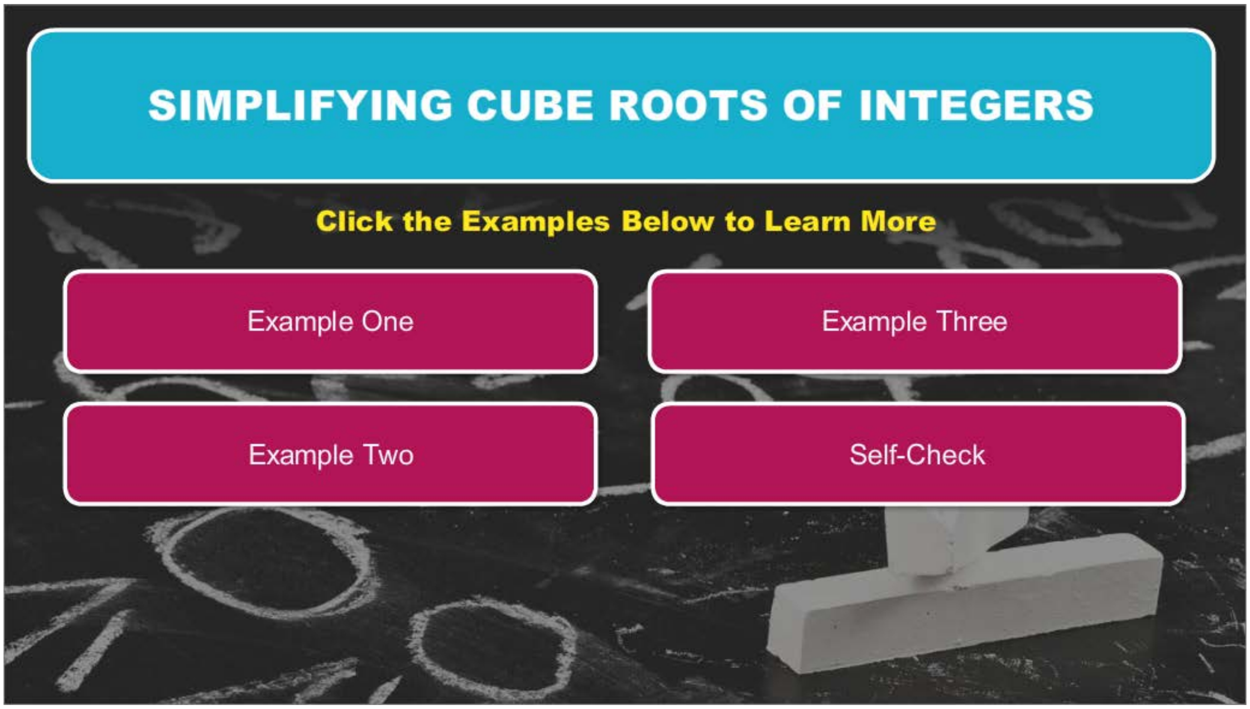
Module 4: Radical Expressions
Topic 1 Content: Simplifying Cube Roots of Integers

Introduction

Hi there! I'm so glad you could join me for this lesson in Algebra I. In this lesson, you will apply the Product Property of Radicals to simplify cube roots of integers. Your factoring skills will prove useful during this lesson.

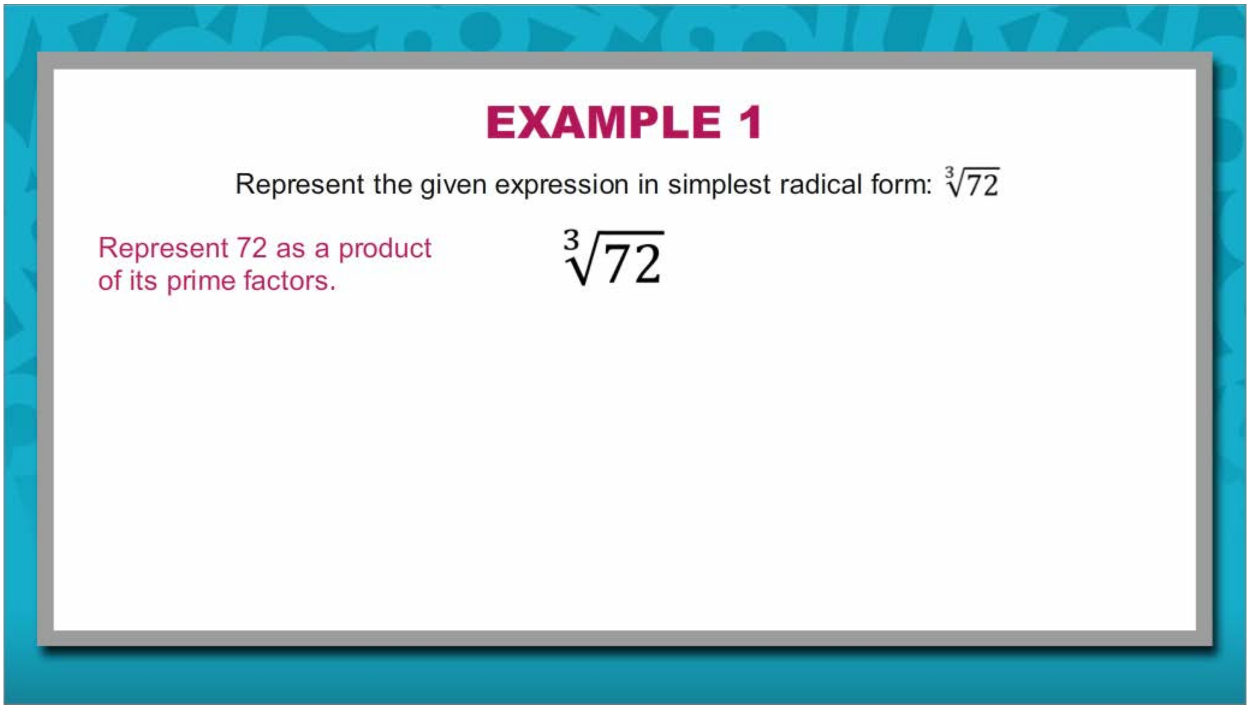
Simplifying Cube Roots of Integers



Click the examples below to learn more.

- Example One
- Example Two
- Example Three
- Self-Check

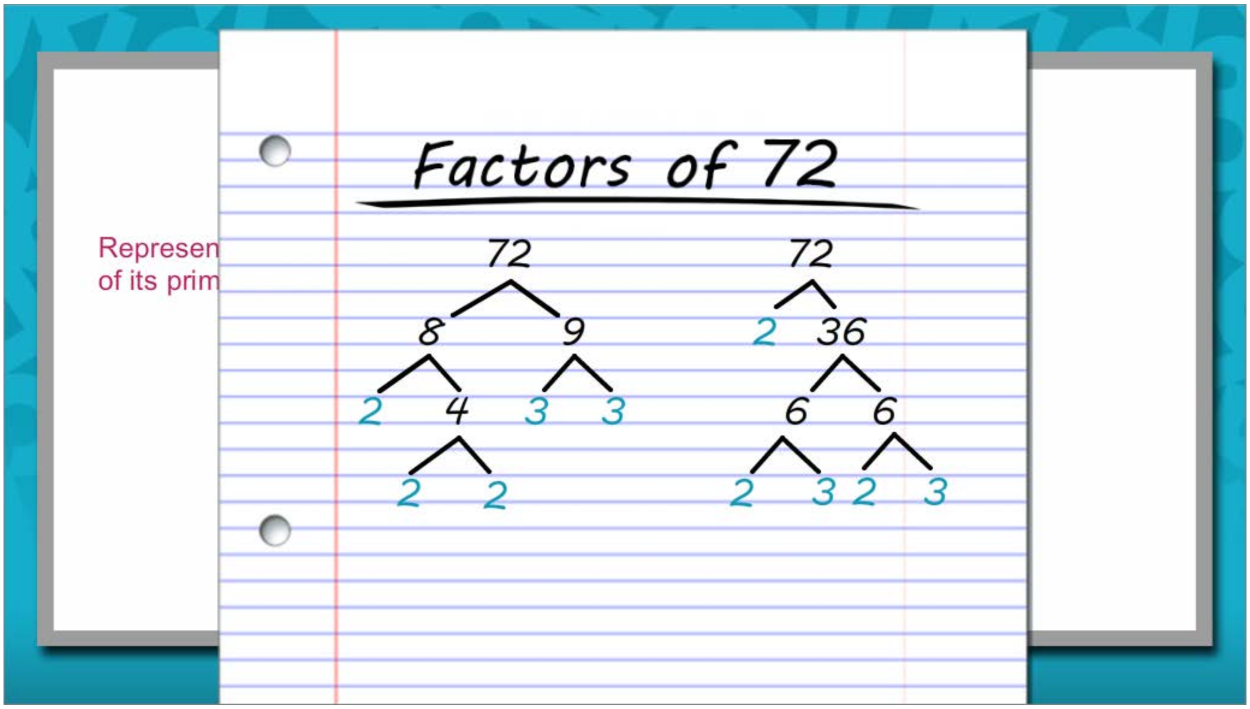
Example 1



Represent the given expression in simplest radical form: $\sqrt[3]{72}$

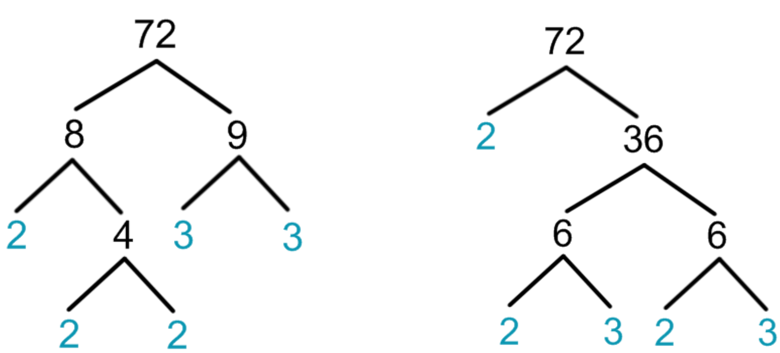
A cube root expression is in simplest form when the radicand has no perfect cube factors other than one. To represent $\sqrt[3]{72}$ in simplest radical form, start by completing the prime factorization of 72. You may find it helpful to use a factor tree.

Example 1 (continued)



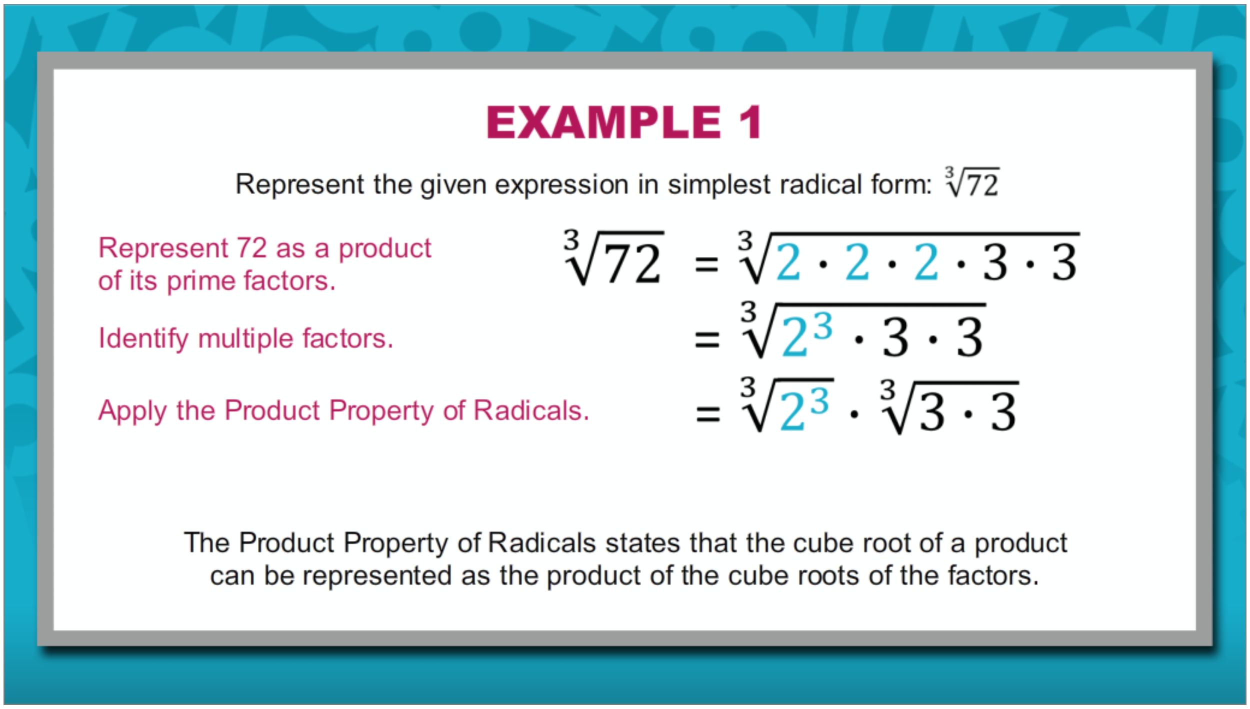
The branches of your factor tree will depend on the initial factors of 72 that you choose to begin with. The prime factors, however, will be the same.

