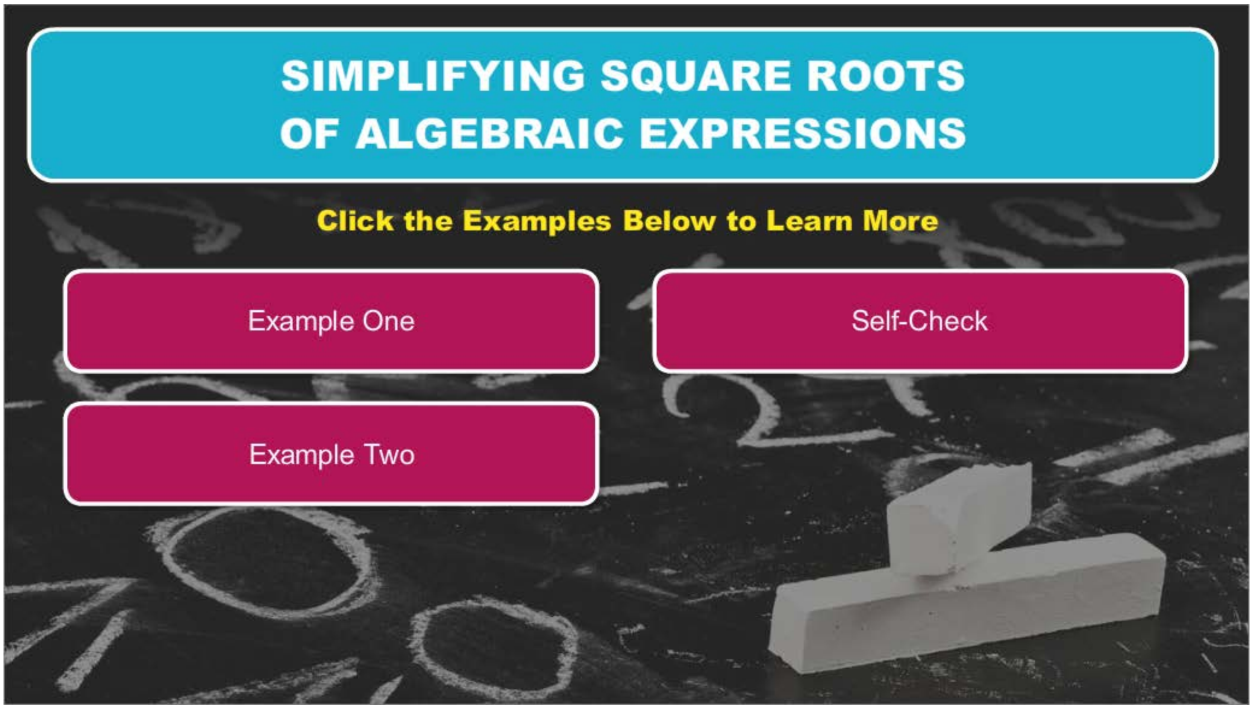
Module 4: Radical Expressions

Topic 1 Content: Simplifying Square Roots of Algebraic Expressions

Introduction

Hi there! I'm so glad you could join me for this lesson in Algebra I, where you will learn how to simplify square roots of monomial expressions. The Product Property of Radicals will be the key to successfully completing this lesson.

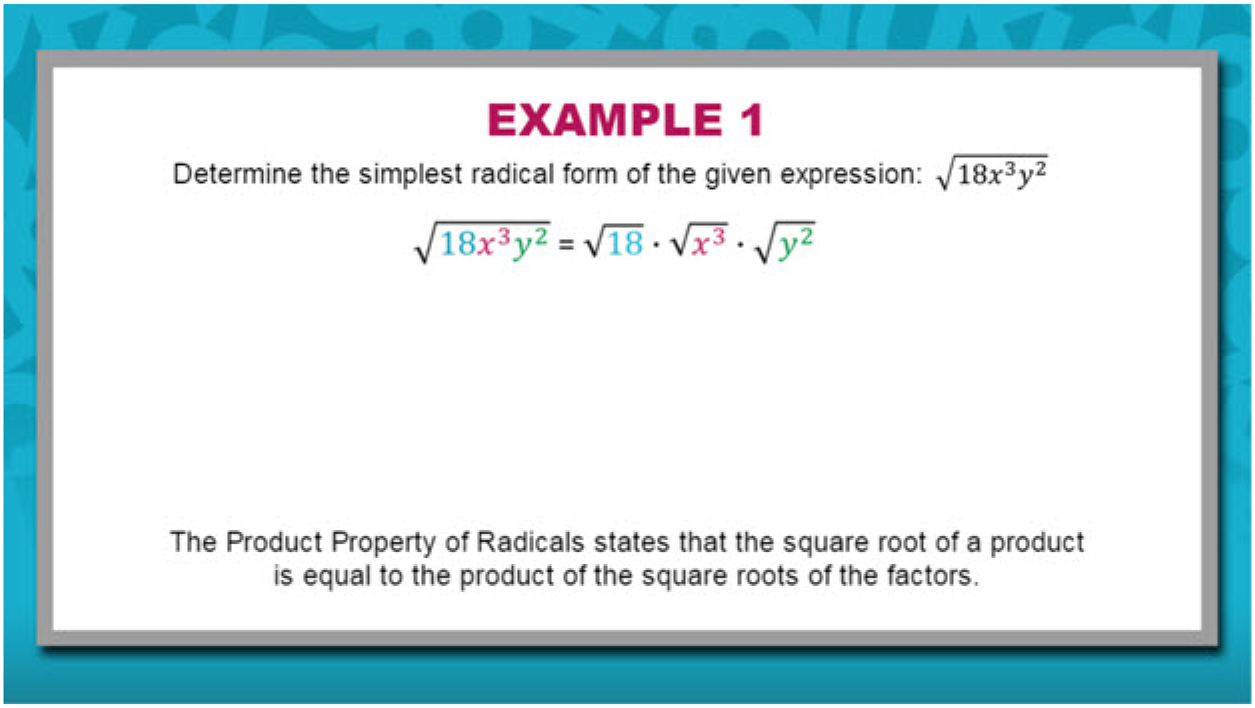
Simplifying Square Roots of Algebraic Expressions



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Example 1

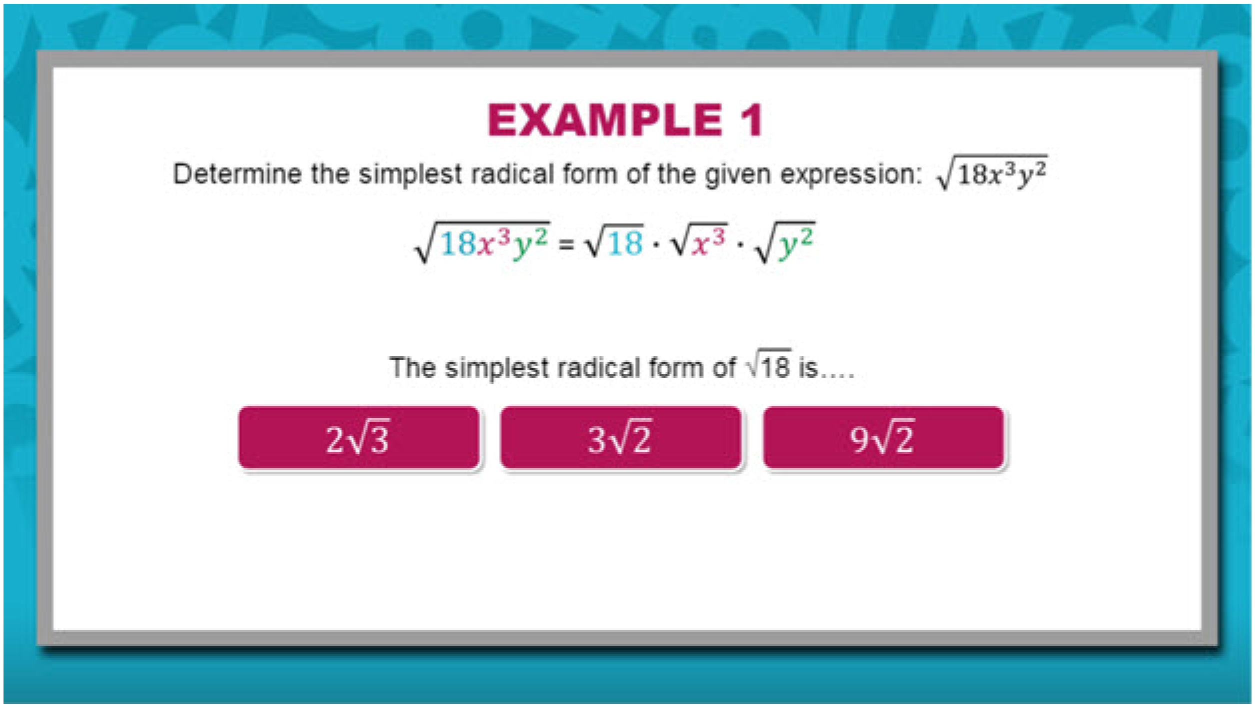


Determine the simplest radical form of the given expression.

$$\sqrt{18x^3y^2}$$

The radicand of the given expression is a monomial consisting of the product of 18, x^3, and y^2. Recall that the Product Property of Radicals states that the square root of a product is equal to the product of the square roots of the factors. This means that $\sqrt{18x^3y^2}$ can be represented as the product of $\sqrt{18}$, $\sqrt{x^3}$, and $\sqrt{y^2}$.

