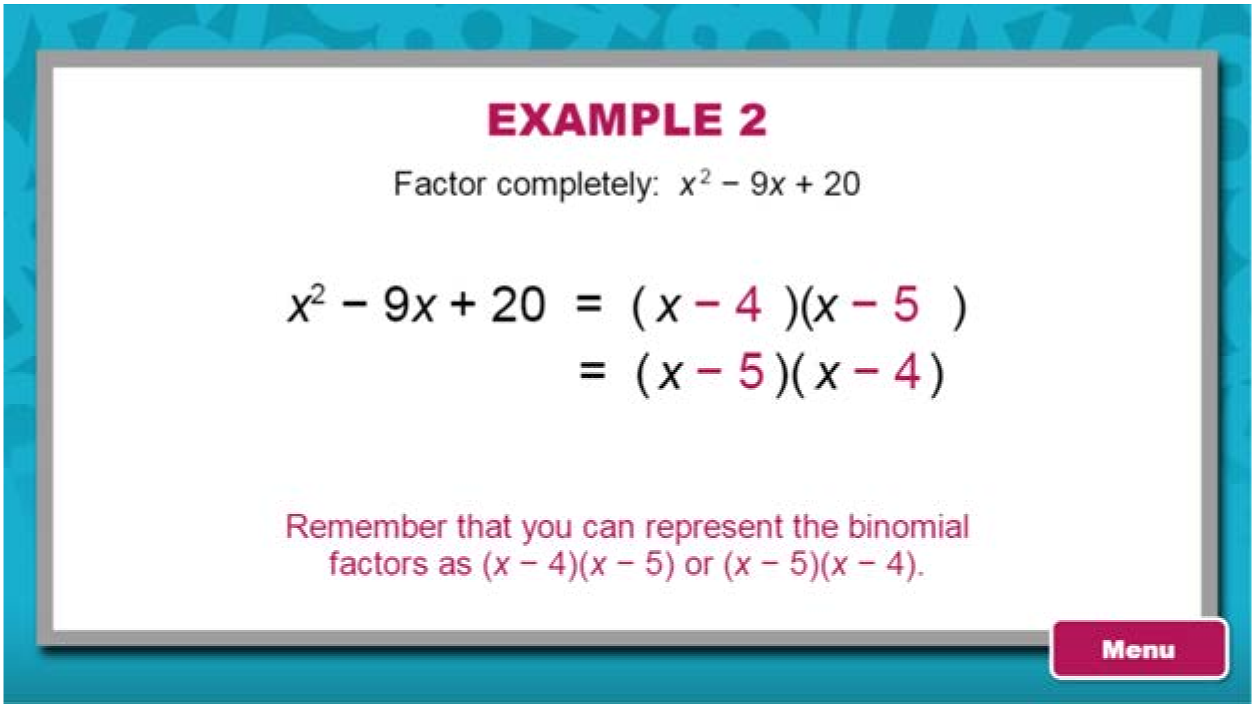
Which pair of factors has a sum of −9?

Example 2 (continued)



The necessary factors are -4 and -5.

Example 2 (continued)