Two possible factor trees of 72:



The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

Example 1 (continued)



Now that you know the prime factorization of 72, you can represent $\sqrt[3]{72}$ as $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$.

$$\sqrt[3]{72} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$

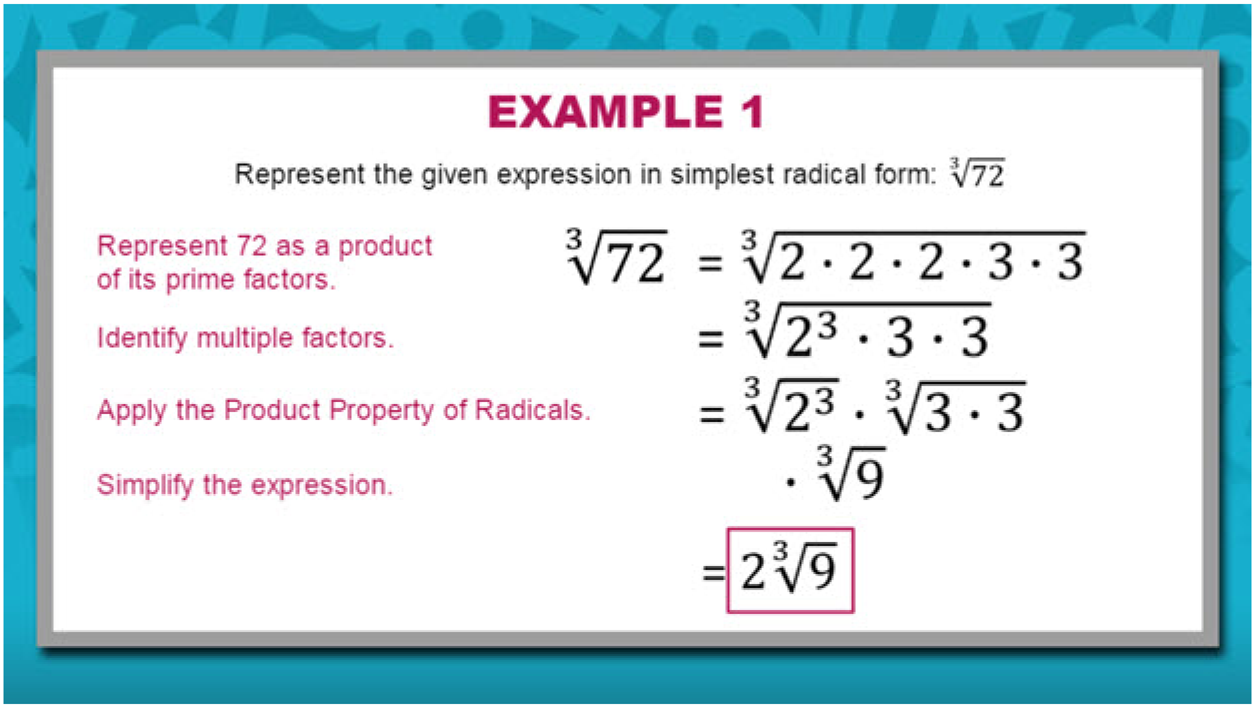
The next step is to identify any multiple factors. Because you are simplifying a cube root expression, the goal is to identify multiple factors that can be rewritten as a cube. $2 \cdot 2 \cdot 2$ can be represented as 2^3.

$$= \sqrt[3]{2^3 \cdot 3 \cdot 3}$$

You can apply the Product Property of Radicals to represent the cube root of a product as the product of the cube roots of the factors. So $\sqrt[3]{72}$ can be represented as $\sqrt[3]{2^3} \cdot \sqrt[3]{3 \cdot 3}$.

$$= \sqrt[3]{2^3} \cdot \sqrt[3]{3 \cdot 3}$$

Example 1 (continued)



Now simplify the expression. Begin with the first cube root factor. Recall that cubing a number and finding the cube root of a number are inverse operations. This means that $\sqrt[3]{2^3} = 2$.

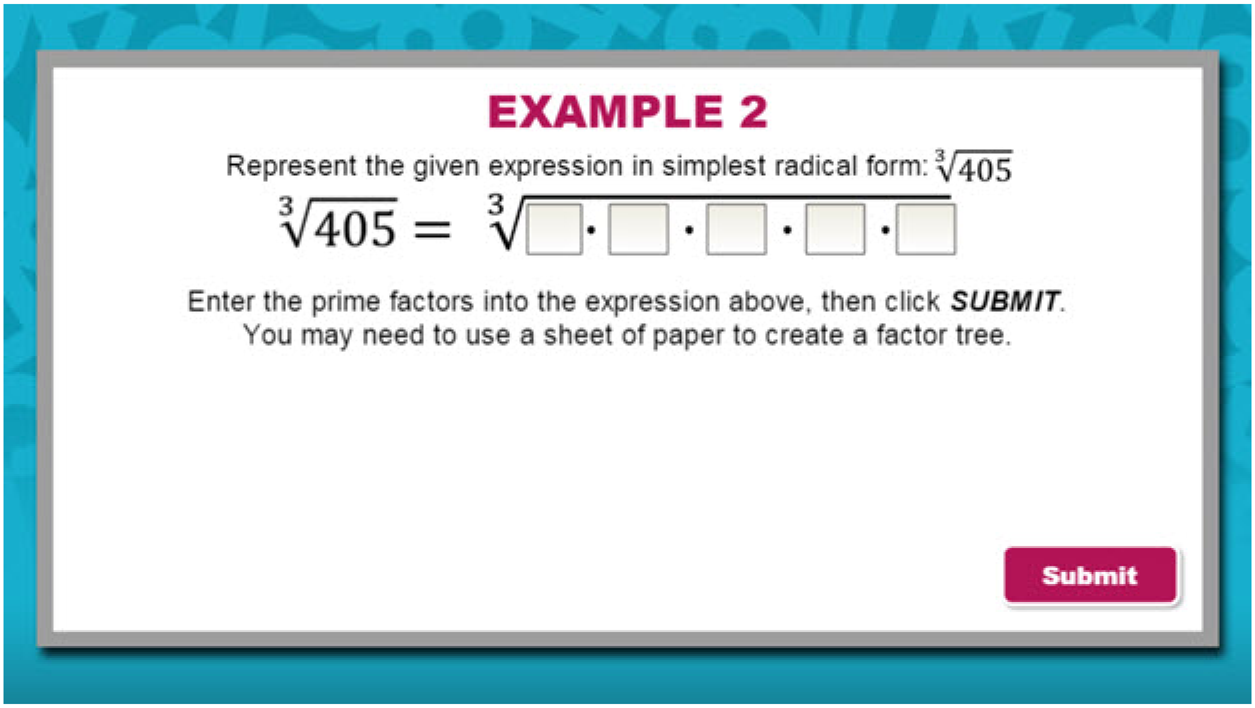
Next simplify the radicand of the second cube root factor by multiplying 3 and 3; $3 \cdot 3 = 9$.

$$= \; 2 \cdot \sqrt[3]{9}$$

The final step is to simplify the resulting expression. Your work is complete. The simplest radical form of $\sqrt[3]{72}$ is $2\sqrt[3]{9}$.

$$= \; 2\sqrt[3]{9}$$

Example 2



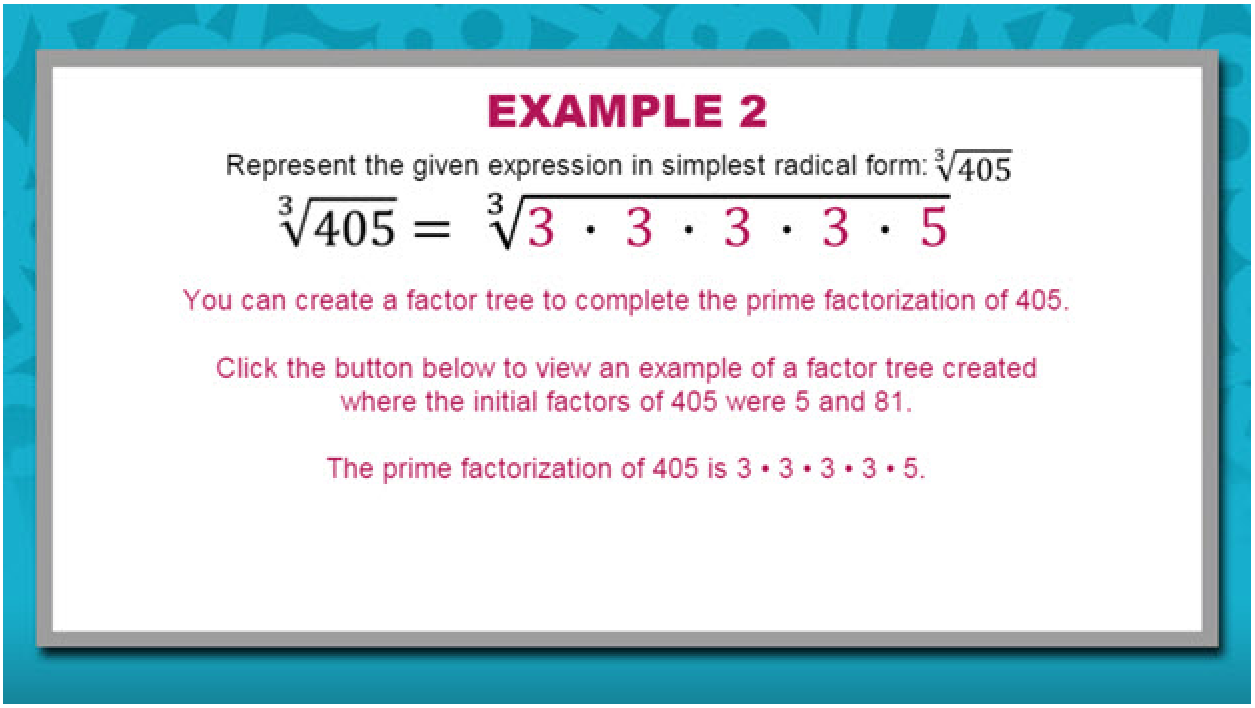
Represent the given expression in simplest radical form: $\sqrt[3]{405}$

To simplify the expression, begin by representing the radicand as a product of its prime factors.

$$\sqrt[3]{405} = \sqrt[3]{?\cdot?\cdot?\cdot?\cdot?}$$

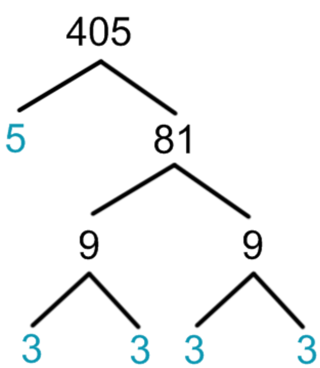
Enter the prime factors into the expression above, then click ***SUBMIT***. You may need to use a sheet of paper to create a factor tree.

Example 2 (continued)



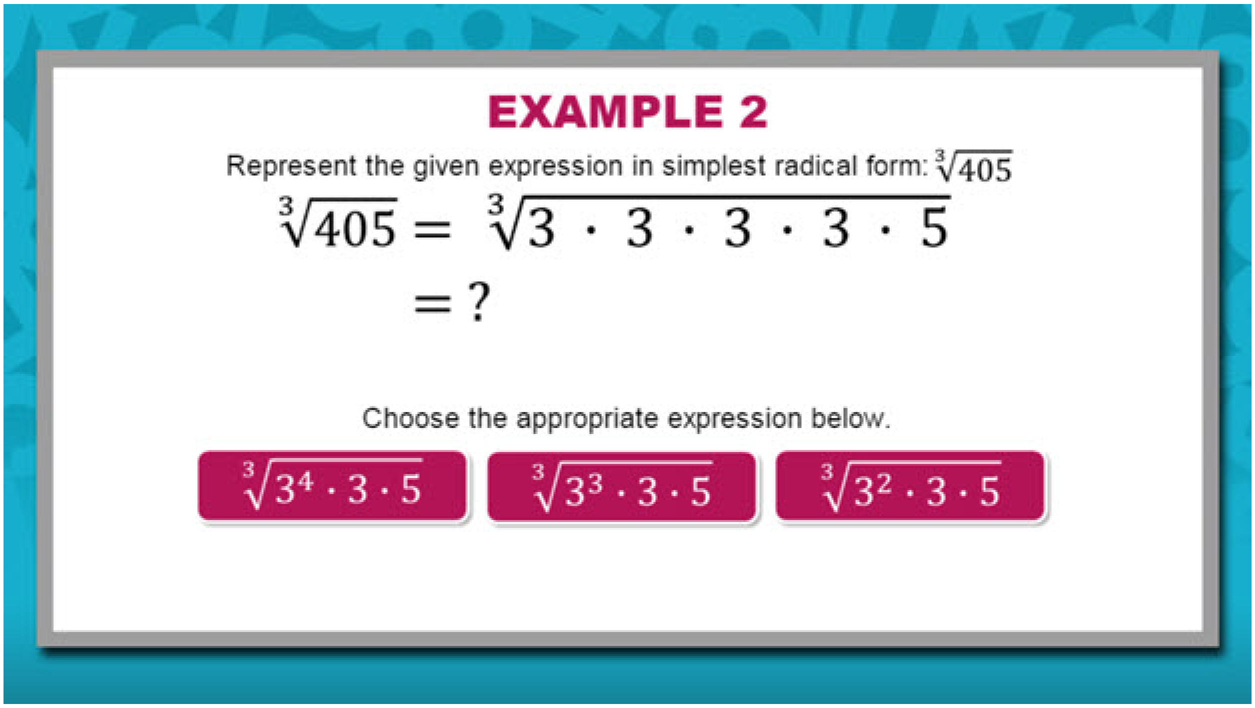
$$\sqrt[3]{405} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}$$

You can create a factor tree to complete the prime factorization of 405. Take a look at the example of a factor tree created when the initial factors of 405 were 5 and 81.



The prime factorization of 405 is $3 \cdot 3 \cdot 3 \cdot 3 \cdot 5$.

Example 2 (continued)



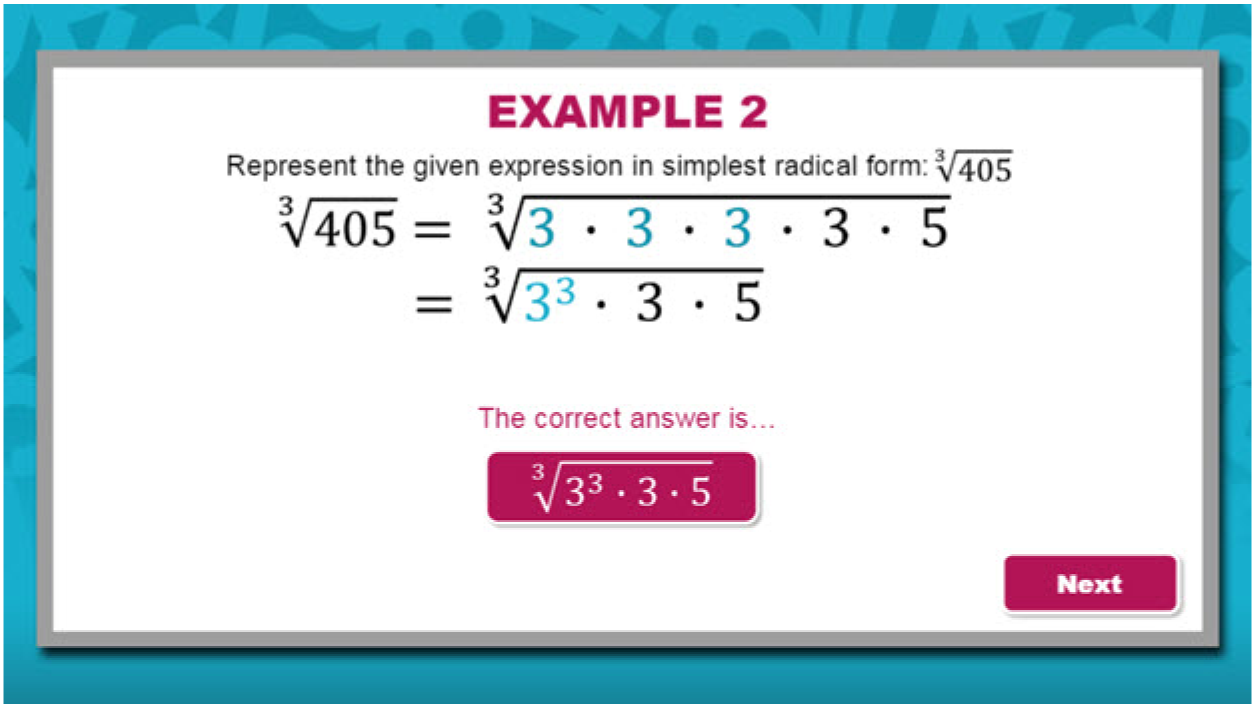
Now identify multiple factors that can be rewritten as a cube.

$$\sqrt[3]{405} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} = ?$$

Choose the appropriate expression below.