Example 1 (continued)

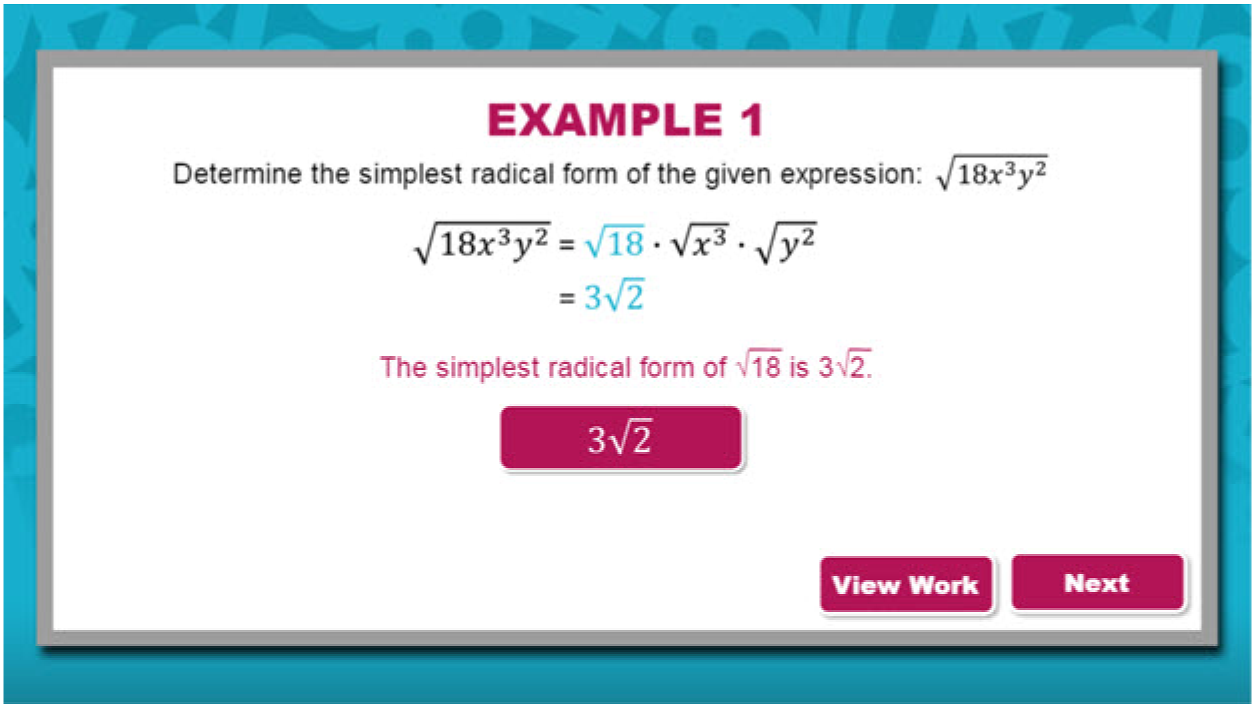


The next step in simplifying the expression is to simplify each square root factor. Begin with $\sqrt{18}$.

The simplest radical form of $\sqrt{18}$ is ...

A) $2\sqrt{3}$
B) $3\sqrt{2}$
C) $9\sqrt{2}$

Example 1 (continued)

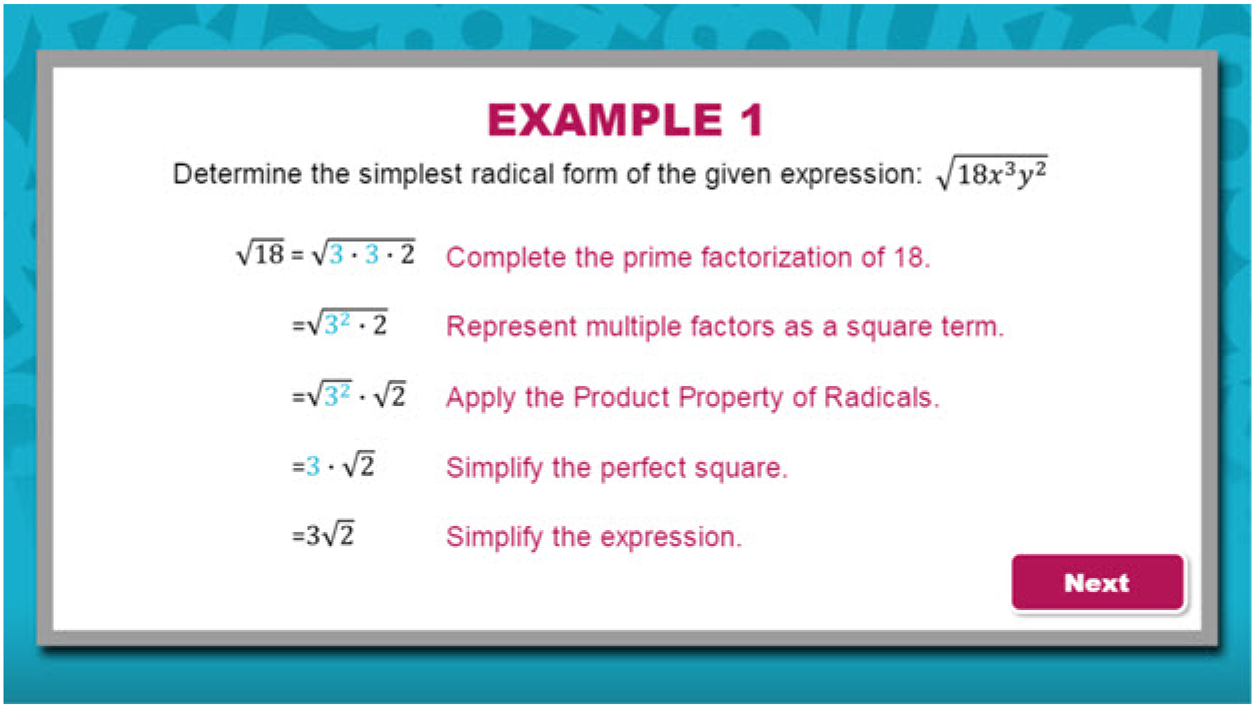


$$
\begin{aligned}
\sqrt{18x^3y^2} &= \sqrt{18} \cdot \sqrt{x^3} \cdot \sqrt{y^2} \\
&= 3\sqrt{2}
\end{aligned}
$$

The simplest radical form of $\sqrt{18}$ is $3\sqrt{2}$.

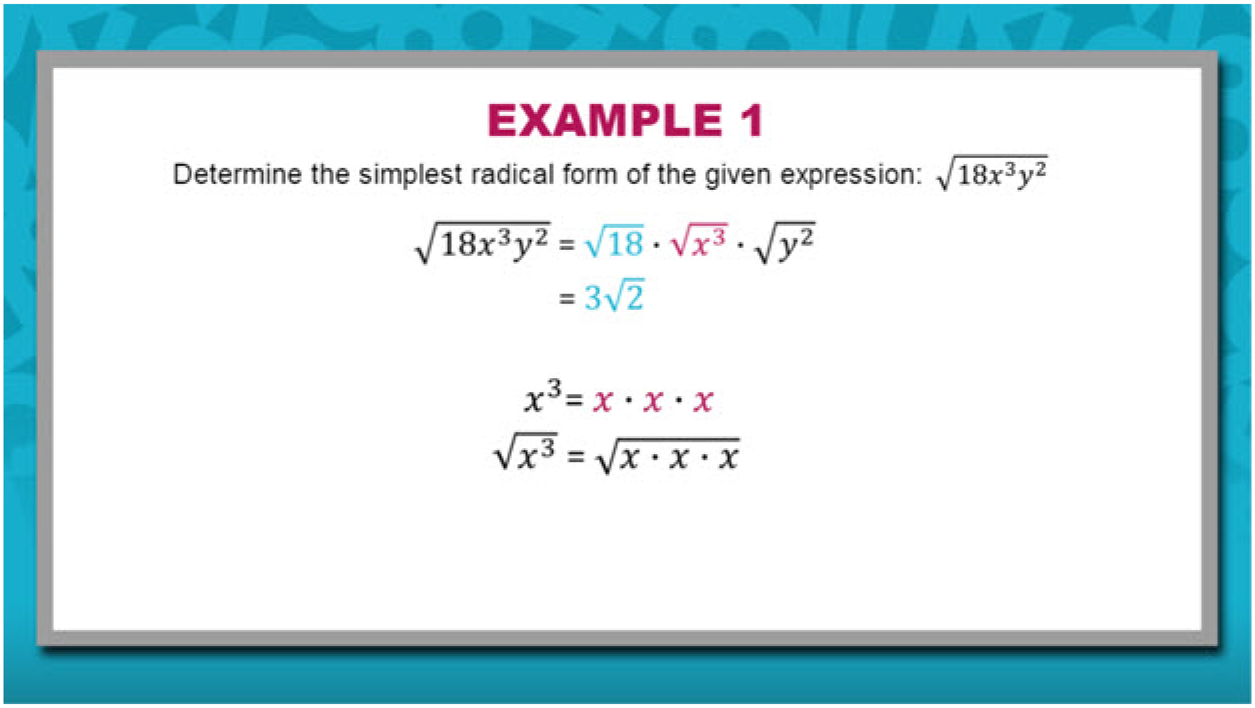
You can now represent $\sqrt{18}$ as $3\sqrt{2}$.

Example 1 (continued)



$\sqrt{18} = \sqrt{3 \cdot 3 \cdot 2}$	Complete the prime factorization of 18.
$= \sqrt{3^2 \cdot 2}$	Represent multiple factors as a square term.
$= \sqrt{3^2} \cdot \sqrt{2}$	Apply the Product Property of Radicals.
$= 3 \cdot \sqrt{2}$	Simplify the perfect square.
$= 3\sqrt{2}$	Simplify the expression.

Example 1 (continued)

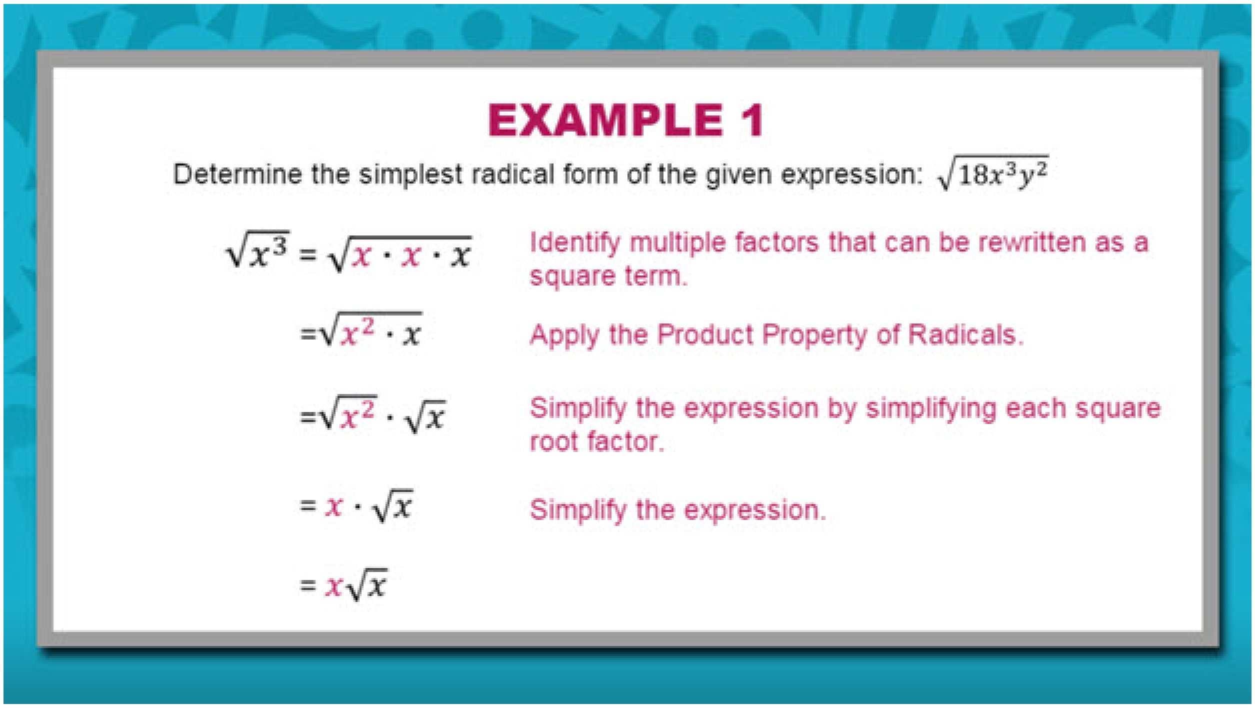


Now that you know that $\sqrt{18}$ simplifies to $3\sqrt{2}$, you can move on and simplify $\sqrt{x^3}$. Even though the radicand is an algebraic expression, the process to simplifying it is the same as simplifying a numerical expression.