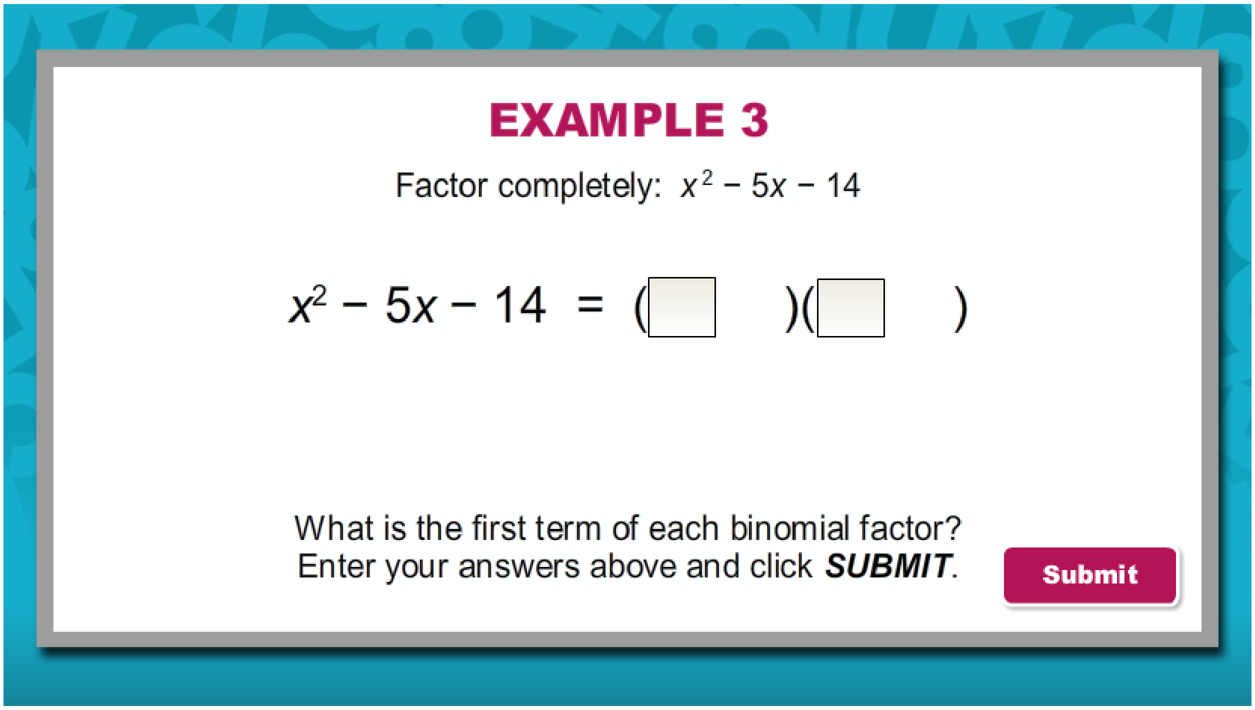


Now that you have identified the necessary factors, you can insert them into the parentheses.

$$x^2 - 9x + 20 = (x - 4)(x - 5)$$

Remember that you can represent the binomial factors as $(x - 4)(x - 5)$ or $(x - 5)(x - 4)$.