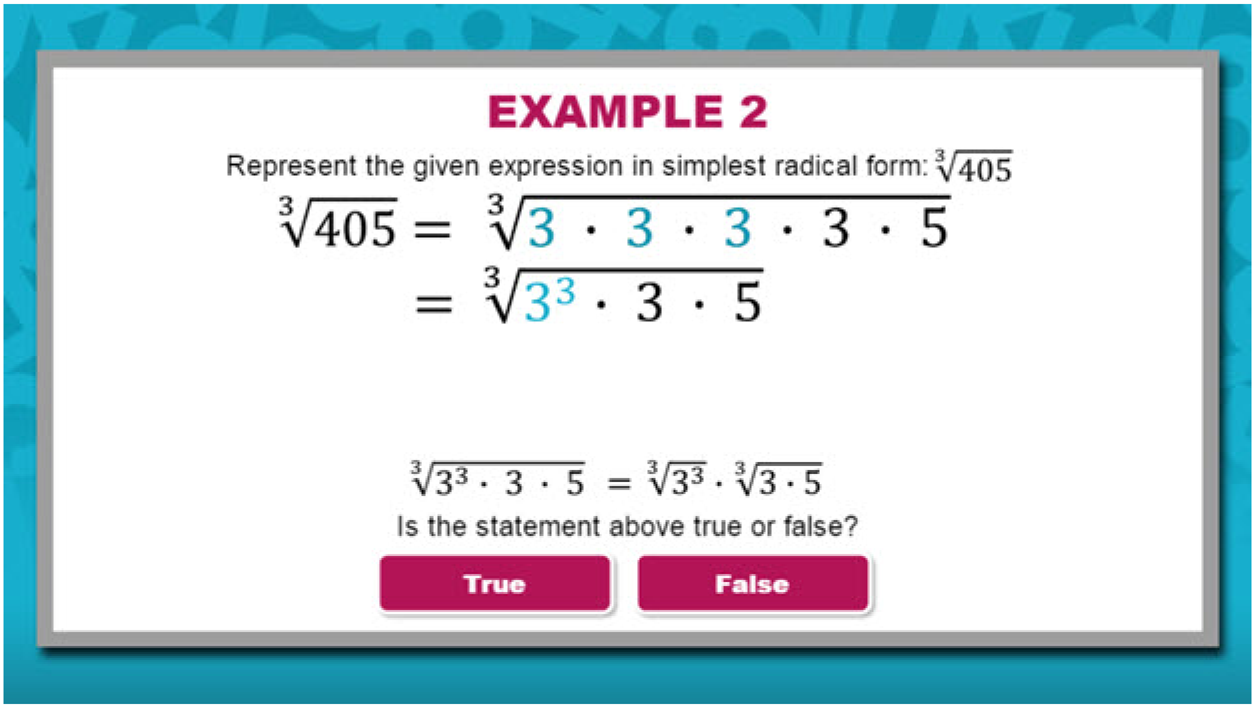
A) $\sqrt[3]{3^4 \cdot 3 \cdot 5}$
B) $\sqrt[3]{3^3 \cdot 3 \cdot 5}$
C) $\sqrt[3]{3^2 \cdot 3 \cdot 5}$

Example 2 (continued)



$$\begin{aligned}\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\ &= \sqrt[3]{3^3 \cdot 3 \cdot 5}\end{aligned}$$

Example 2 (continued)



$$\begin{aligned}\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\ &= \sqrt[3]{3^3 \cdot 3 \cdot 5}\end{aligned}$$

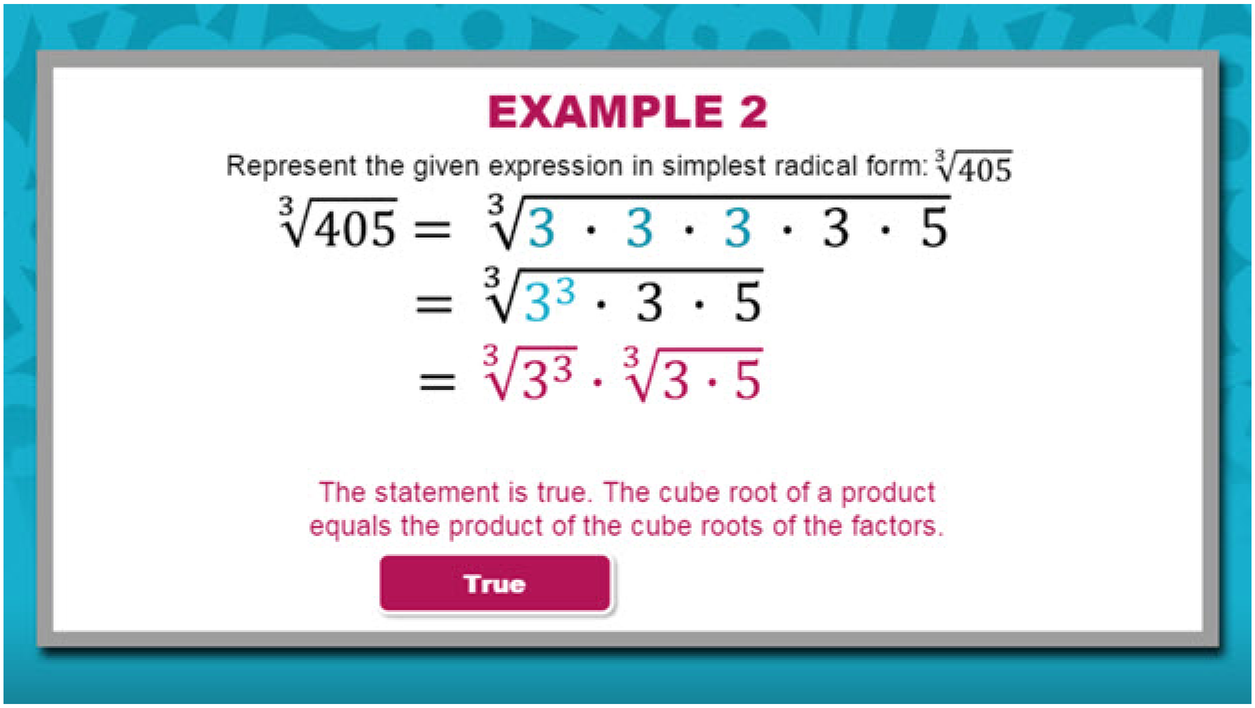
Next apply the Product Property of Radicals to represent the cube root of the product as the product of the cube roots of the factors.

$$\sqrt[3]{3^3 \cdot 3 \cdot 5} = \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5}$$

Is the expression above true or false?

A) True
B) False

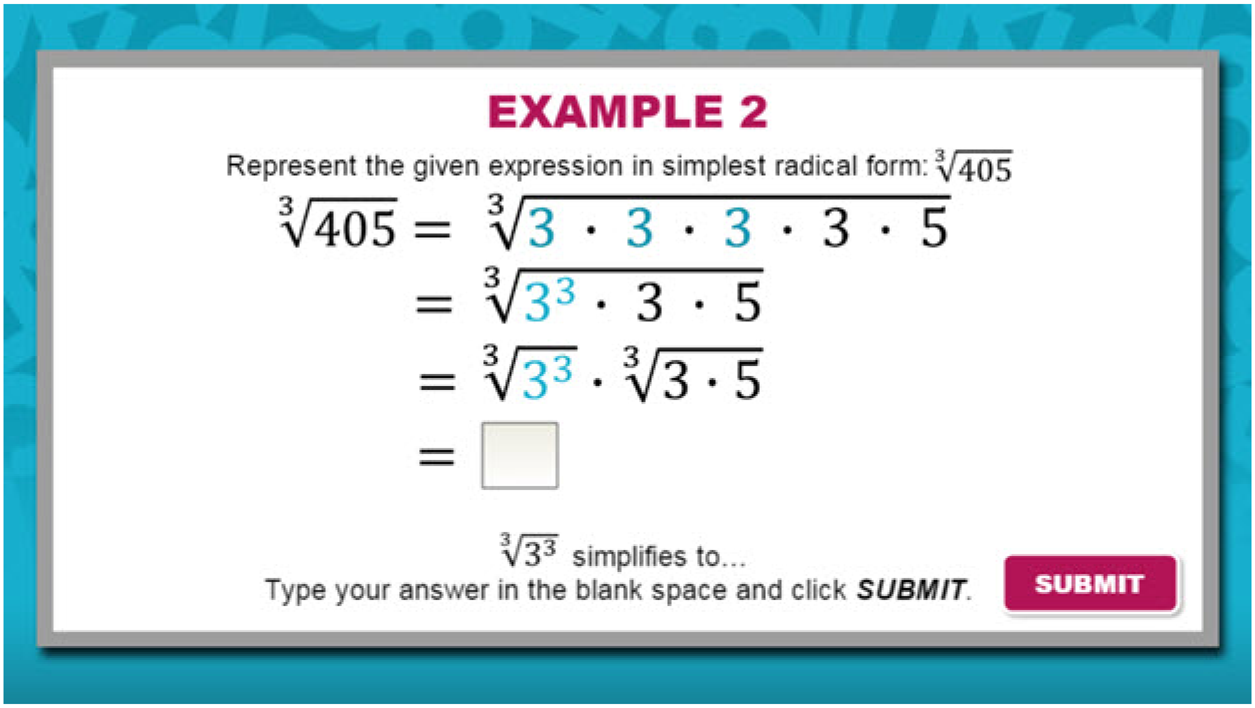
Example 2 (continued)



$$
\begin{aligned}
\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5}
\end{aligned}
$$

The expression is true. The cube root of the product equals the product of the cube roots of the factors.

Example 2 (continued)



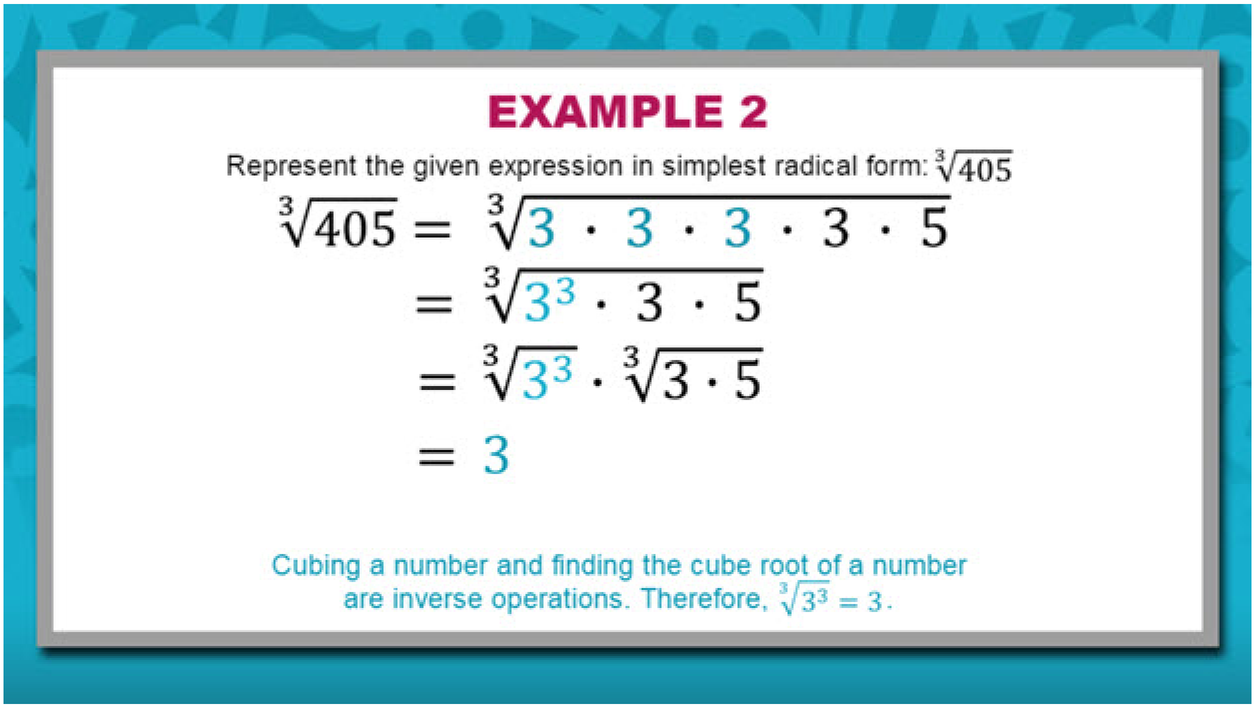
$$
\begin{aligned}
\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5} \\
&=?
\end{aligned}
$$

Now simplify the expression.

$\sqrt[3]{3^3}$ simplifies to …

Type your answer in the blank space and click ***SUBMIT***.