Begin by representing x^3 as the product of its prime factors. $x^3 = x \cdot x \cdot x$

Now that you know the prime factorization of x^3, you can represent $\sqrt{x^3}$ as $\sqrt{x \cdot x \cdot x}$.

Example 1 (continued)



The next step is to identify multiple factors that can be rewritten as a square term; $x \cdot x$ can be represented as x^2.

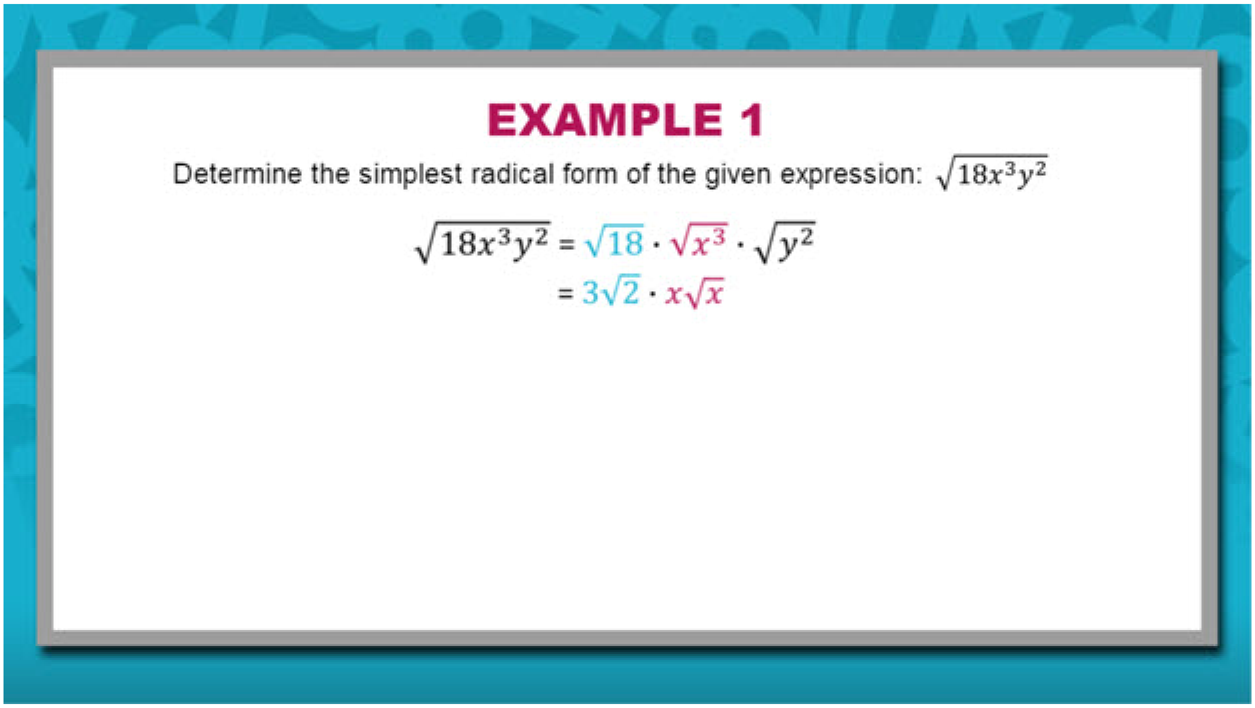
So, $\sqrt{x^3}$ can be represented as $\sqrt{x^2 \cdot x}$.

Next, apply the Product Property of Radicals to represent $\sqrt{x^2 \cdot x}$ as $\sqrt{x^2} \cdot \sqrt{x}$.

Now, simplify the expression. Begin with the first square root factor. Recall that squaring a term and finding the square root of a term are inverse operations. This means that $\sqrt{x^2}$ is x. Move on to the second square root factor. The radicand contains no perfect square factors other than 1. Therefore, $\sqrt{x}$ is in simplest radical form.

So, $\sqrt{x^3}$ can be represented as $x \cdot \sqrt{x}$. After simplifying the expression, you find that the simplest radical form of $\sqrt{x^3}$ is $x\sqrt{x}$.

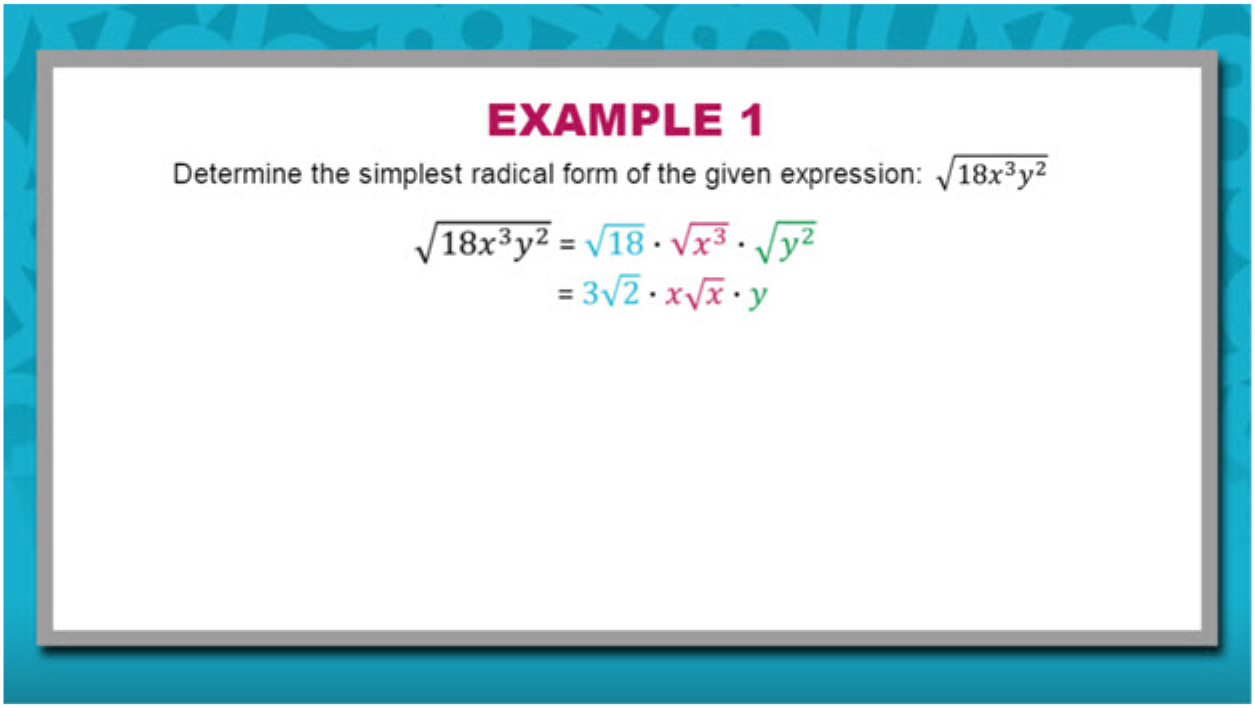
Example 1 (continued)



$$\begin{aligned} \sqrt{18x^3y^2} &= \sqrt{18} \cdot \sqrt{x^3} \cdot \sqrt{y^2} \\ &= 3\sqrt{2} \cdot x\sqrt{x} \end{aligned}$$

In the expression, you can now represent $\sqrt{x^3}$ as $x\sqrt{x}$.

Example 1 (continued)

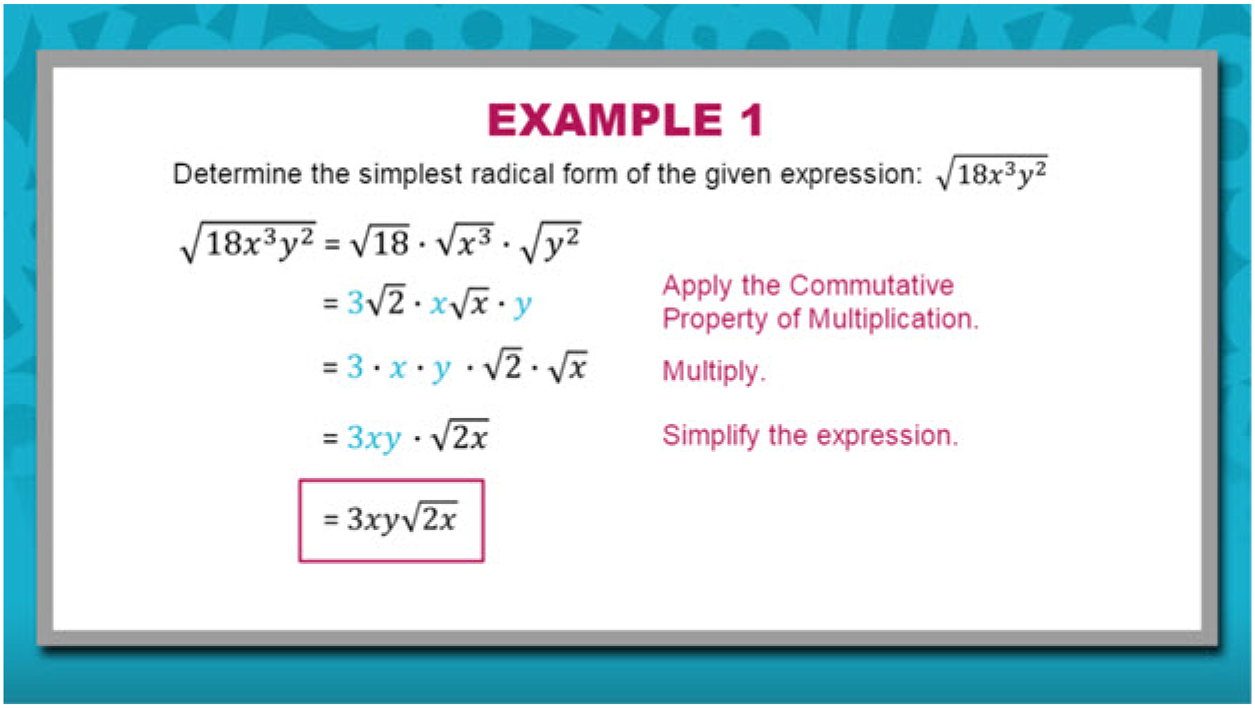


Move on to simplify $\sqrt{y^2}$. The radicand only consists of a square term. So you can immediately simplify the expression. Squaring a term and finding the square root of a term are inverse operations. This means that $\sqrt{y^2}$ is y.

$$\begin{aligned}\sqrt{18x^3y^2} &= \sqrt{18} \cdot \sqrt{x^3} \cdot \sqrt{y^2} \\ &= 3\sqrt{2} \cdot x\sqrt{x} \cdot y\end{aligned}$$

In the expression, you can represent $\sqrt{y^2}$ as y. Now that you have simplified the square root factors, determine the product.