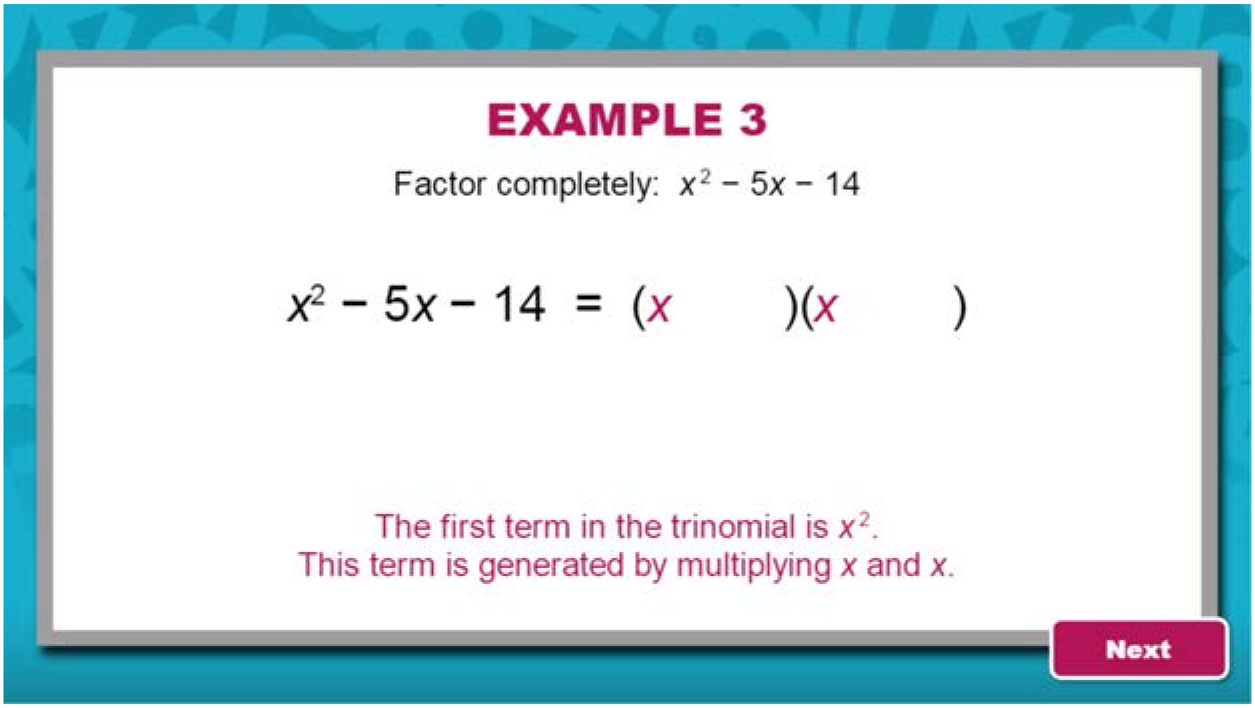
Example 3



Factor completely: $x^2 - 5x - 14$.

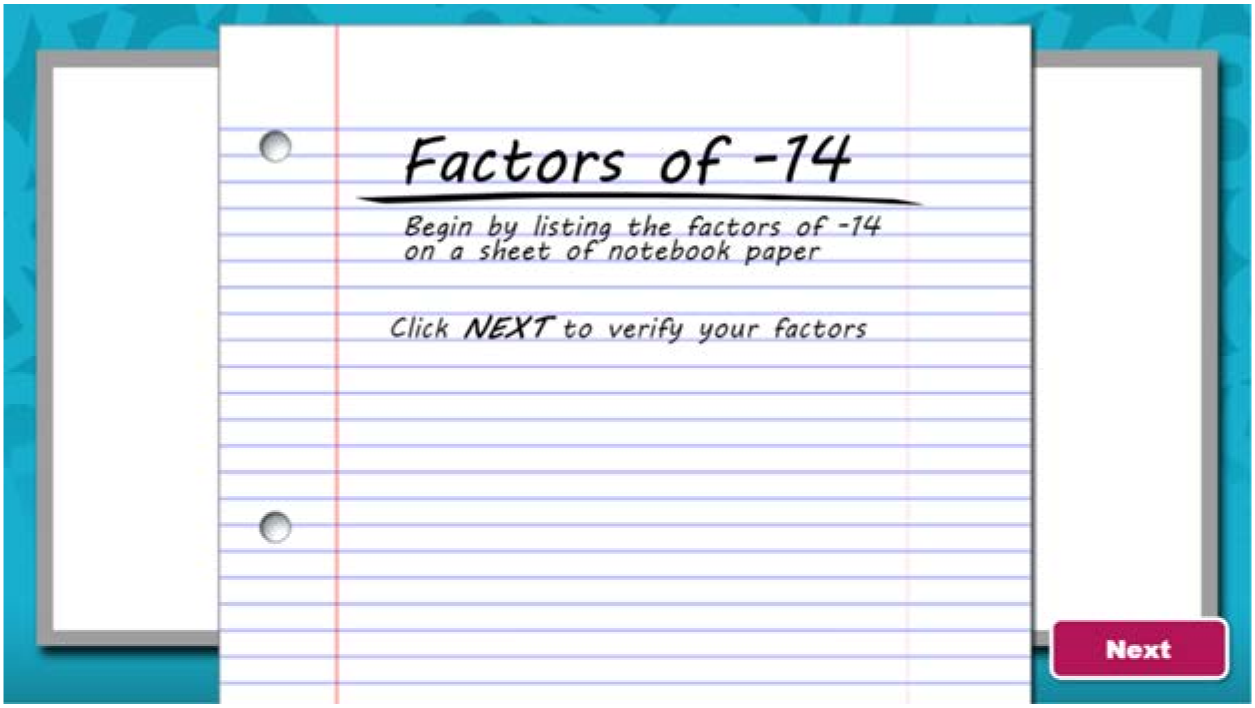
What is the first term of each binomial factor?