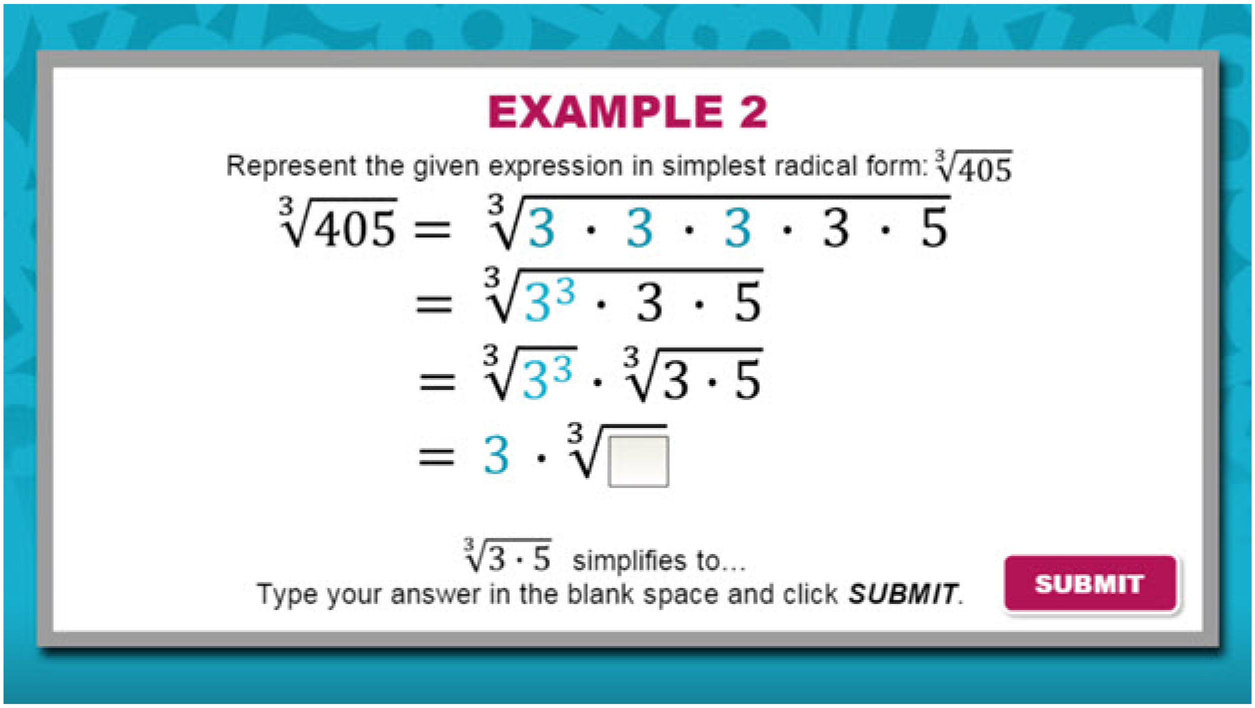
Example 2 (continued)



$$\begin{aligned}\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\ &= \sqrt[3]{3^3 \cdot 3 \cdot 5} \\ &= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5} \\ &= 3\end{aligned}$$

Cubing a number and finding the cube root of a number are inverse operations. Therefore, $\sqrt[3]{3^3} = 3$.

Example 2 (continued)

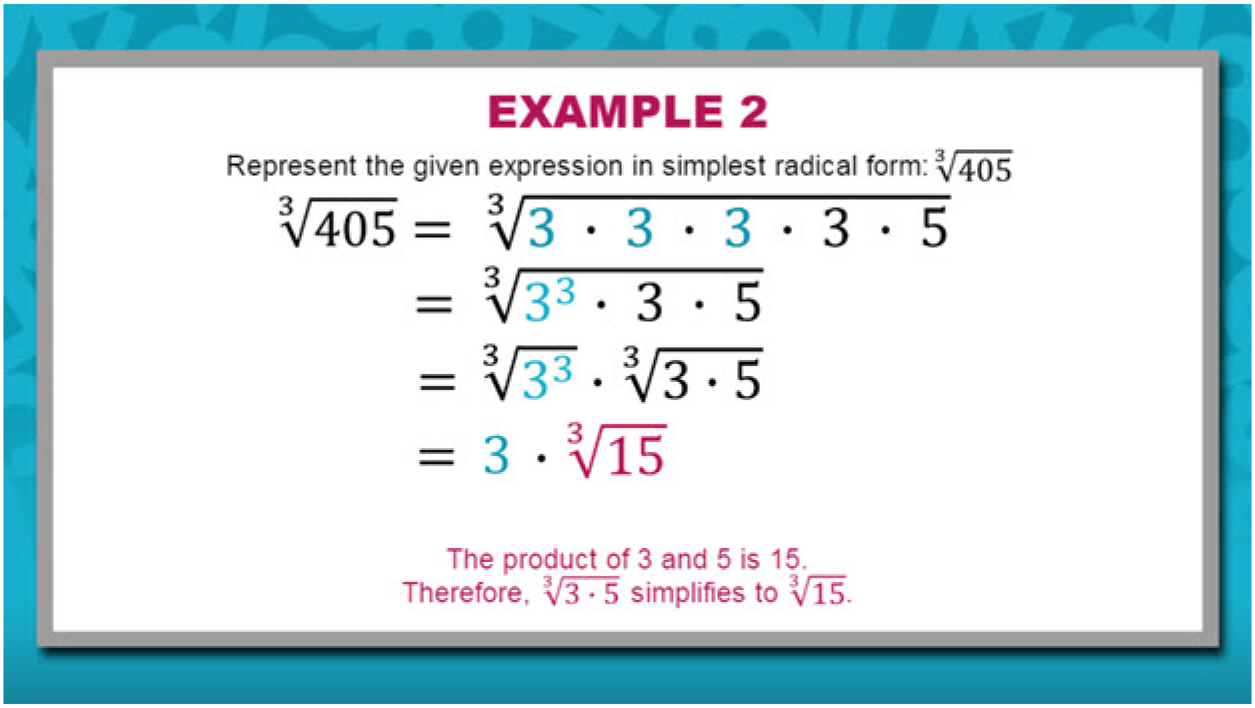


$$
\begin{aligned}
\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5} \\
&= 3 \cdot \sqrt[3]{?}
\end{aligned}
$$

$\sqrt[3]{3 \cdot 5}$ simplifies to….

Type your answer in the blank space and click ***SUBMIT***.

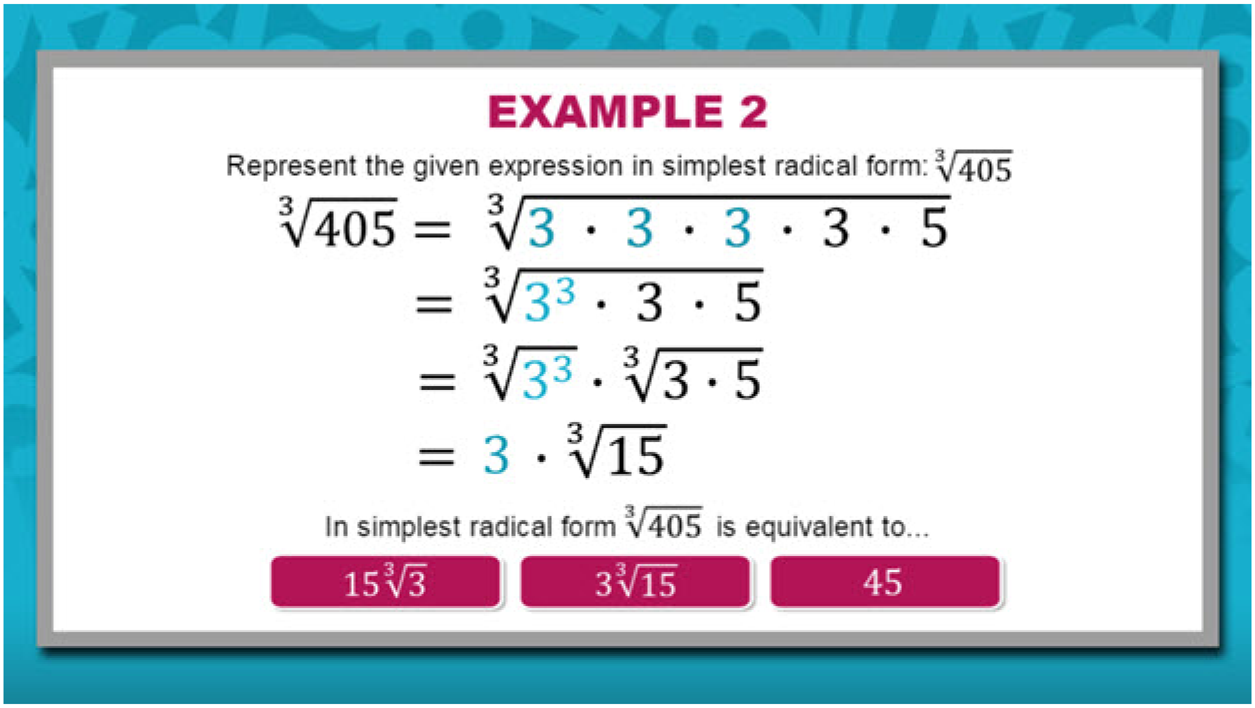
Example 2 (continued)



$$
\begin{aligned}
\sqrt[3]{405} &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3 \cdot 3 \cdot 5} \\
&= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5} \\
&= 3 \cdot \sqrt[3]{15}
\end{aligned}
$$

The product of 3 and 5 is 15. Therefore, $\sqrt[3]{3 \cdot 5}$ simplifies to $\sqrt[3]{15}$.

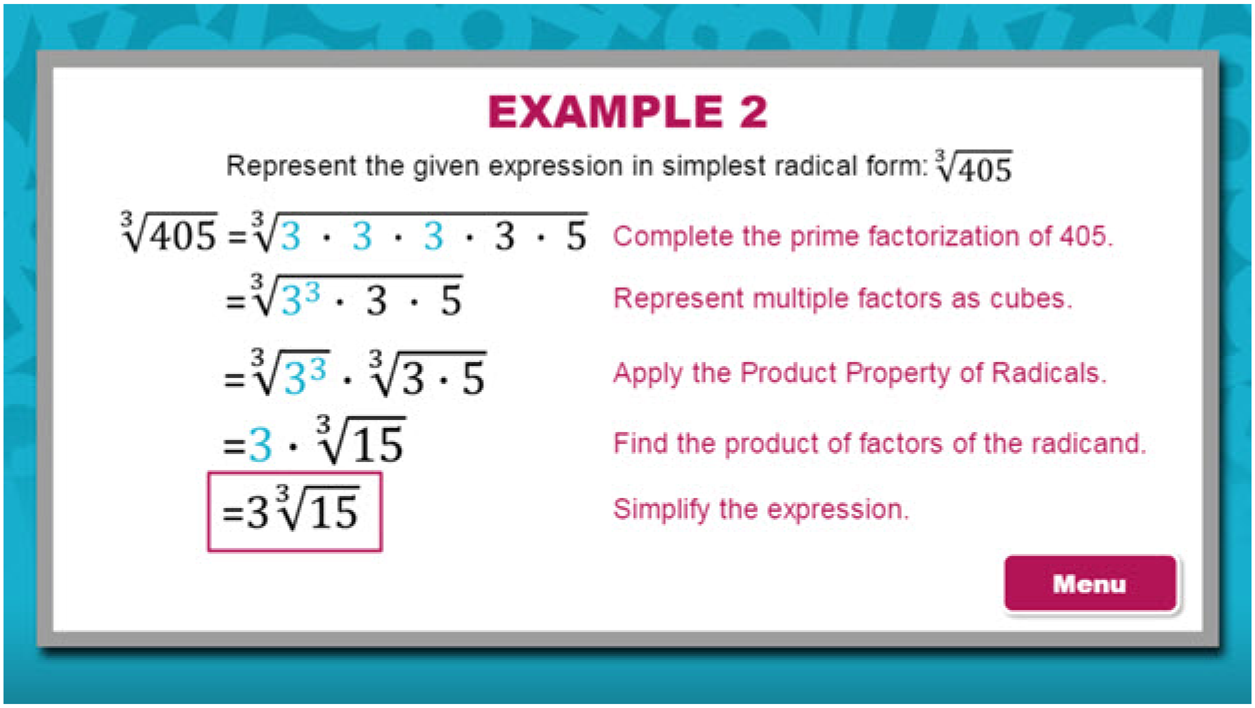
Example 2 (continued)



Therefore, in simplest radical form $\sqrt[3]{405}$ is equivalent to…

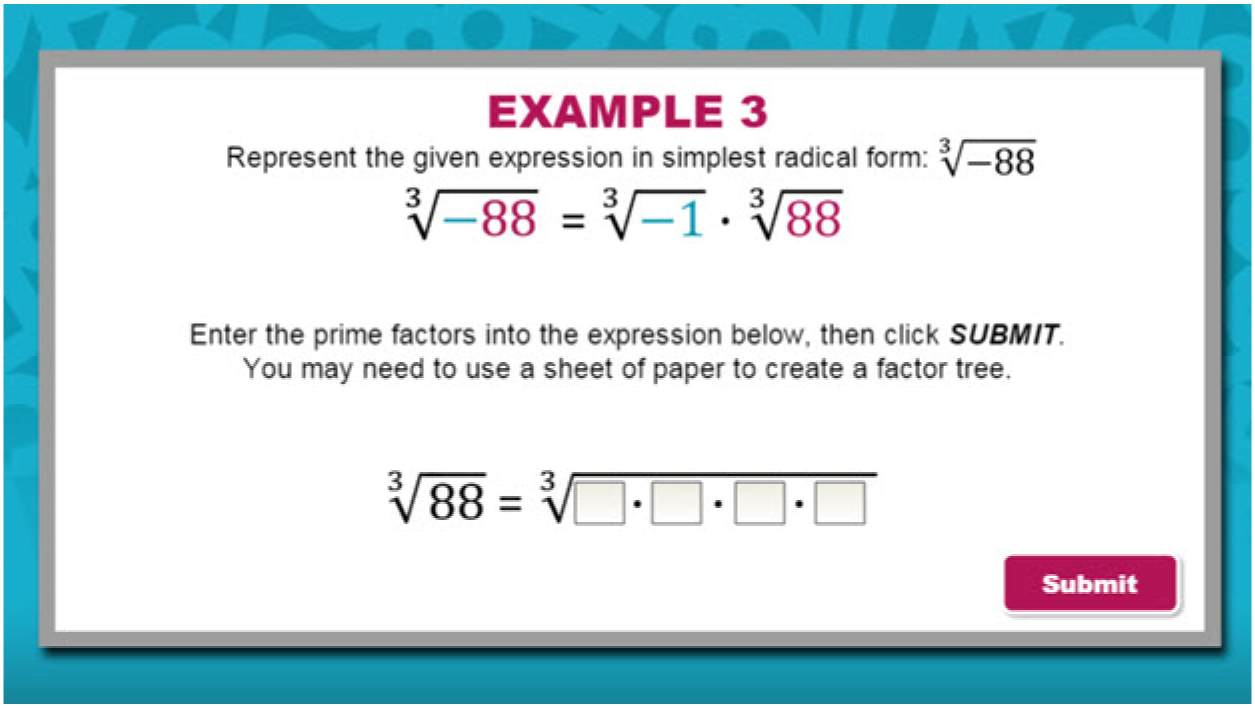
A) $15\sqrt[3]{3}$
B) $3\sqrt[3]{15}$
C) 45

Example 2 (continued)



$\sqrt[3]{405} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}$	Complete the prime factorization of 405.
$= \sqrt[3]{3^3 \cdot 3 \cdot 5}$	Represent multiple factors as cubes.
$= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 5}$	Apply the Product Property of Radicals
$= 3 \cdot \sqrt[3]{15}$	Find the product of the factors of the radicand.
$= 3\sqrt[3]{15}$	Simplify the expression.