Example 1 (continued)

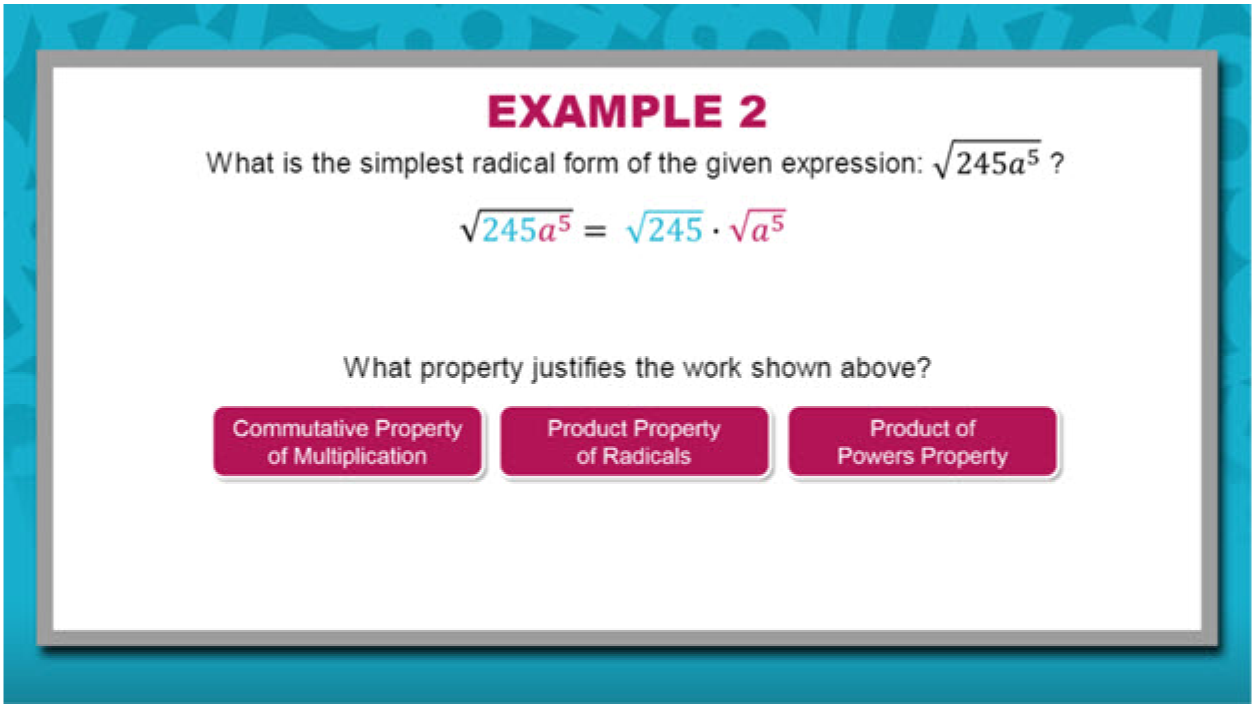


Recall that the Commutative Property of Multiplication allows you to change the order of multiplication. Apply the property in this example by multiplying 3, x, and y first. Then, multiply $\sqrt{2}$ and $\sqrt{x}$.

The product of 3, x, and y can be represented as $3xy$. The product of $\sqrt{2}$ and $\sqrt{x}$ can be represented as $\sqrt{2x}$.

Lastly, simplify the resulting expression. The simplest radical form of $\sqrt{18x^3y^2}$ is $3xy\sqrt{2x}$.

Example 2



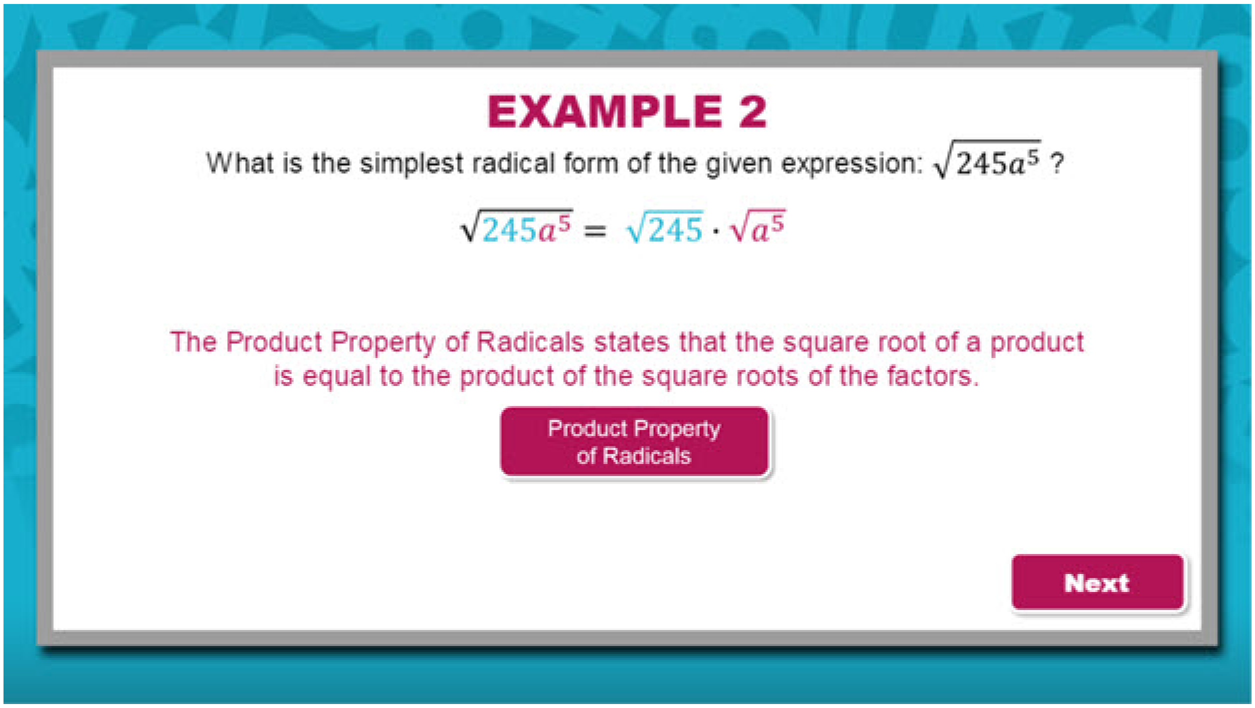
What is the simplest radical form of the given expression: $\sqrt{245a^5}$?

$$\sqrt{245a^5} = \sqrt{245} \cdot \sqrt{a^5}$$

What property justifies the work shown above?

A) Commutative Property of Multiplication
B) Product Property of Radicals
C) Product of Powers Property

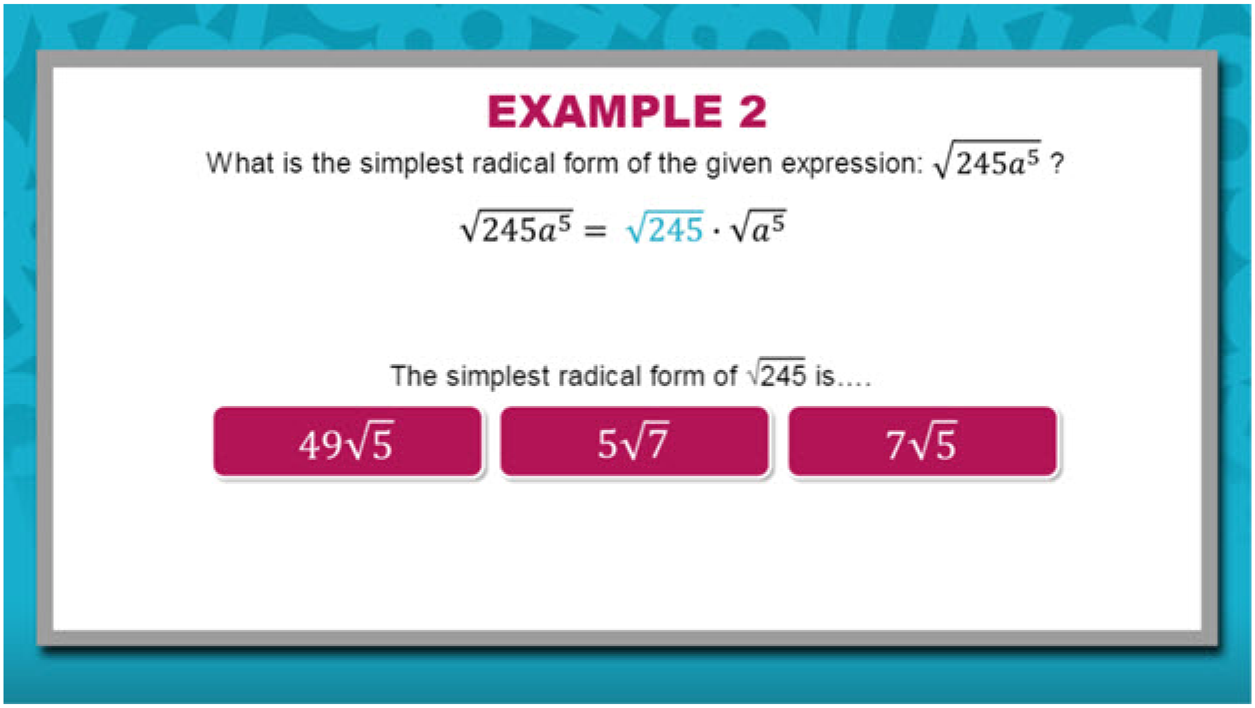
Example 2 (continued)