Example 3 (continued)



The first term in the trinomial is x^2. This term is generated by multiplying x and x.

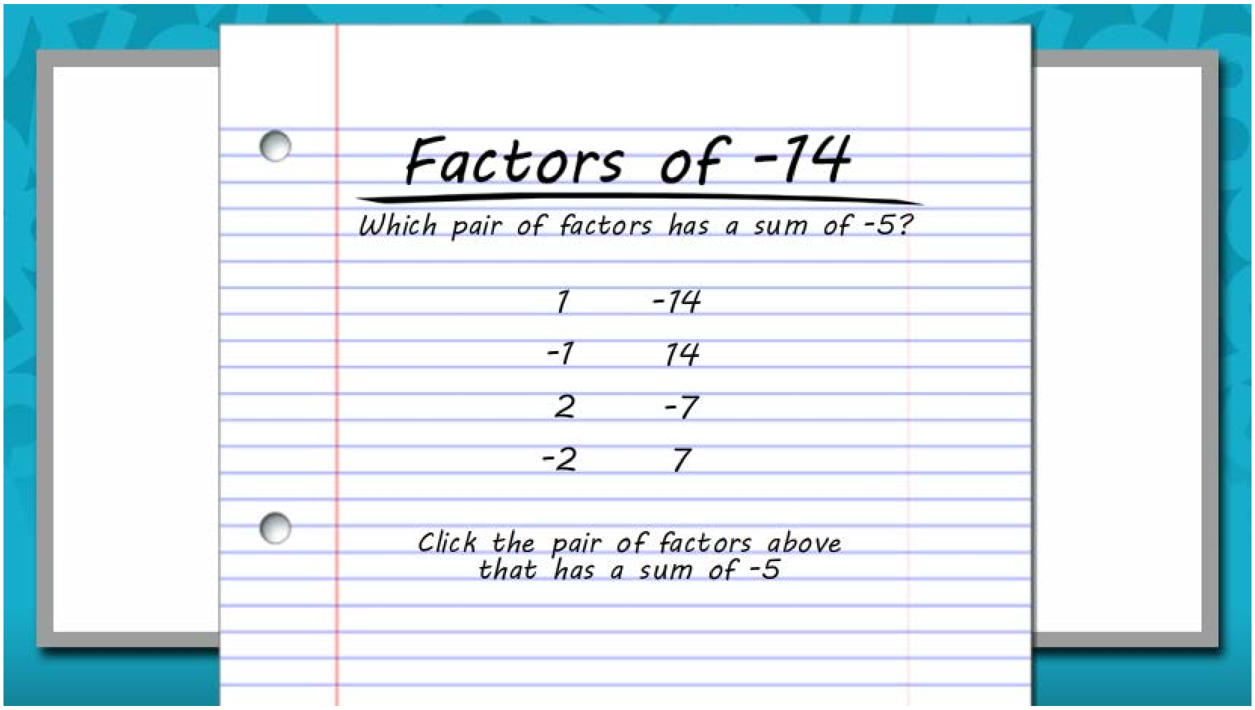
Example 3 (continued)



Now you must determine the factors of -14, that have a sum of -5.

List the factors of -14.

Example 3 (continued)