Example 3



Represent the given expression in simplest radical form: $\sqrt[3]{-88}$

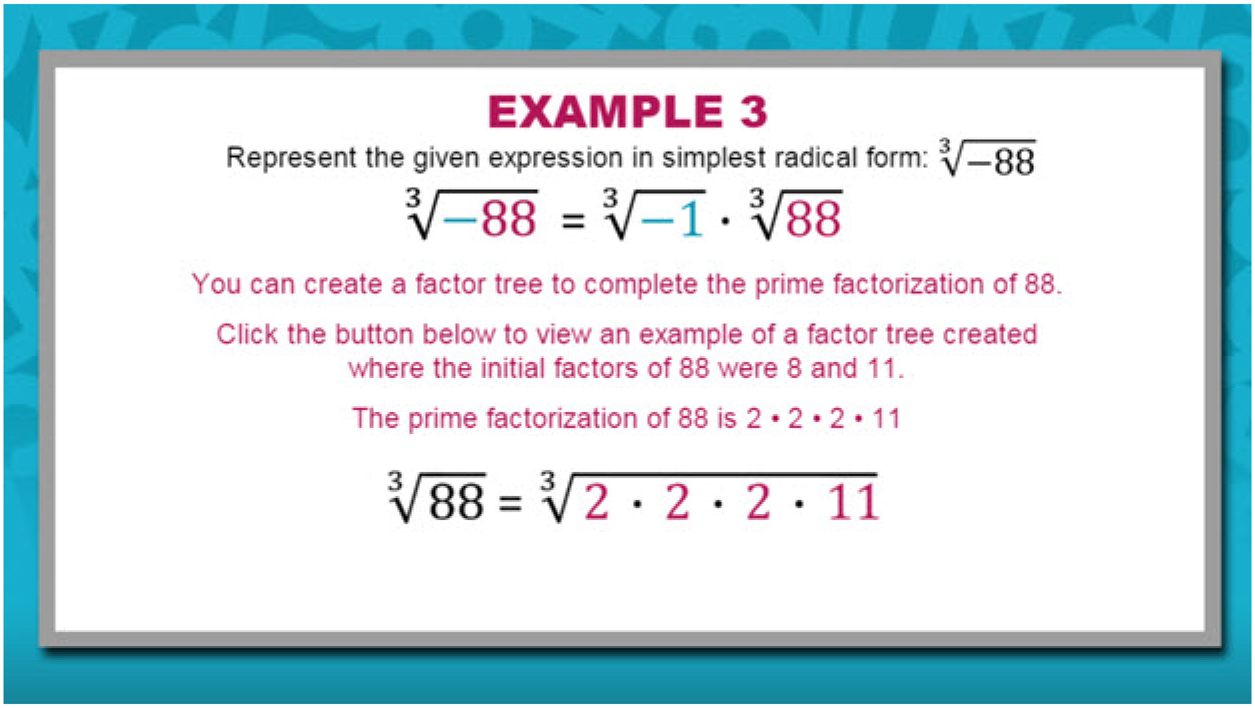
When simplifying the cube root of a negative number, you can be certain that one of the factors of the radicand is -1. So in the given expression, start by representing $\sqrt[3]{-88}$ as the product of $\sqrt[3]{-1}$ and $\sqrt[3]{88}$.

Next, begin to simplify $\sqrt[3]{88}$ by representing the radicand as the product of its prime factors.

$\sqrt[3]{88} = \sqrt[3]{? \cdot ? \cdot ? \cdot ?}$

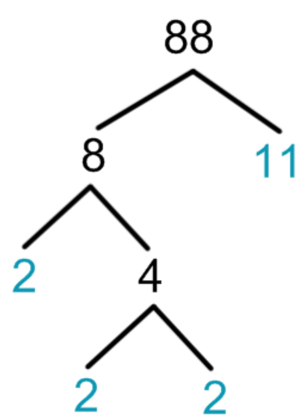
Enter the prime factors into the expression above, then click ***SUBMIT***. You may need to use a sheet of paper to create a factor tree.

Example 3 (continued)



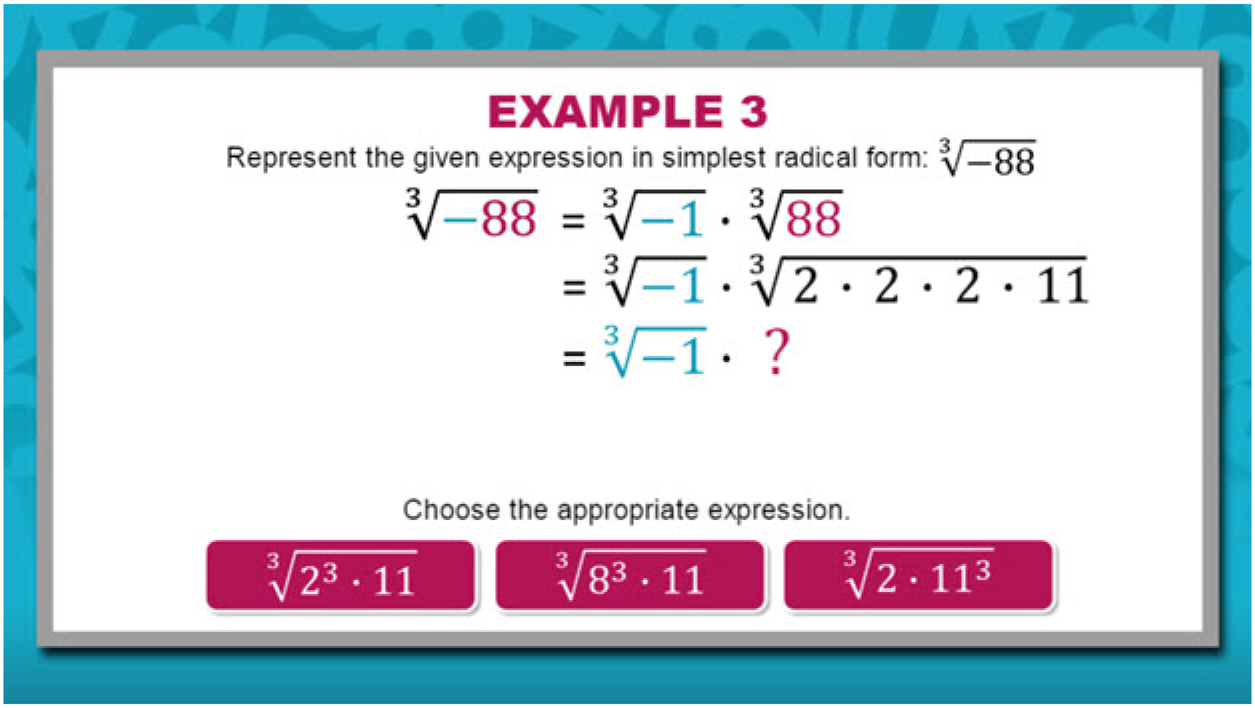
$$\sqrt[3]{88} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11}$$

You can create a factor tree to complete the prime factorization of 88. Take a look at the example of a factor tree created when the initial factors of 88 were 8 and 11.



The prime factorization of 88 is $2 \cdot 2 \cdot 2 \cdot 11$.

Example 3 (continued)



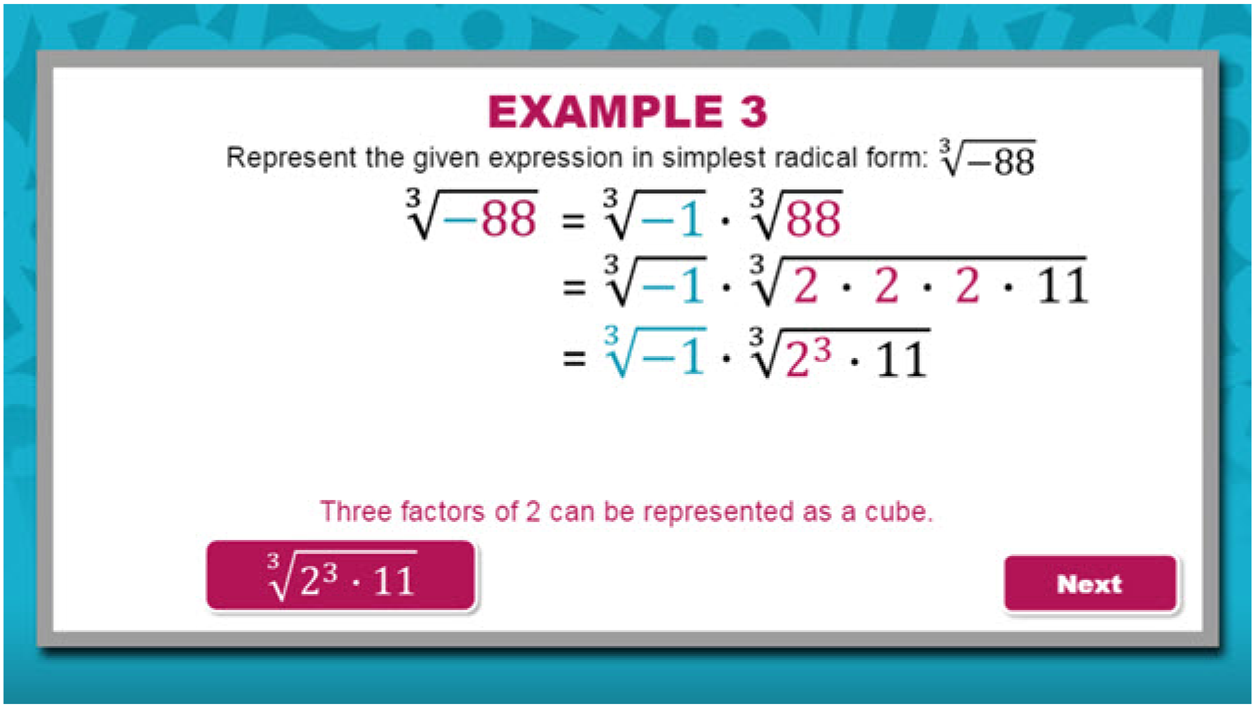
Now identify multiple factors that can be rewritten as a cube.

$$\sqrt[3]{88} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} = ?$$

Choose the appropriate expression.

A) $\sqrt[3]{2 \cdot 11^3}$
B) $\sqrt[3]{8^3 \cdot 11}$
C) $\sqrt[3]{2^3 \cdot 11}$

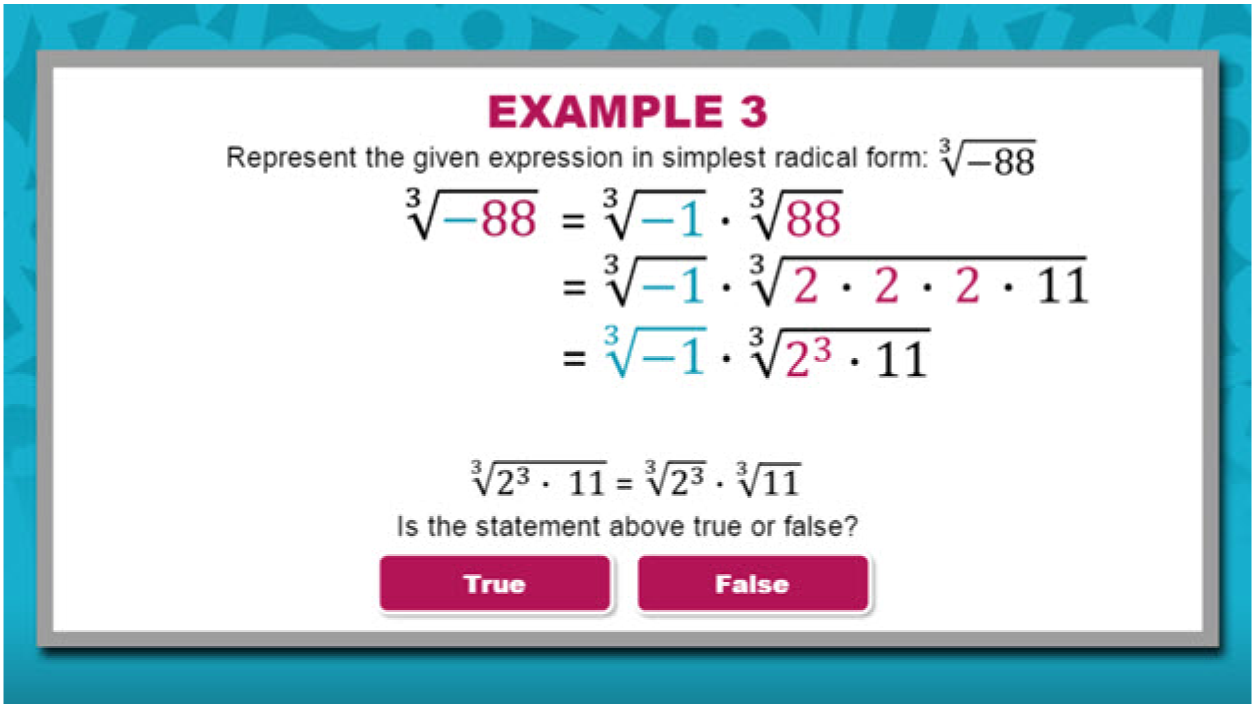
Example 3 (continued)



$$\begin{aligned}\sqrt[3]{88} &= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\ &= \sqrt[3]{2^3 \cdot 11}\end{aligned}$$

Three factors of 2 can be represented as a cube.

Example 3 (continued)



$$
\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11}
\end{aligned}
$$

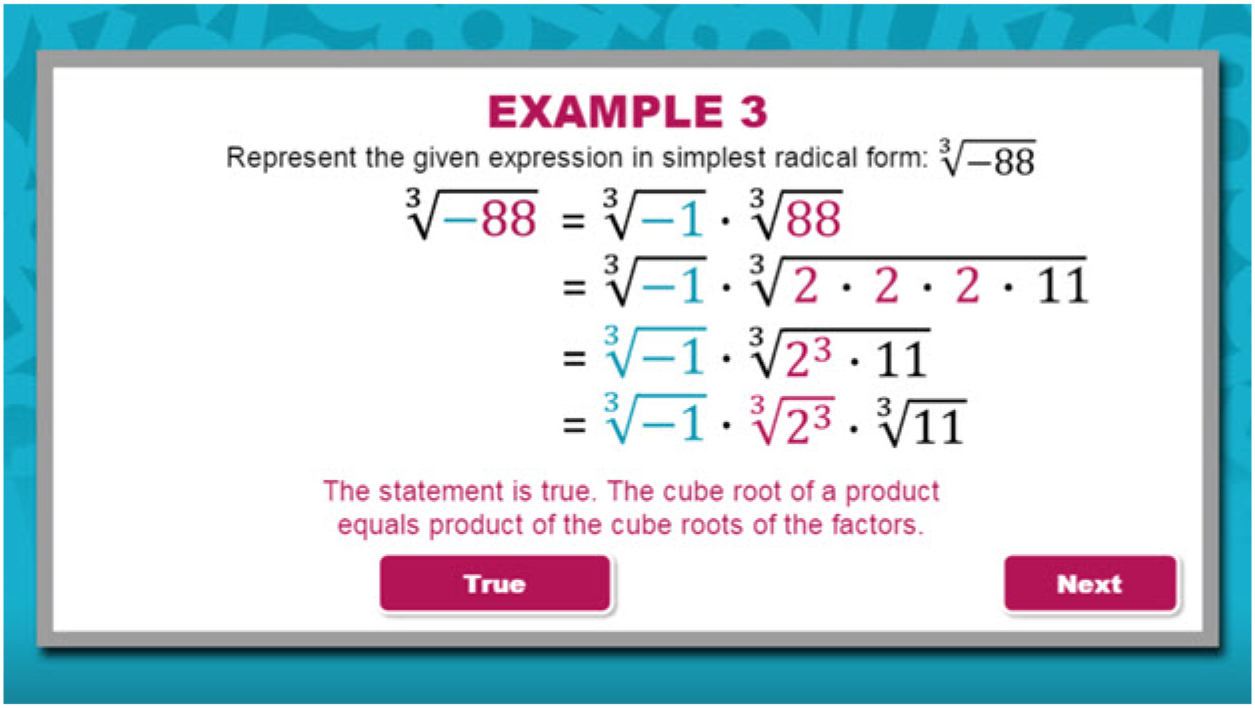
Next apply the Product Property of Radicals to represent the cube root of the product as the product of the cube roots of the factors.

$$\sqrt[3]{2^3 \cdot 11} = \sqrt[3]{2^3} \cdot \sqrt[3]{11}$$

Is the expression above true or false?