The Product Property of Radicals states that the square root of a product is equal to the product of the square roots of the factors.

$$\sqrt{245a^5} = \sqrt{245} \cdot \sqrt{a^5}$$

Example 2 (continued)

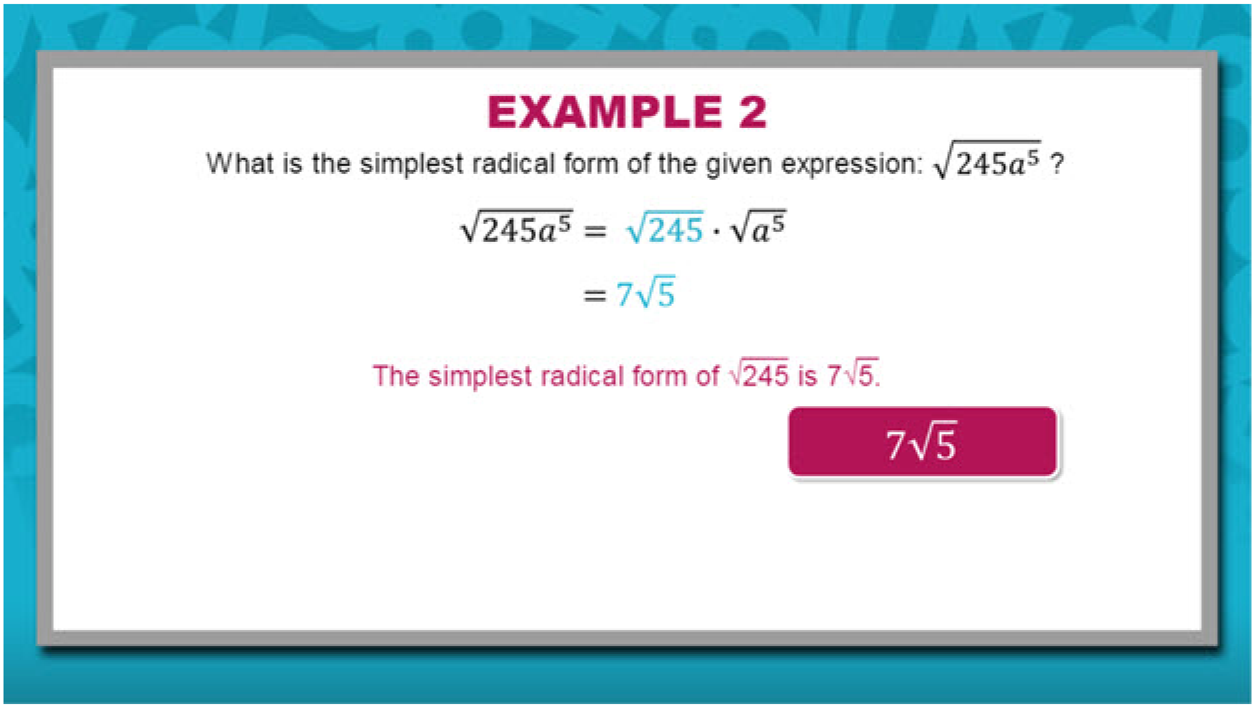


The next step is to simplify each square root factor. Begin with $\sqrt{245}$.

The simplest radical form of $\sqrt{245}$ is ...

A) $49\sqrt{5}$
B) $5\sqrt{7}$
C) $7\sqrt{5}$

Example 2 (continued)

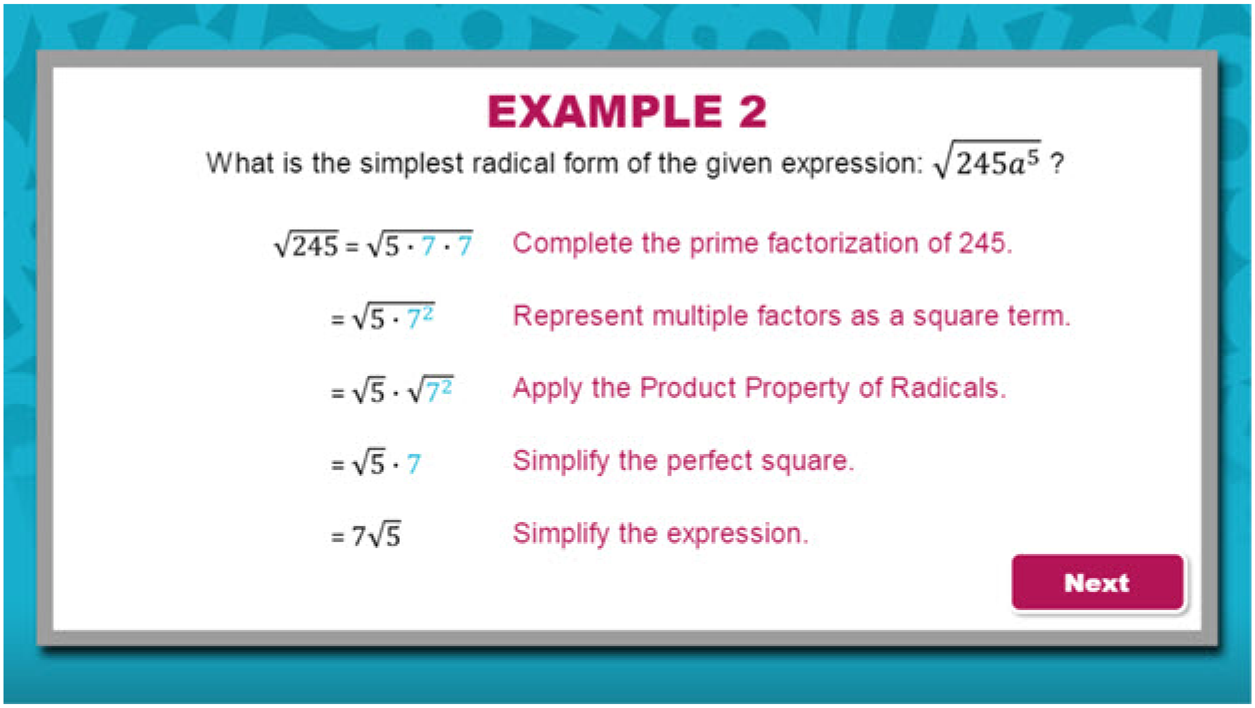


$$\begin{aligned}\sqrt{245a^5} &= \sqrt{245} \cdot \sqrt{a^5} \\ &= 7\sqrt{5}\end{aligned}$$

The simplest radical form of $\sqrt{245}$ is $7\sqrt{5}$.

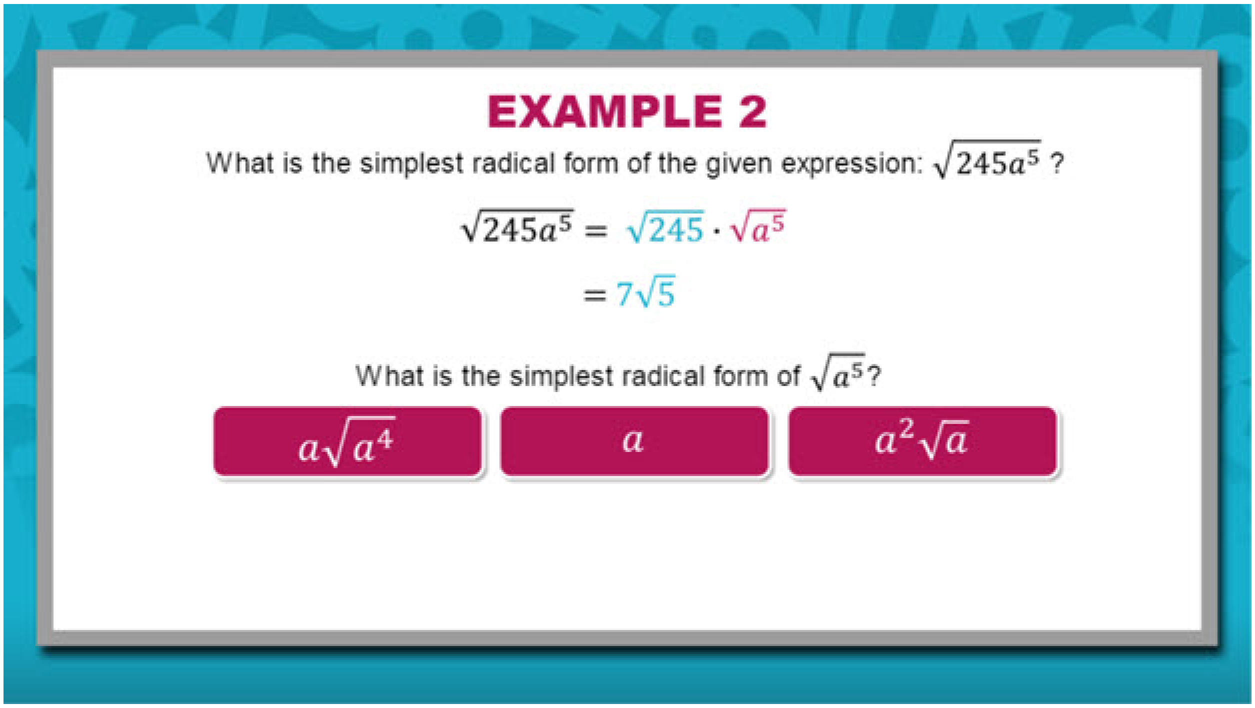
In the expression, you can now represent $\sqrt{245}$ as $7\sqrt{5}$.

Example 2 (continued)



$\sqrt{245} = \sqrt{5 \cdot 7 \cdot 7}$	Complete the prime factorization of 245.
$= \sqrt{5 \cdot 7^2}$	Represent multiple factors as a square term.
$= \sqrt{5} \cdot \sqrt{7^2}$	Apply the Product Property of Radicals.
$= \sqrt{5} \cdot 7$	Simplify the perfect square.
$= 7\sqrt{5}$	Simplify the expression.