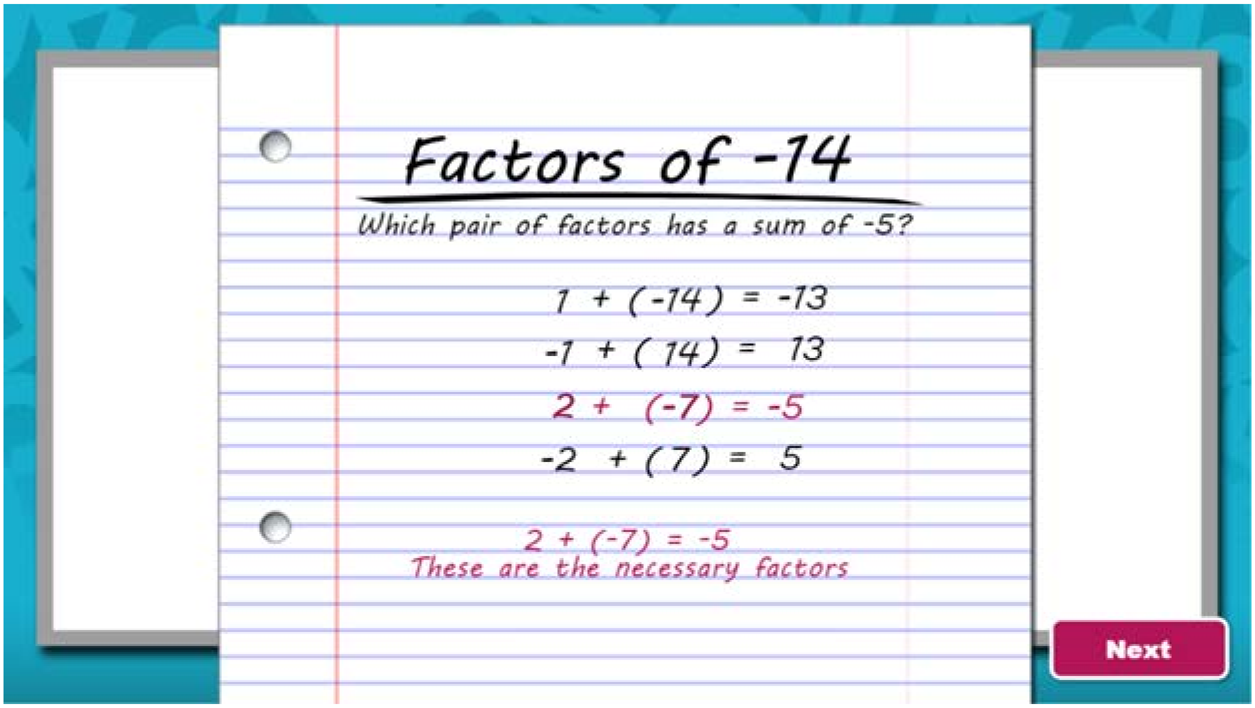


Your list of factors should resemble the following.

Factors of -14	
1	-14
-1	14
2	-7
-2	7

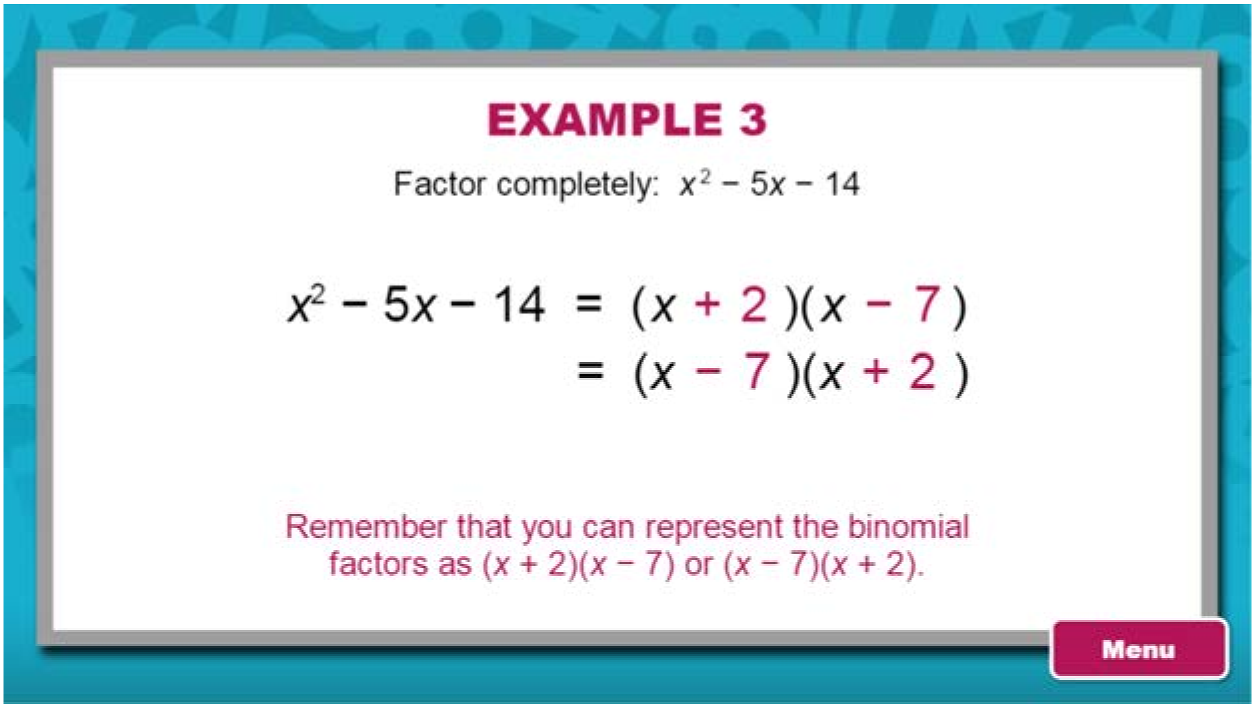
Which pair of factors has a sum of -5?