A) True
B) False

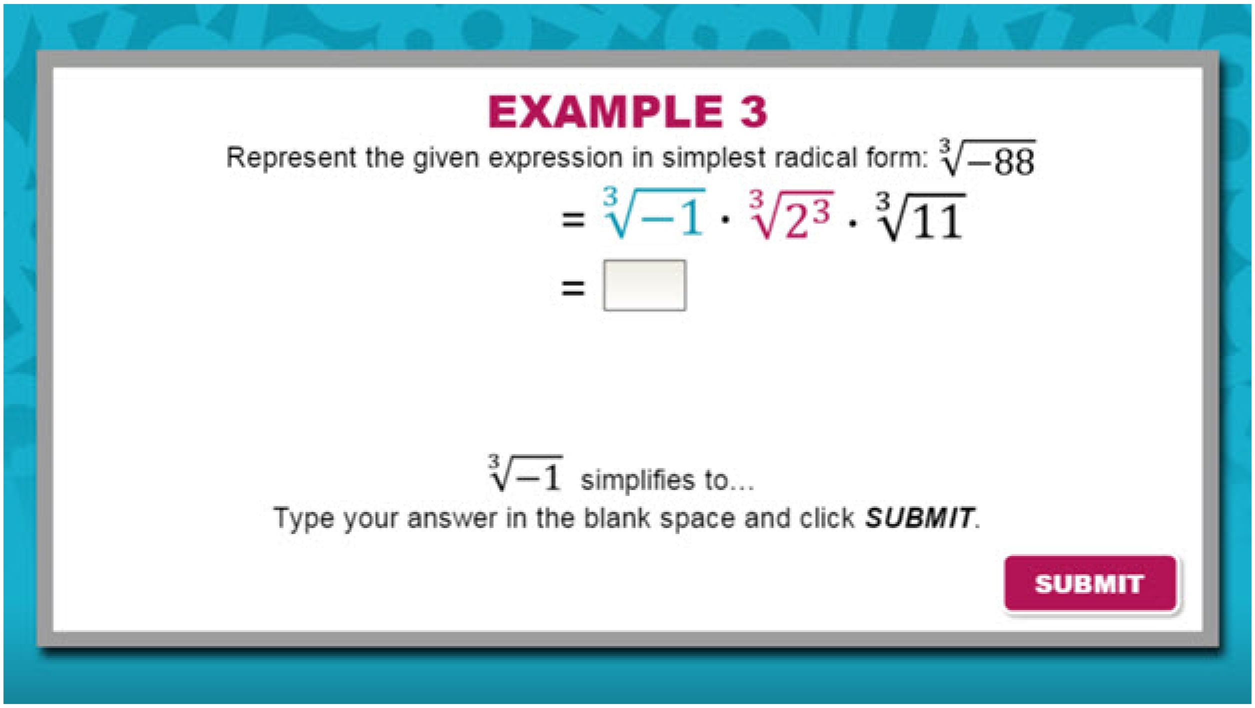
Example 3 (continued)



$$
\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11}
\end{aligned}
$$

The expression is true. The cube root of a product is equal to the product of the cube roots of the factors

Example 3 (continued)



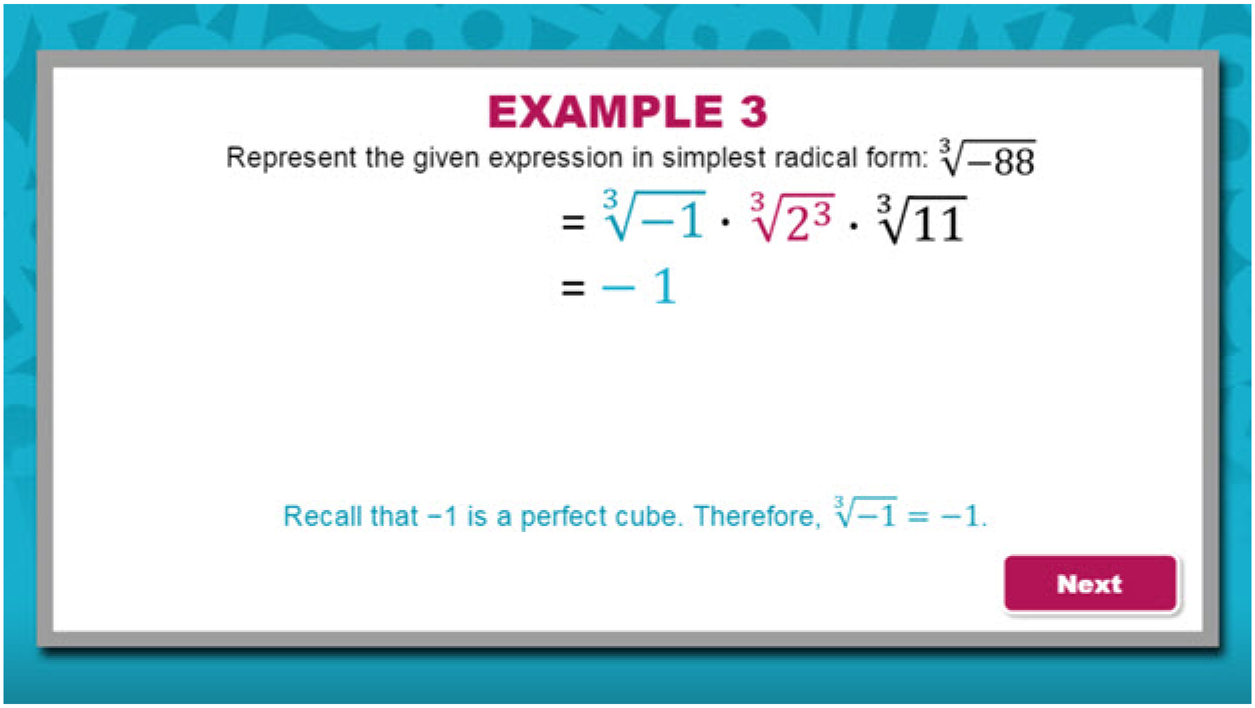
$$
\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11} \\
&= ?
\end{aligned}
$$

Now simplify the expression.

$\sqrt[3]{-1}$ simplifies to …

Type your answer in the blank space and click ***SUBMIT***.

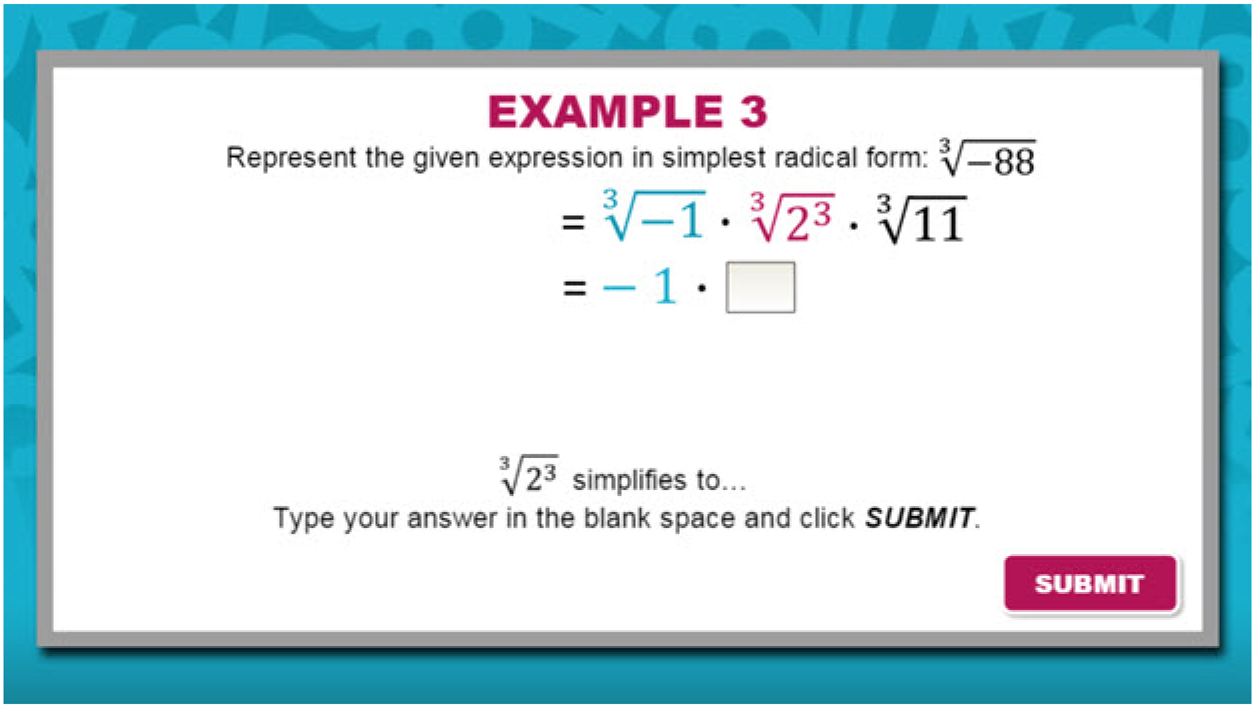
Example 3 (continued)



$$\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11} \\
&= -1
\end{aligned}$$

Recall that -1 is a perfect cube. Therefore, $\sqrt[3]{-1} = -1$.

Example 3 (continued)

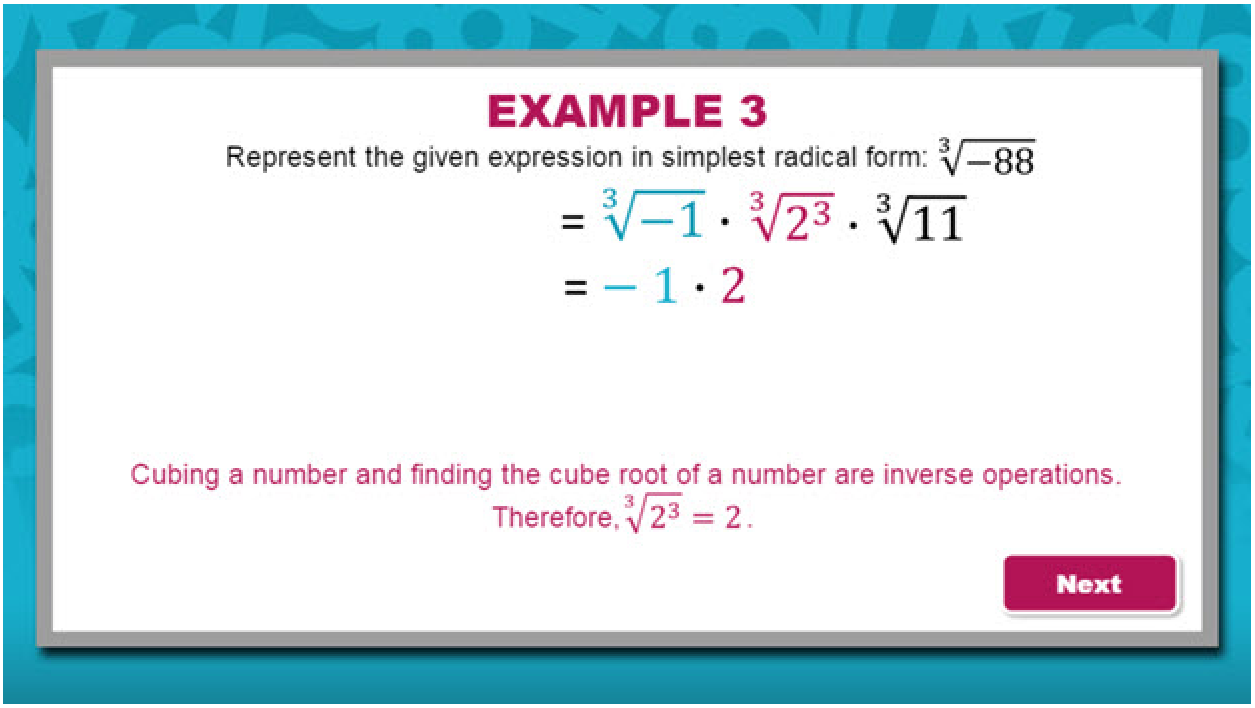


$$
\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11} \\
&= -1 \cdot ?
\end{aligned}
$$

$\sqrt[3]{2^3}$ simplifies to …

Type your answer in the blank space and click ***SUBMIT***.