Example 2 (continued)

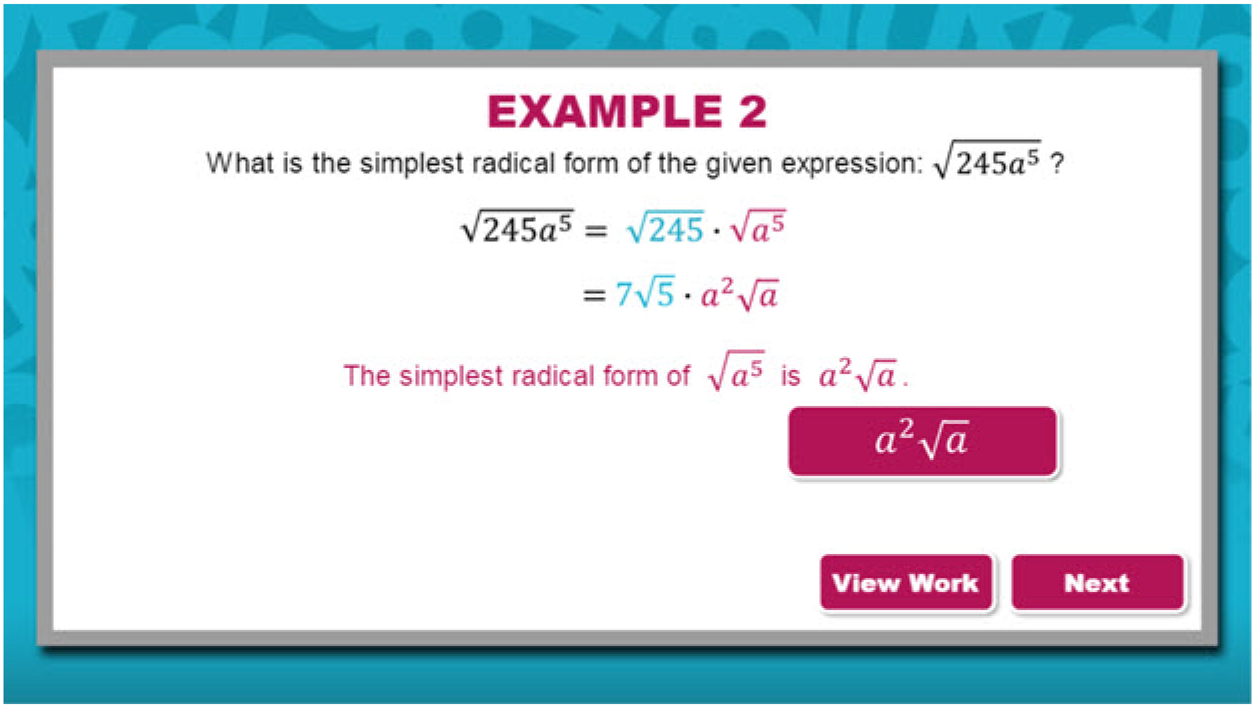


Move on to simplify $\sqrt{a^5}$.

What is the simplest radical form of $\sqrt{a^5}$?

A) $a\sqrt{a^4}$
B) a
C) $a^2\sqrt{a}$

Example 2 (continued)

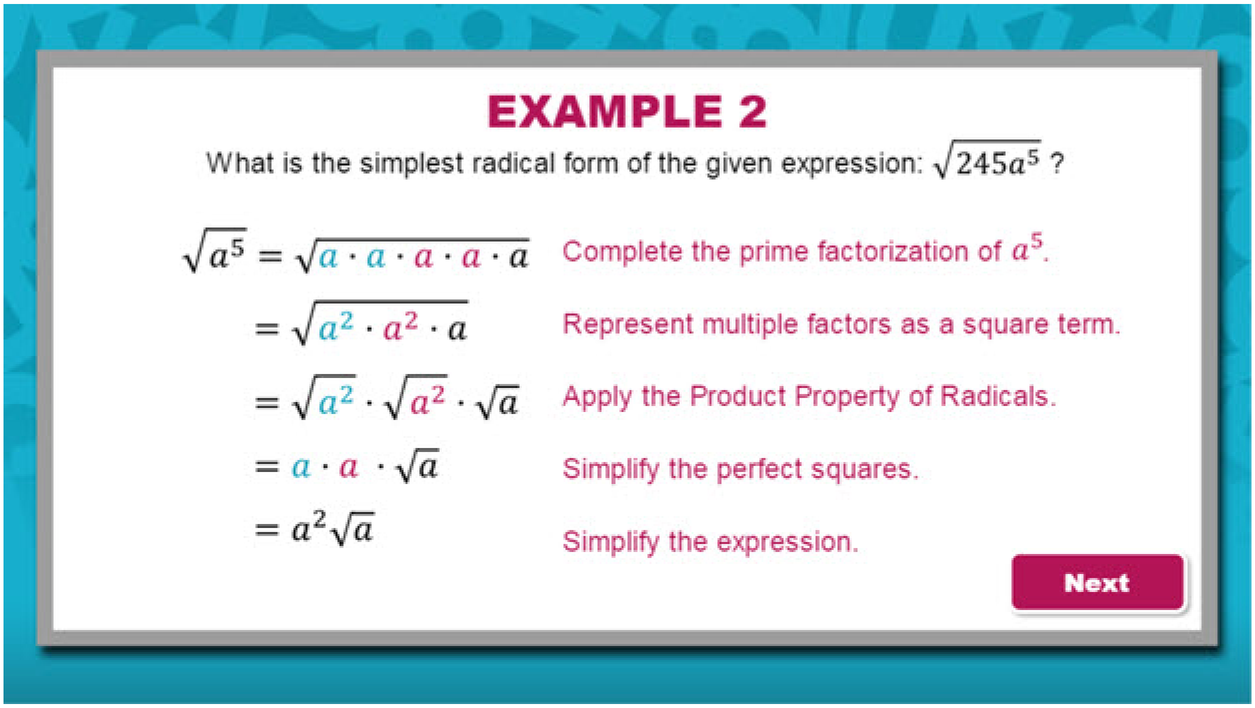


$$\begin{aligned}\sqrt{245a^5} &= \sqrt{245} \cdot \sqrt{a^5} \\ &= 7\sqrt{5} \cdot a^2\sqrt{a}\end{aligned}$$

The simplest radical form of $\sqrt{a^5}$ is $a^2\sqrt{a}$.

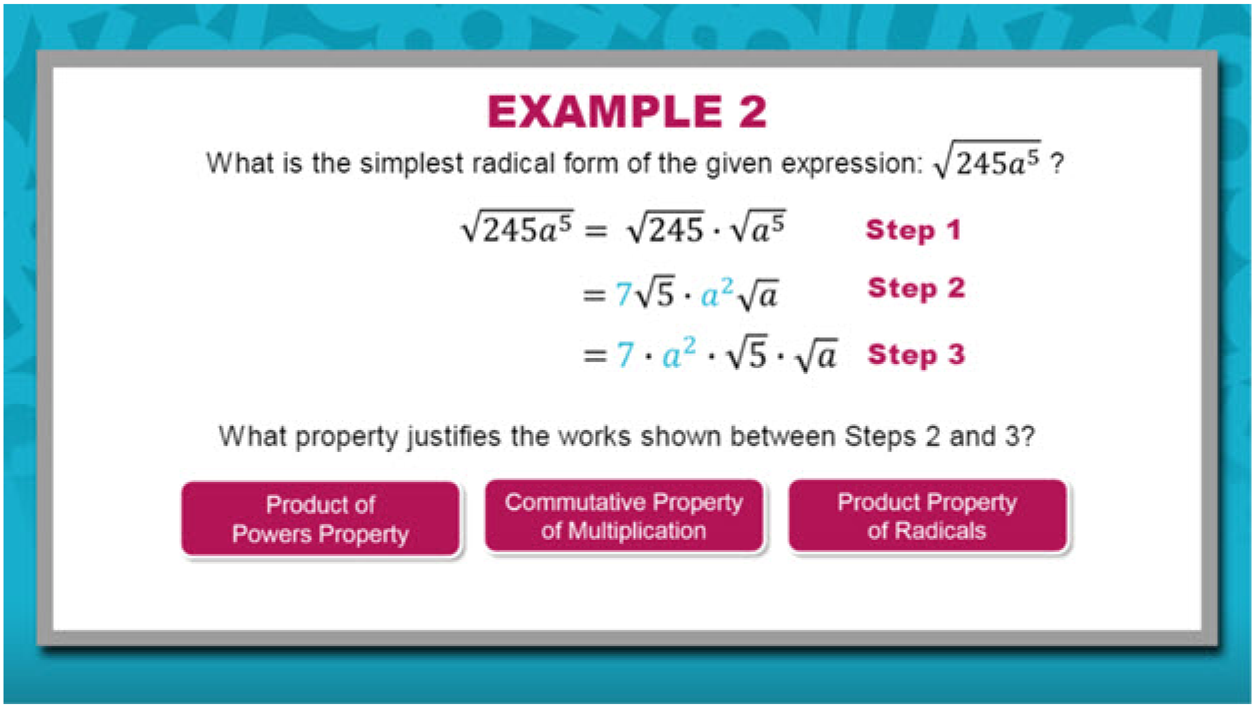
In the expression, you can now represent $\sqrt{a^5}$ as $a^2\sqrt{a}$.

Example 2 (continued)



$\sqrt{a^5} = \sqrt{a \cdot a \cdot a \cdot a \cdot a}$	Complete the prime factorization of a^5.
$= \sqrt{a^2 \cdot a^2 \cdot a}$	Represent multiple factors as square terms.
$= \sqrt{a^2} \cdot \sqrt{a^2} \cdot \sqrt{a}$	Apply the Product Property of Radicals.
$= a \cdot a \cdot \sqrt{a}$	Simplify the perfect squares.
$= a^2\sqrt{a}$	Simplify the expression.