Example 3 (continued)



The necessary factors are 2 and -7.

Example 3 (continued)

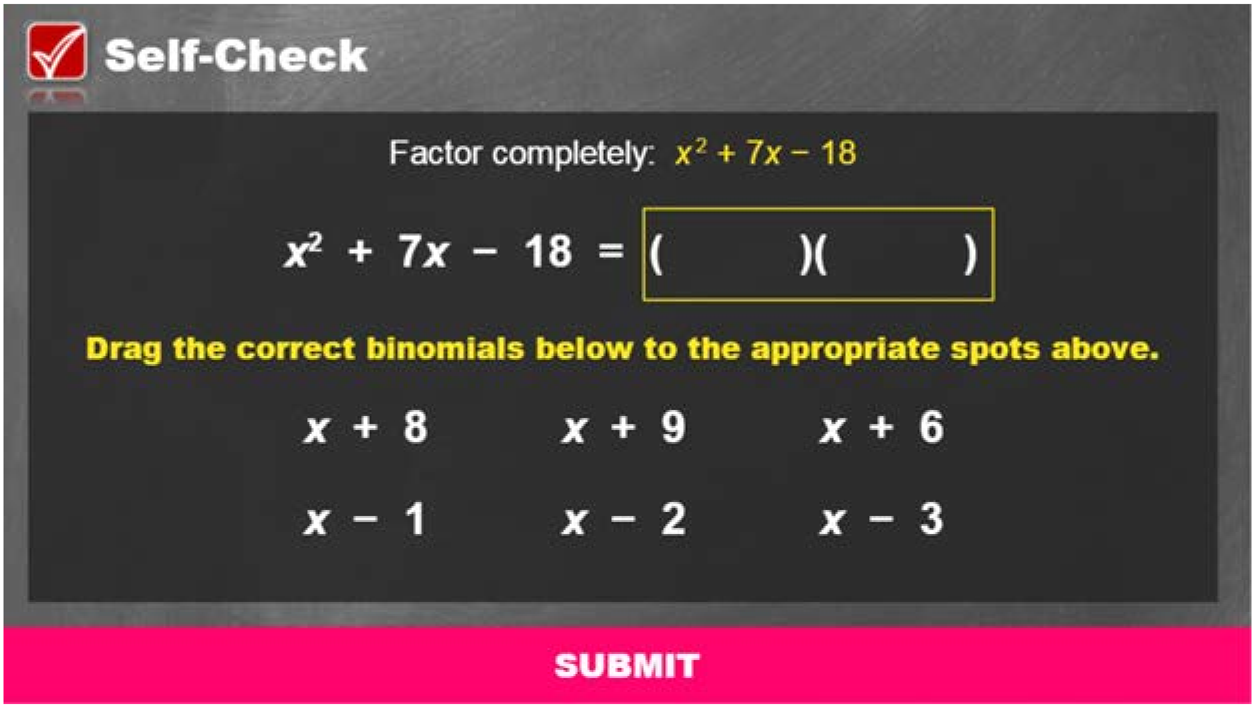


Now that you have identified the factors of -14 that have a sum of -5, you can insert them into the parentheses.