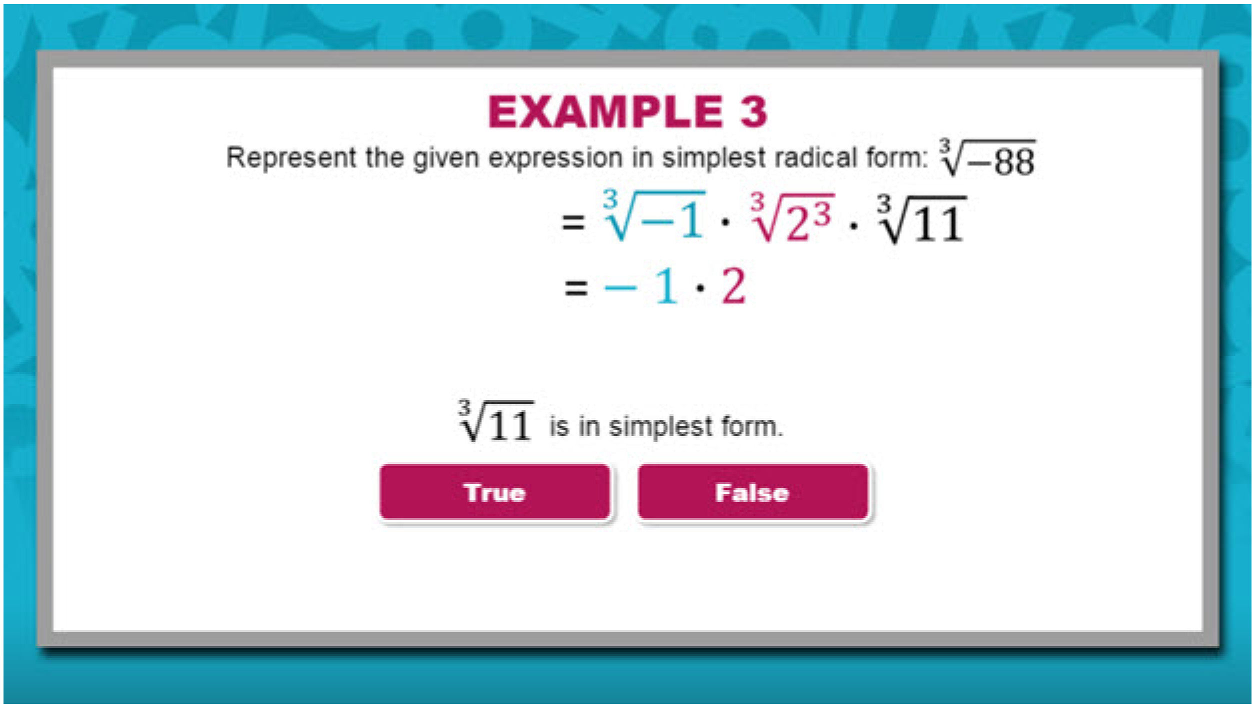
Example 3 (continued)



$$\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11} \\
&= -1 \cdot 2
\end{aligned}$$

Cubing a number and finding the cube root of a number are inverse operations. Therefore, $\sqrt[3]{2^3} = 2$.

Example 3 (continued)

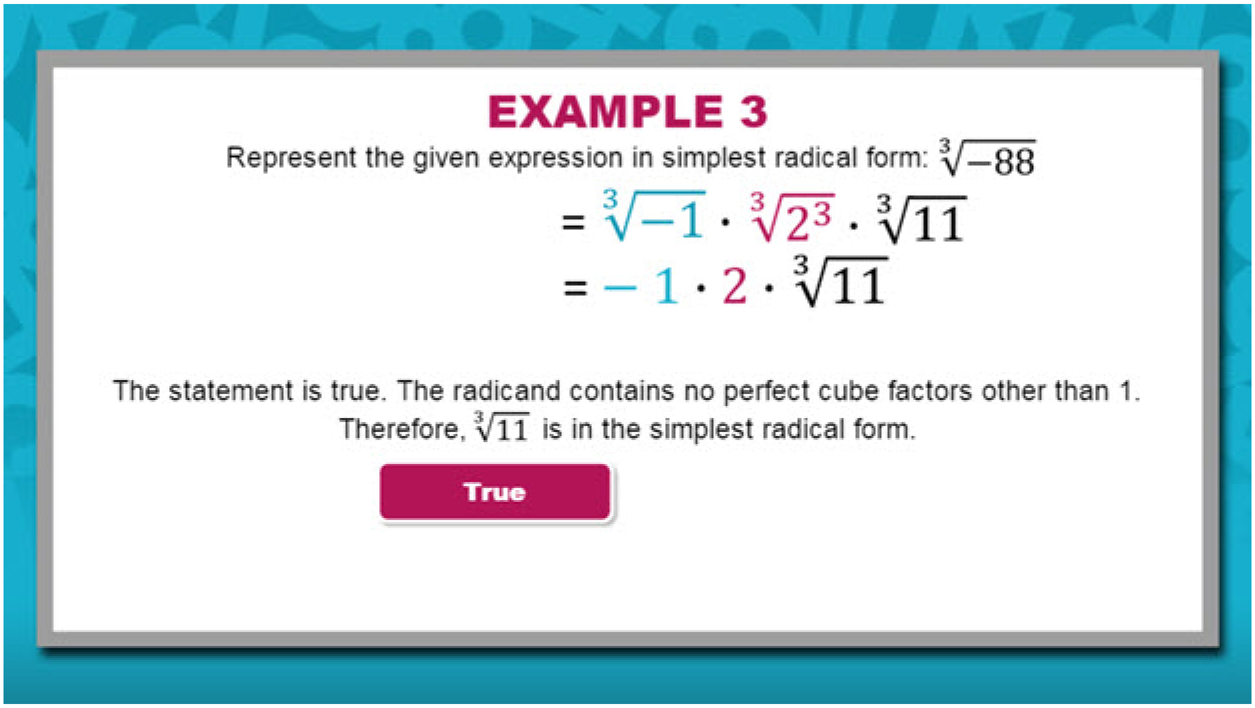


$$\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11} \\
&= -1 \cdot 2
\end{aligned}$$

$\sqrt[3]{11}$ is in simplest form.

A) True
B) False

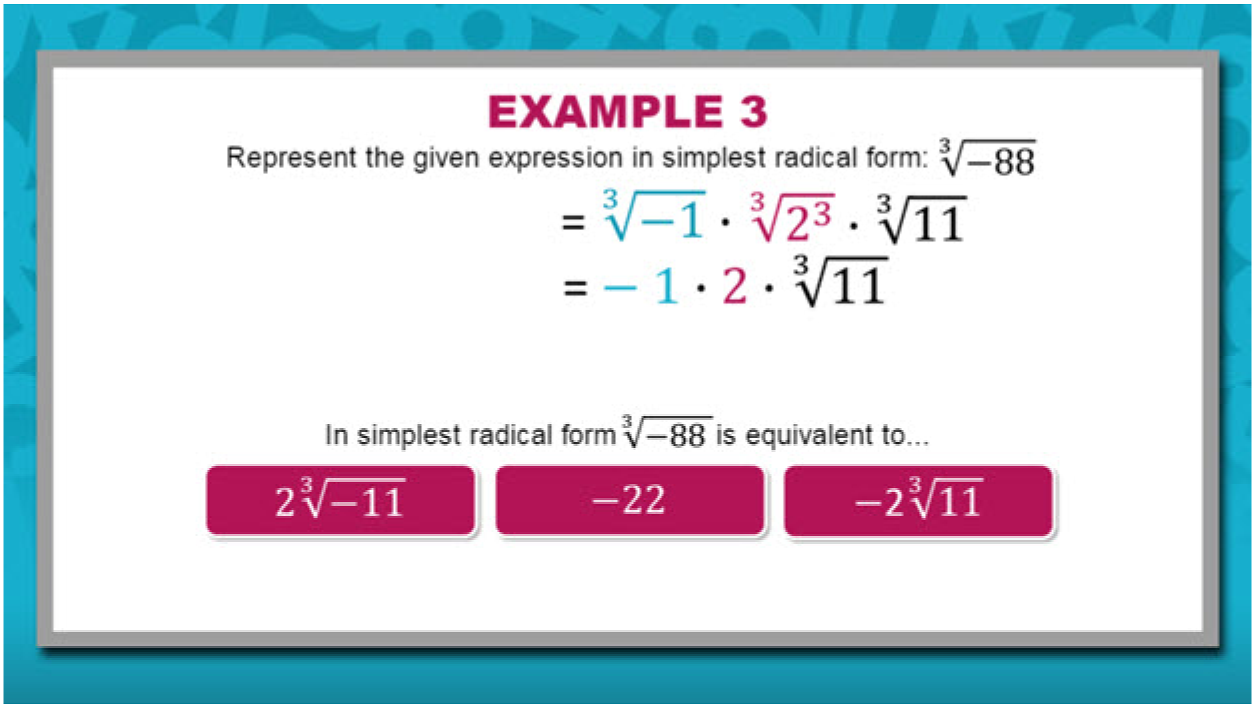
Example 3 (continued)



$$
\begin{aligned}
\sqrt[3]{-88} &= \sqrt[3]{-1} \cdot \sqrt[3]{88} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3 \cdot 11} \\
&= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11} \\
&= -1 \cdot 2 \cdot \sqrt[3]{11}
\end{aligned}
$$

The statement is true. The radicand contains no perfect cube factors other than 1. Therefore, $\sqrt[3]{11}$ is in simplest radical form.

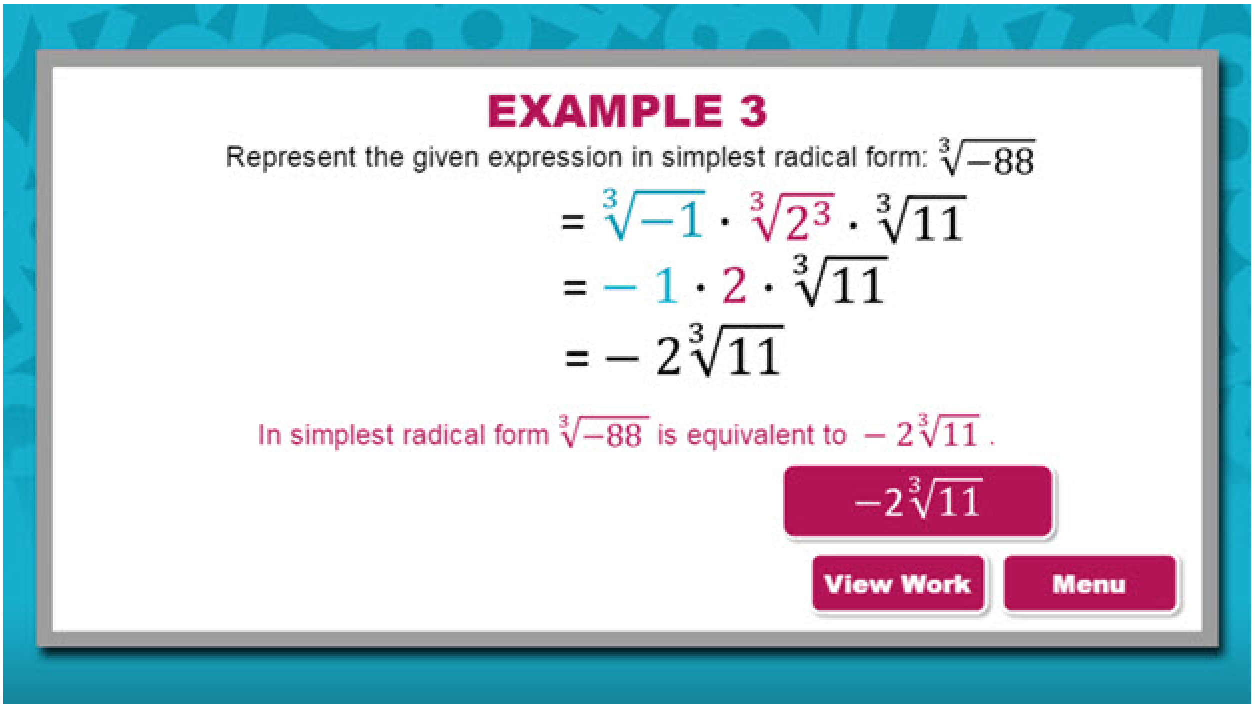
Example 3 (continued)



In simplest radical form $\sqrt[3]{-88}$ is equivalent to…

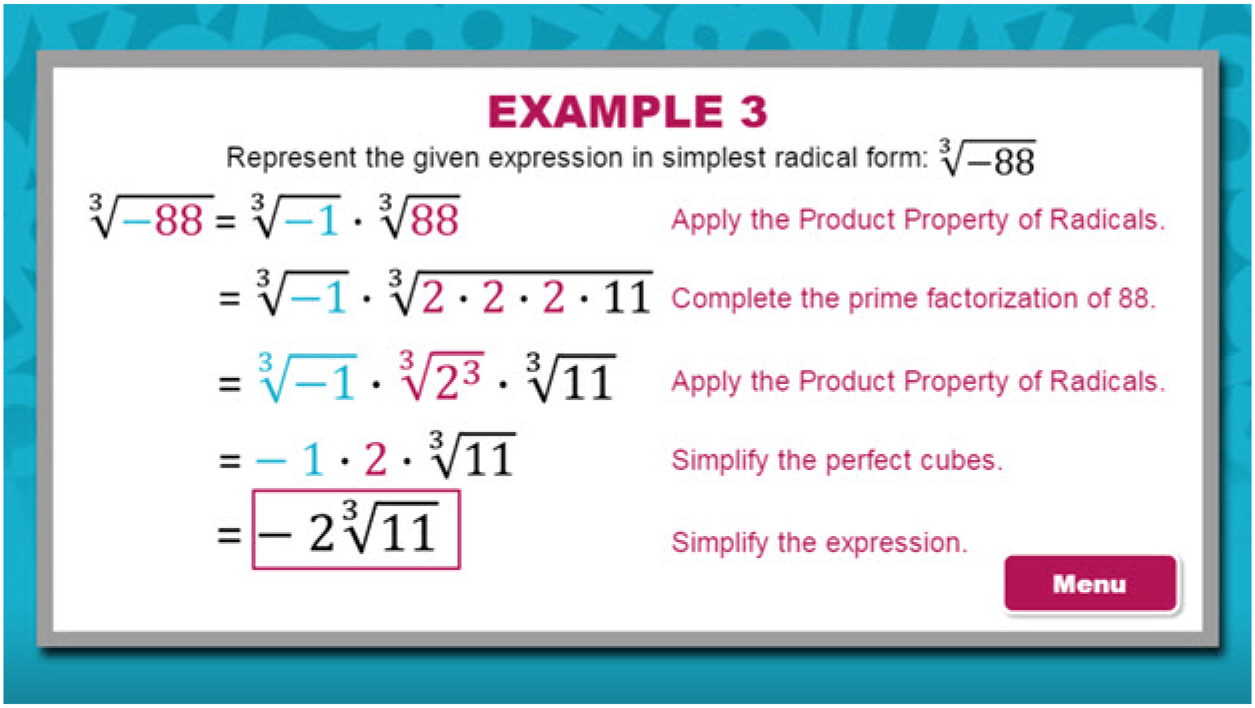
A) $-2\sqrt[3]{11}$
B) -22
C) $2\sqrt[3]{-11}$

Example 3 (continued)