Example 2 (continued)



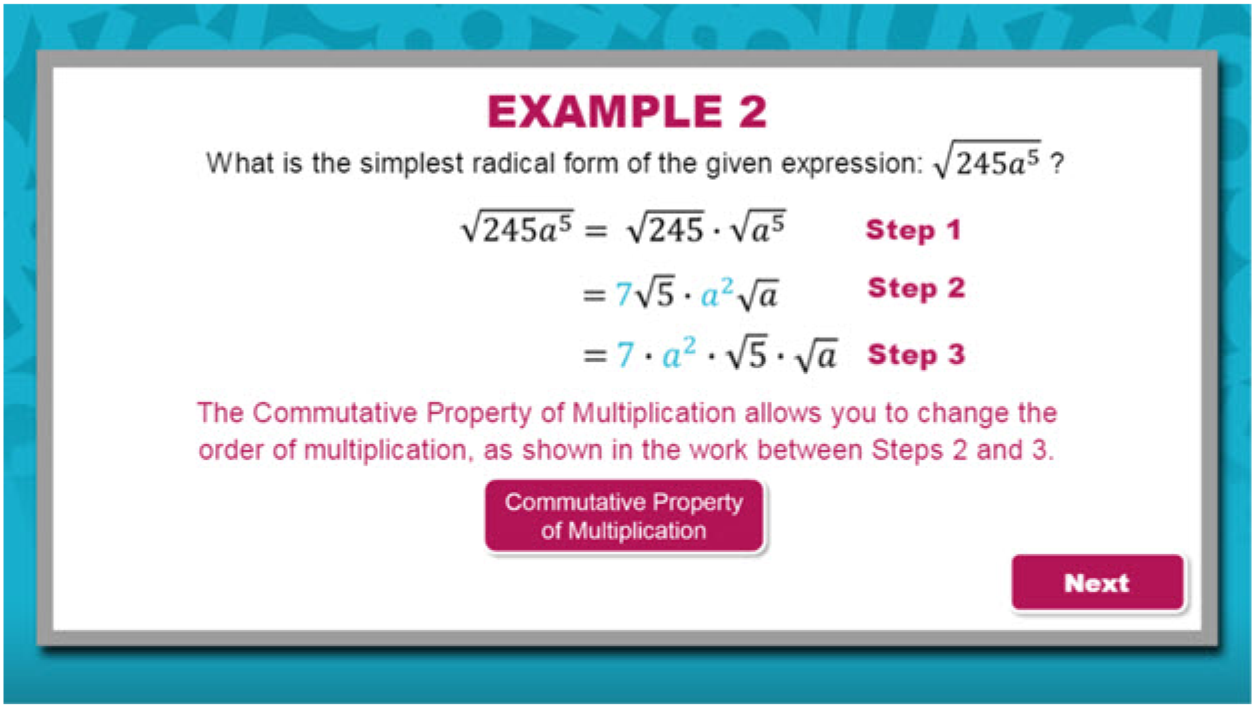
Now that you have simplified the square root factors, determine the product.

$$
\begin{aligned}
\sqrt{245a^5} &= \sqrt{245} \cdot \sqrt{a^5} && \text{Step 1} \\
&= 7\sqrt{5} \cdot a^2\sqrt{a} && \text{Step 2} \\
&= 7 \cdot a^2 \cdot \sqrt{5} \cdot \sqrt{a} && \text{Step 3}
\end{aligned}
$$

What property justifies the work shown between Steps 2 and 3?

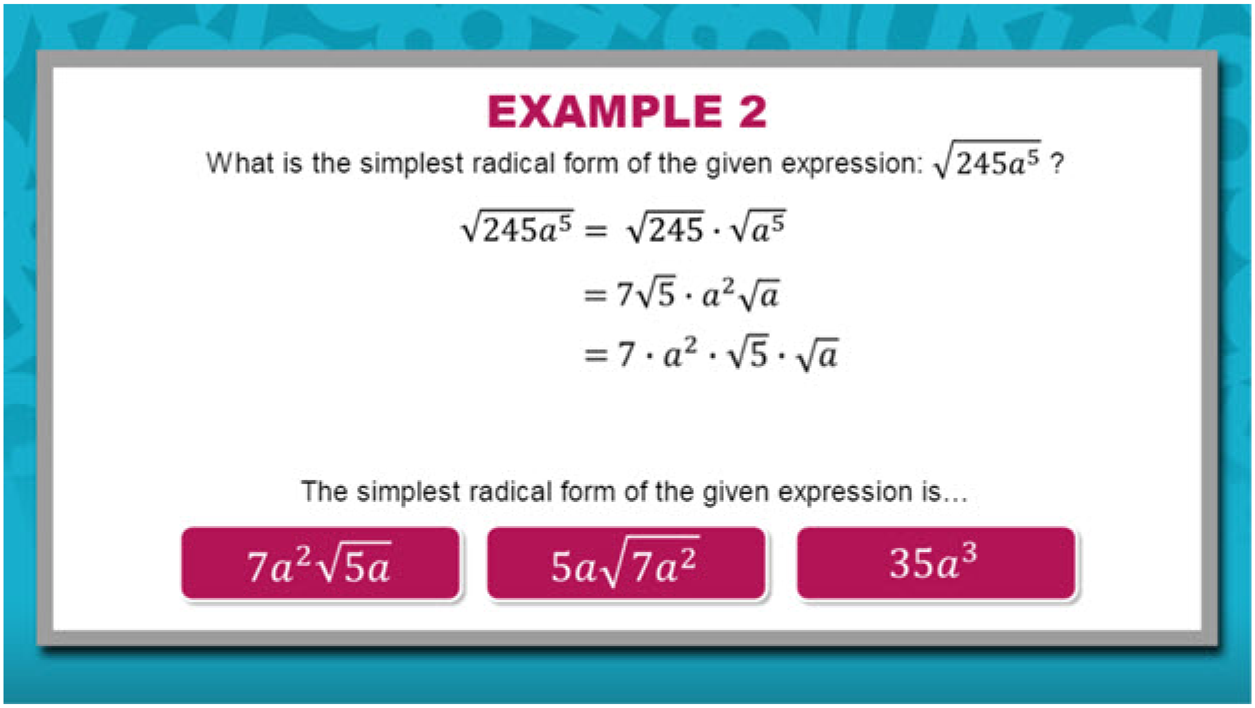
A) Product of Powers Property
B) Commutative Property of Multiplication
C) Product Property of Radicals

Example 2 (continued)



The Commutative Property of Multiplication allows you to change the order of multiplication, as shown in the work between Steps 2 and 3.

Example 2 (continued)

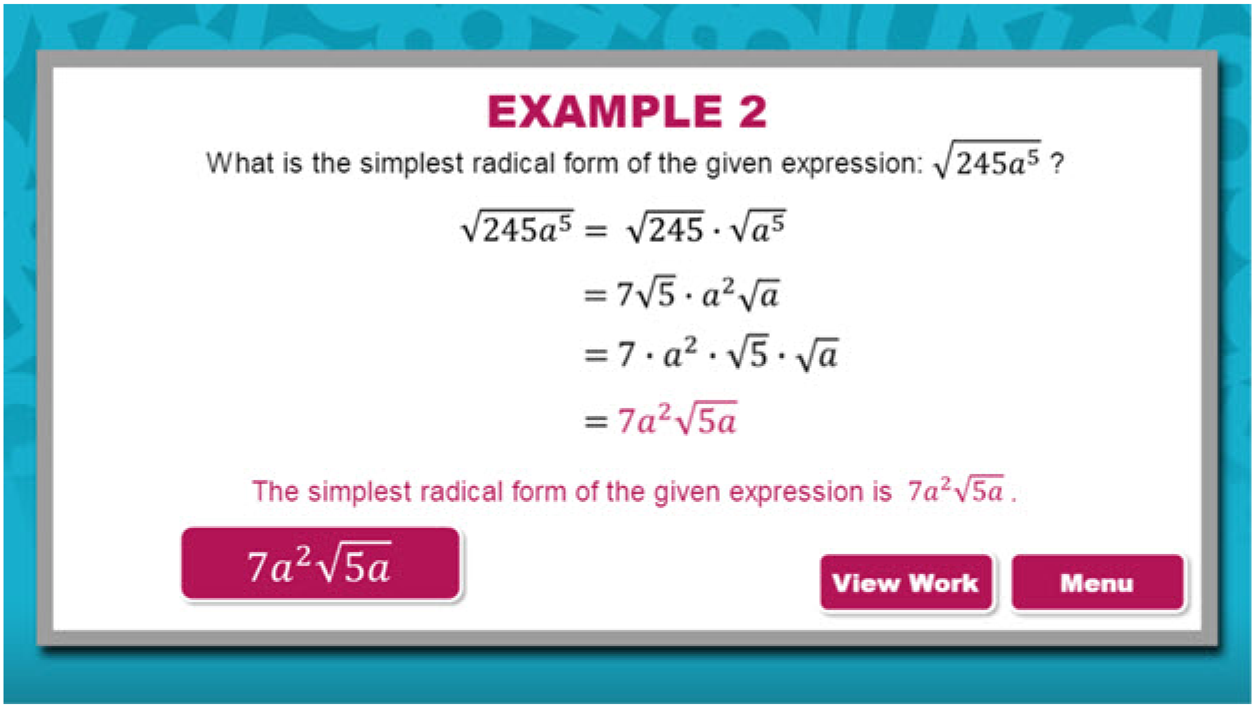


What is the simplest radical form of the given expression: $\sqrt{245a^5}$?

$$\begin{aligned}\sqrt{245a^5} &= \sqrt{245} \cdot \sqrt{a^5} \\ &= 7\sqrt{5} \cdot a^2\sqrt{a} \\ &= 7 \cdot a^2 \cdot \sqrt{5} \cdot \sqrt{a} \\ &= \ ?\end{aligned}$$

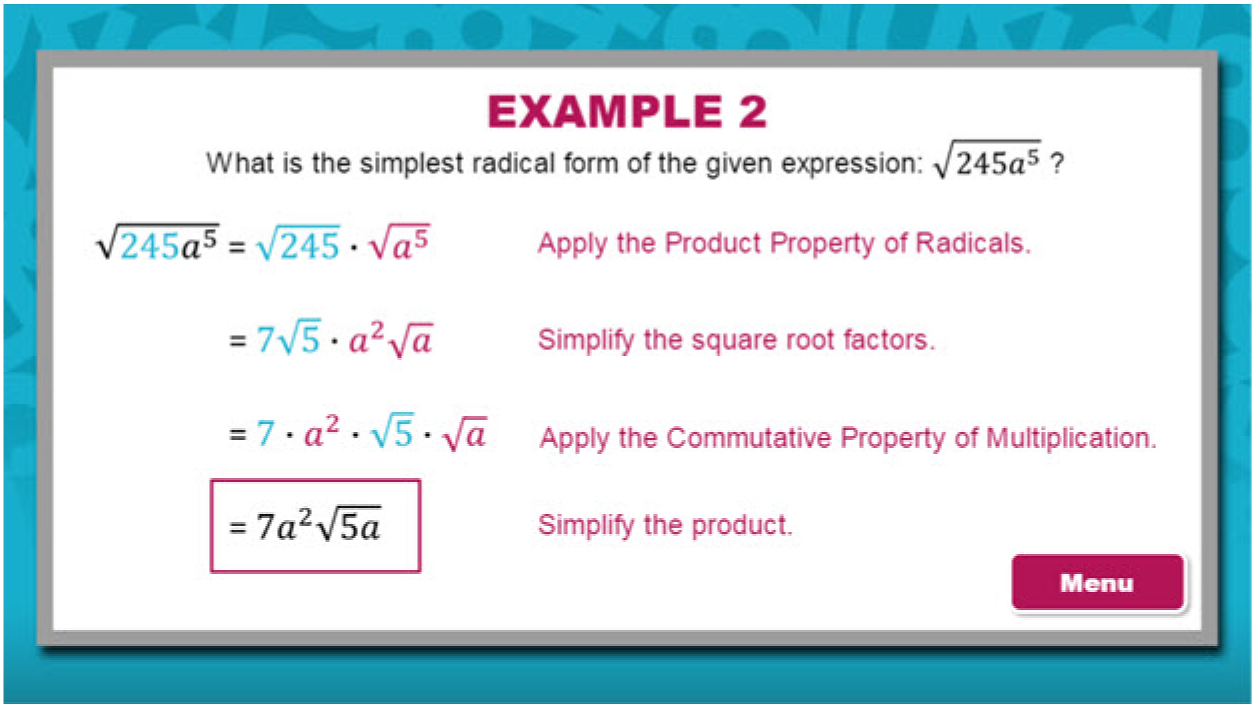
A) $7a^2\sqrt{5a}$
B) $5a\sqrt{7a^2}$
C) $35a^3$