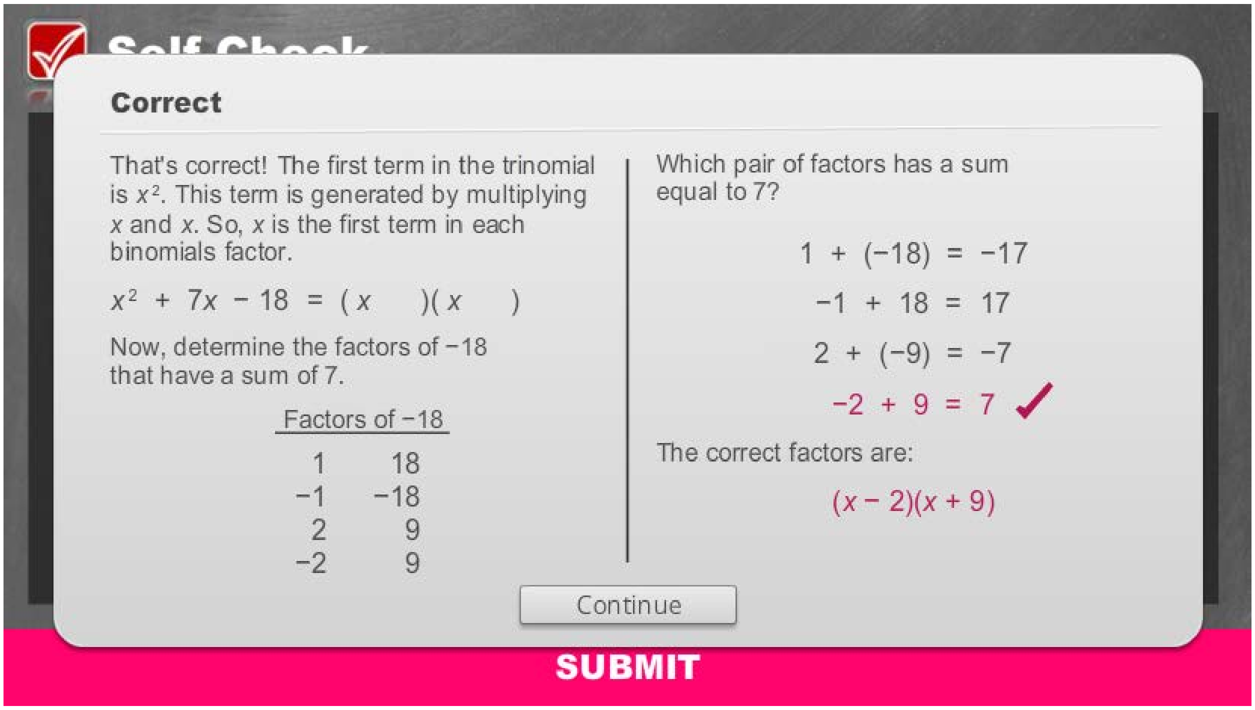
$$x^2 - 9x + 20 = (x + 2)(x - 7)$$

Remember that you can represent the binomial factors as $(x + 2)(x - 7)$ or $(x - 7)(x + 2)$.

Self-Check


Self-Check
Factor completely: $x^2 + 7x - 18$
$x^2 + 7x - 18 = (\quad)(\quad)$
Drag the correct binomials below to the appropriate spots above.
$x + 8$
$x + 9$
$x + 6$
$x - 1$
$x - 2$
$x - 3$
SUBMIT

Self-Check Answer



Conclusion

Congratulations! You have reached the conclusion of this lesson in Algebra I. In this lesson you applied your knowledge of the distributive property to factor second-degree trinomials that have a leading coefficient of one.