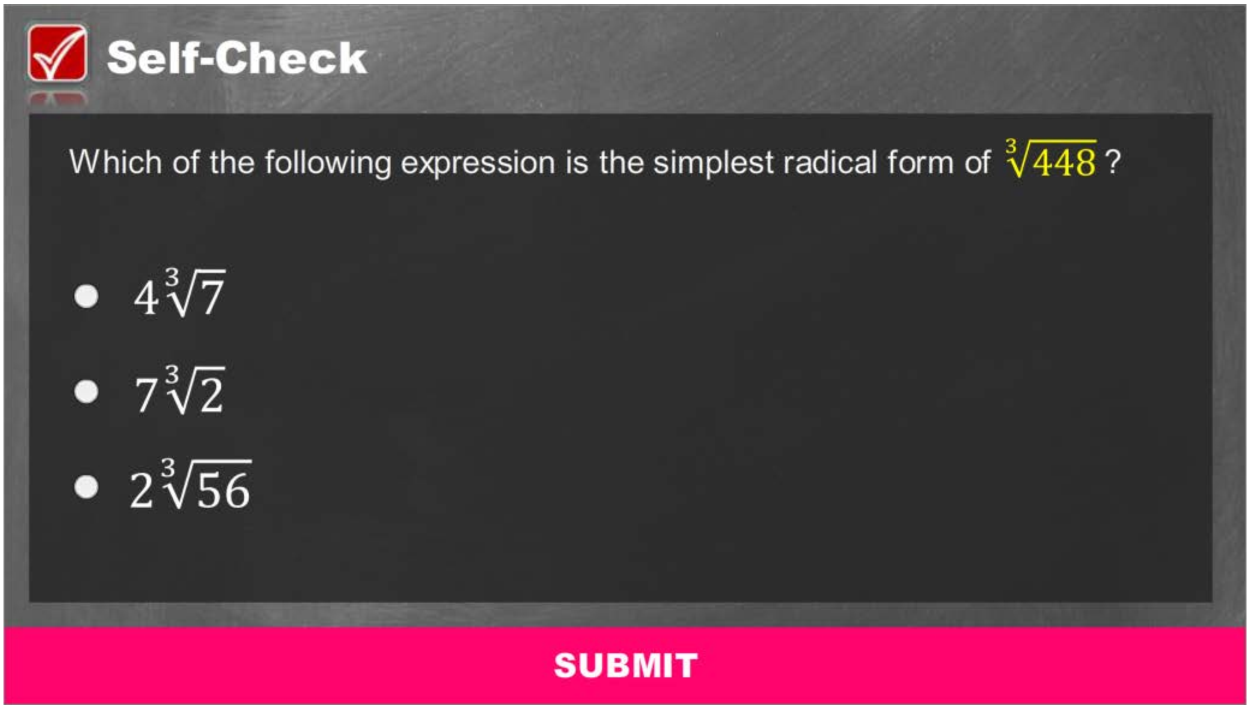
In simplest radical form $\sqrt[3]{-88}$ is equivalent to $-2\sqrt[3]{11}$.

Example 3 (continued)



$\sqrt[3]{-88} = \sqrt[3]{-1} \cdot \sqrt[3]{88}$ Apply the Product Property of Radicals.
$= \sqrt[3]{-1} \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 11}$ Complete the prime factorization of 88.
$= \sqrt[3]{-1} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{11}$ Apply the Product Property of Radicals.
$= -1 \cdot 2 \cdot \sqrt[3]{11}$ Simplify the perfect cubes.
$= -2\sqrt[3]{11}$ Simplify the expression.

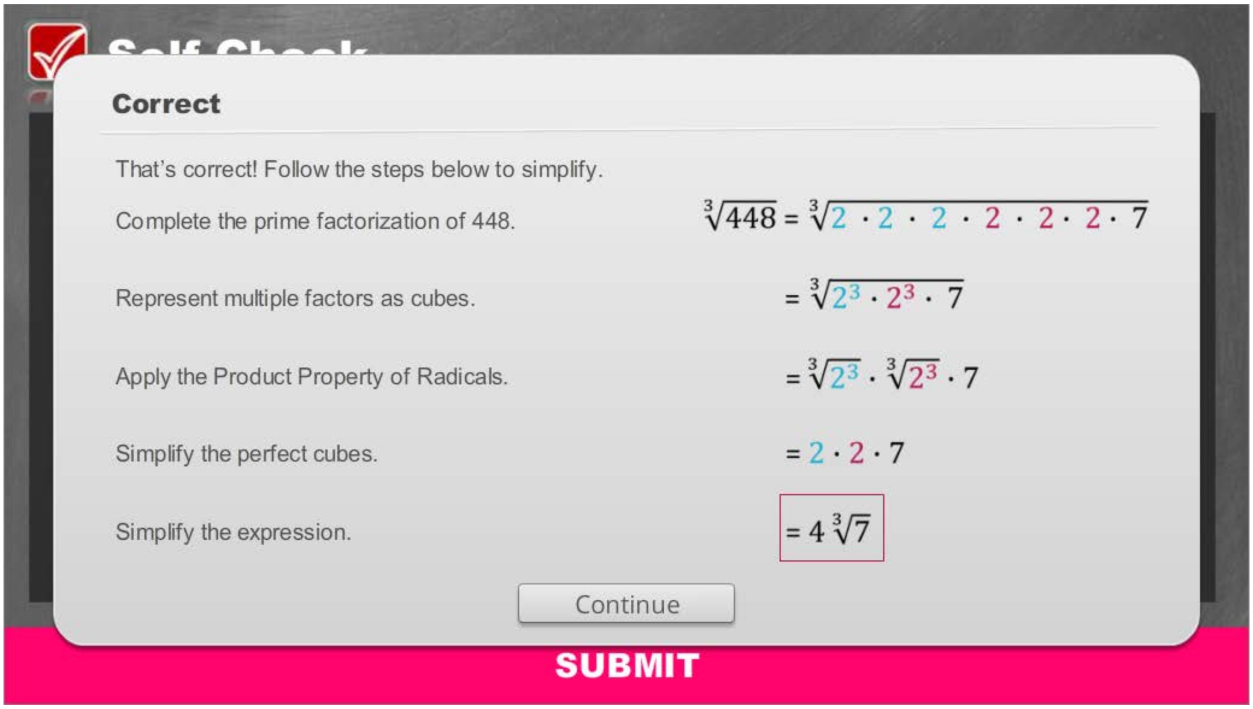
Self-Check 1



Solve the problem in the image above to check your understanding of the content.

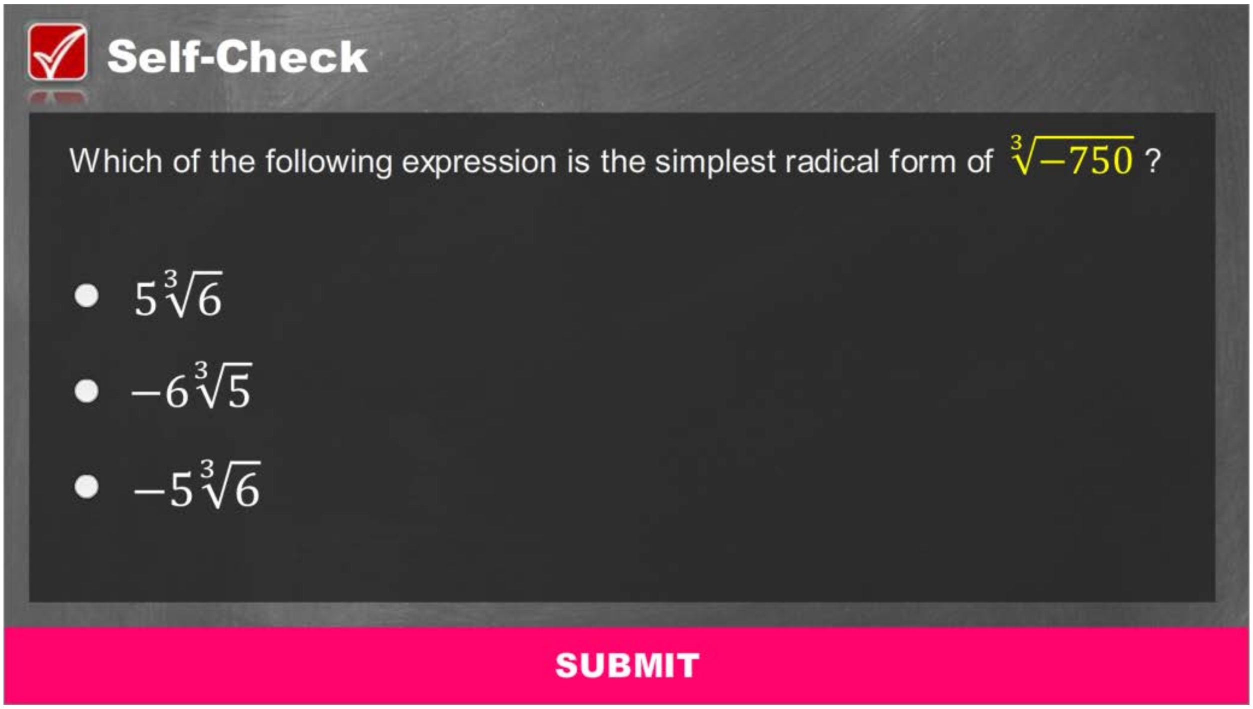
Module 4: Radical Expressions
Topic 1 Content: Simplifying Cube Roots of Integers

Self-Check 1: Answer



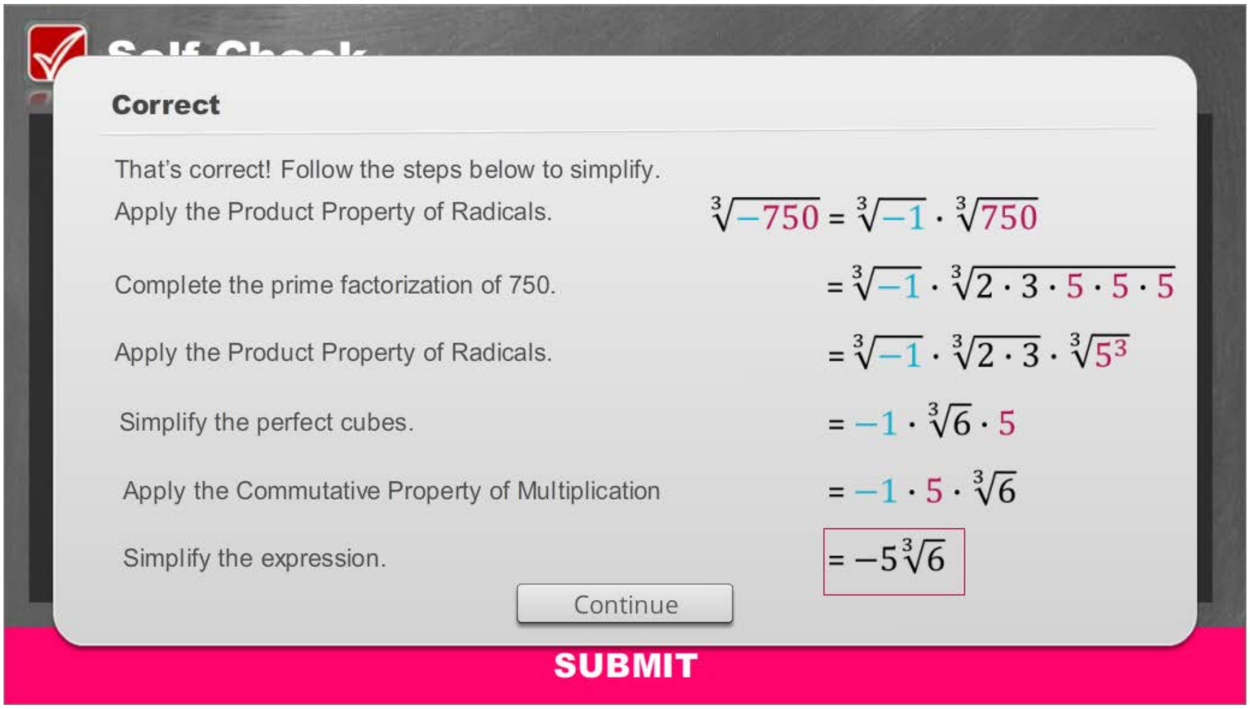
For your reference, the image above shows the correct solution to the self-check problem.

Self-Check 2



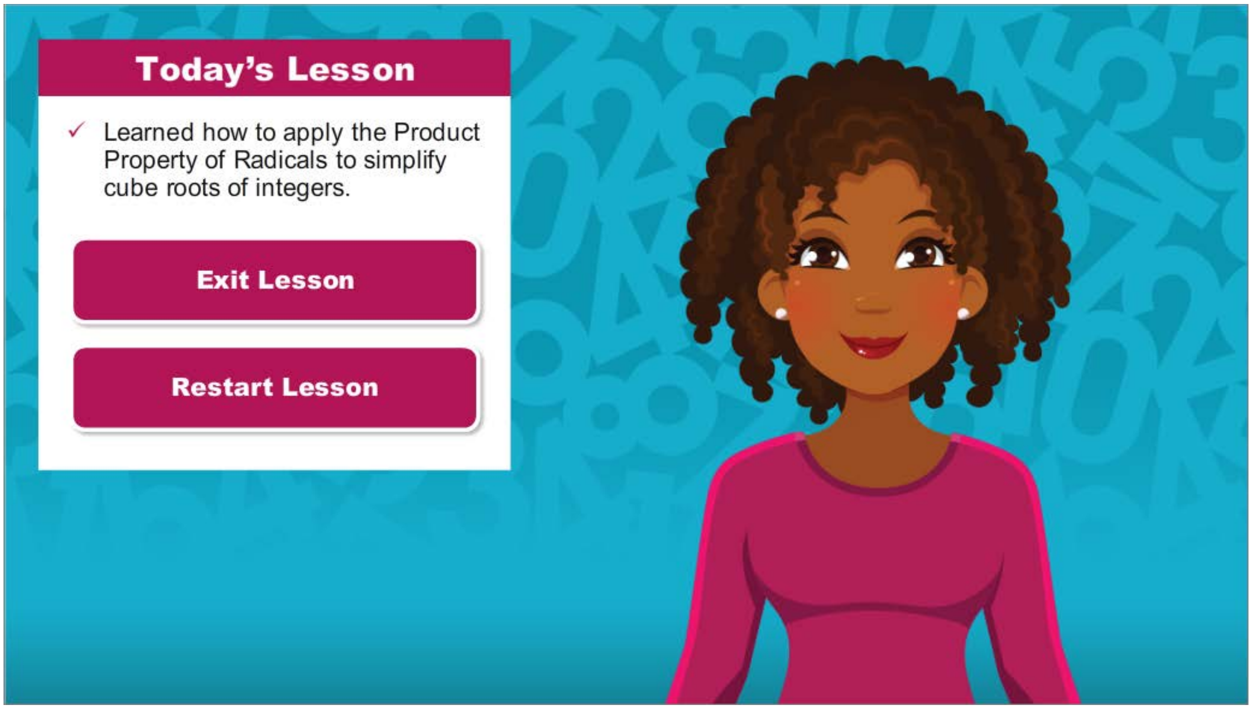
Solve the problem in the image above to check your understanding of the content.

Self-Check 2: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



You have reached the conclusion of this lesson where you learned how to apply the Product Property of Radicals to simplify cube roots of integers.