Example 2 (continued)



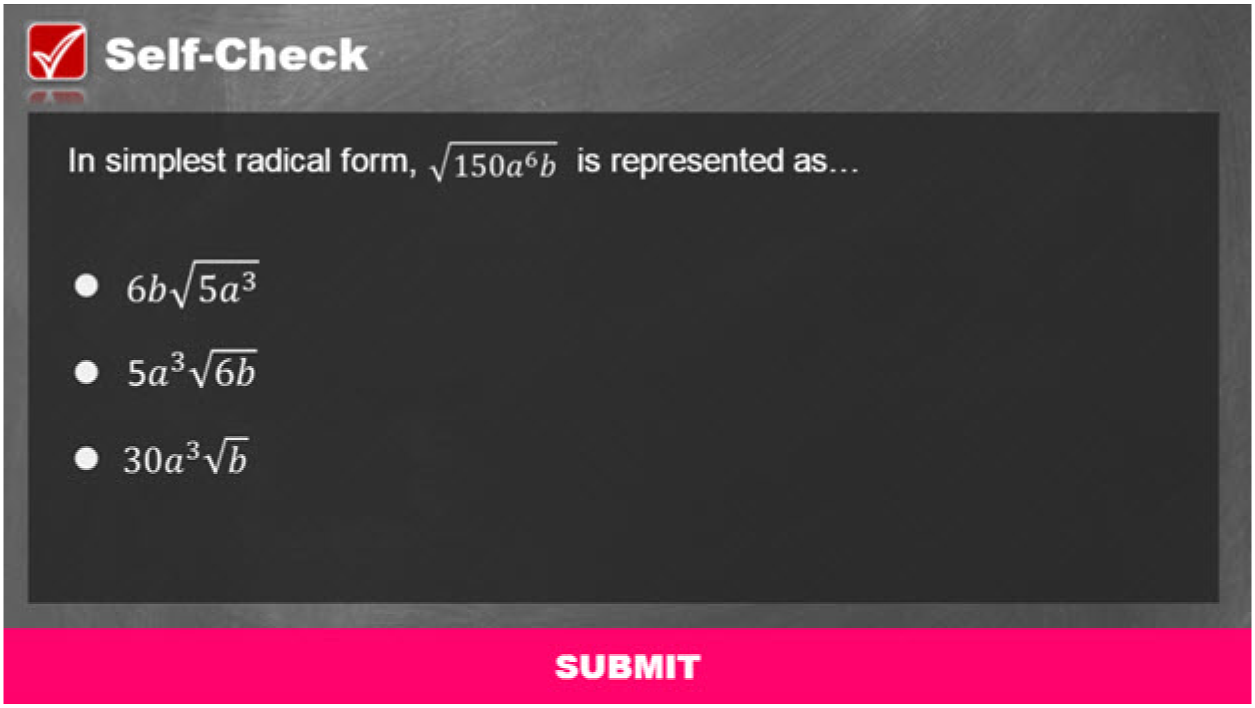
The simplest radical form of the given expression is $7a^2\sqrt{5a}$.

Example 2 (continued)



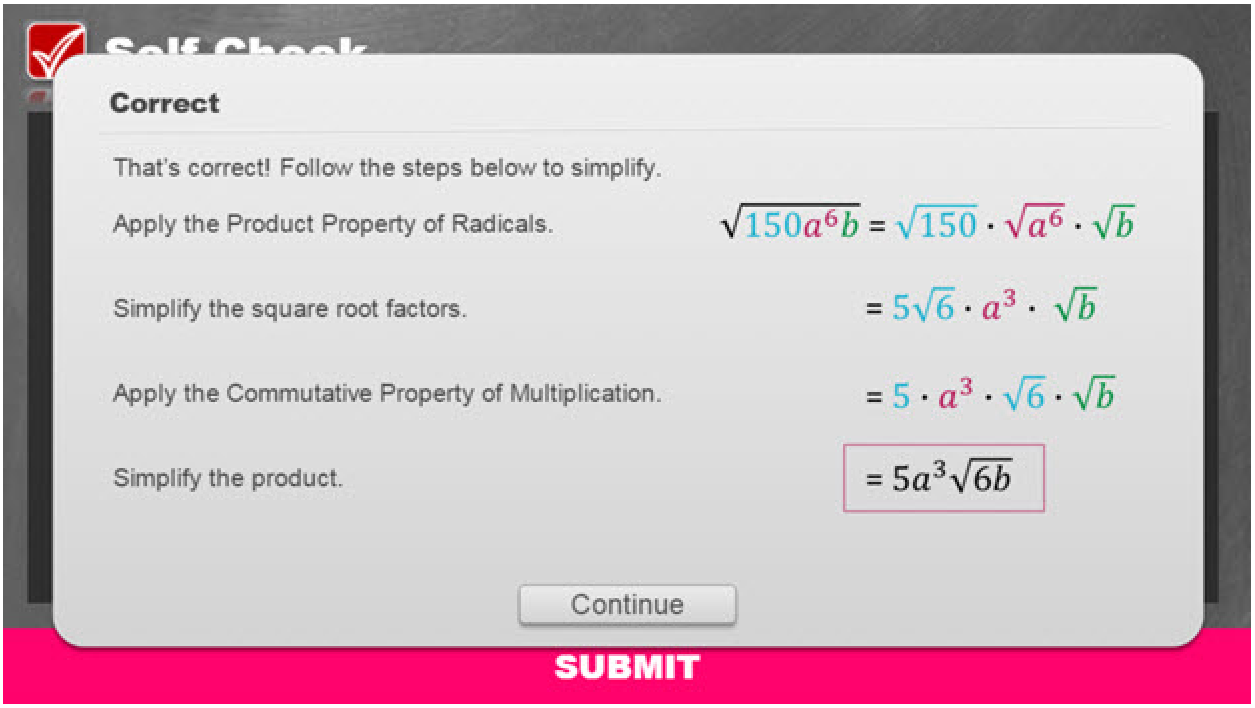
$\sqrt{245a^5} = \sqrt{245} \cdot \sqrt{a^5}$	Apply the Product Property of Radicals.
$= 7\sqrt{5} \cdot a^2\sqrt{a}$	Simplify the square root factors.
$= 7 \cdot a^2 \cdot \sqrt{5} \cdot \sqrt{a}$	Apply the Commutative Property of Multiplication.
$= 7a^2\sqrt{5a}$	Simplify the product.

Self-Check



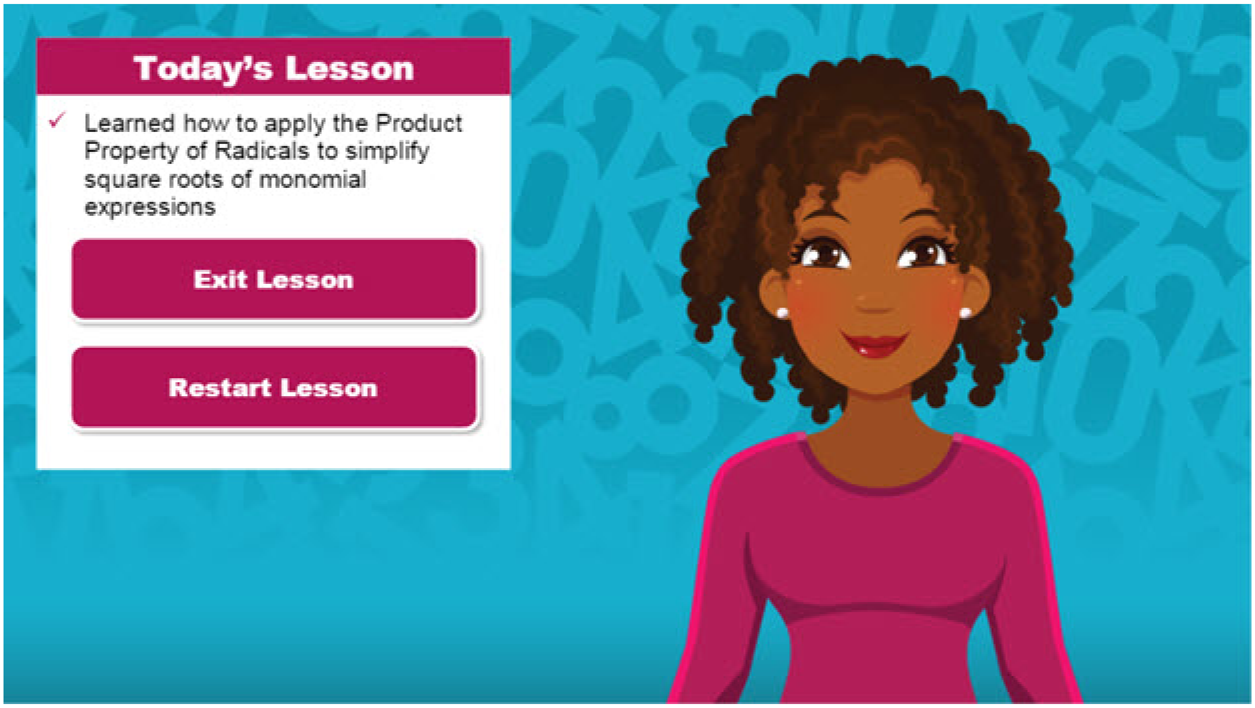
Solve the problem in the image above to check your understanding of the content.

Self-Check: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



You have reached the conclusion of this lesson where you learned how to apply the Product Property of Radicals to simplify square roots of monomial expressions.