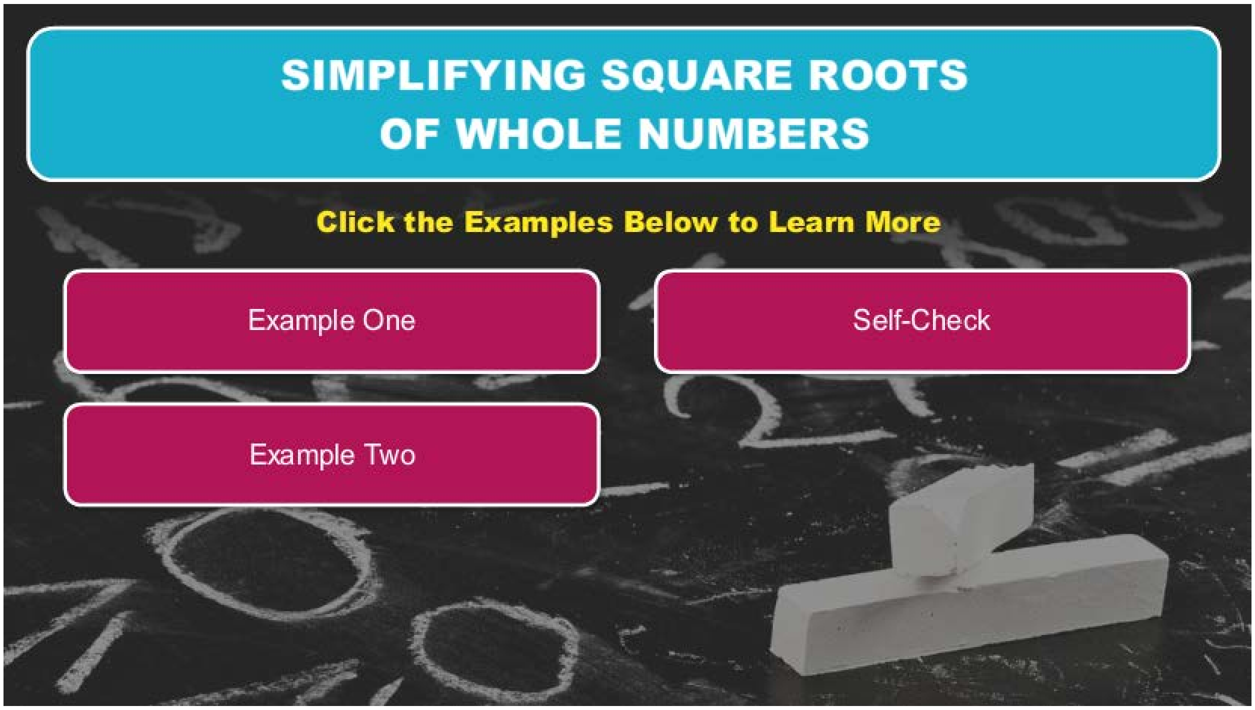
Module 4: Radical Expressions

Topic 1 Content: Simplifying Square Roots of Whole Numbers

Introduction

Hi there! I'm so glad you could join me for this lesson in Algebra I. In this lesson, you will apply the Product Property of Radicals to simplify square roots of whole numbers. Your factoring skills will be useful a resource during this lesson.

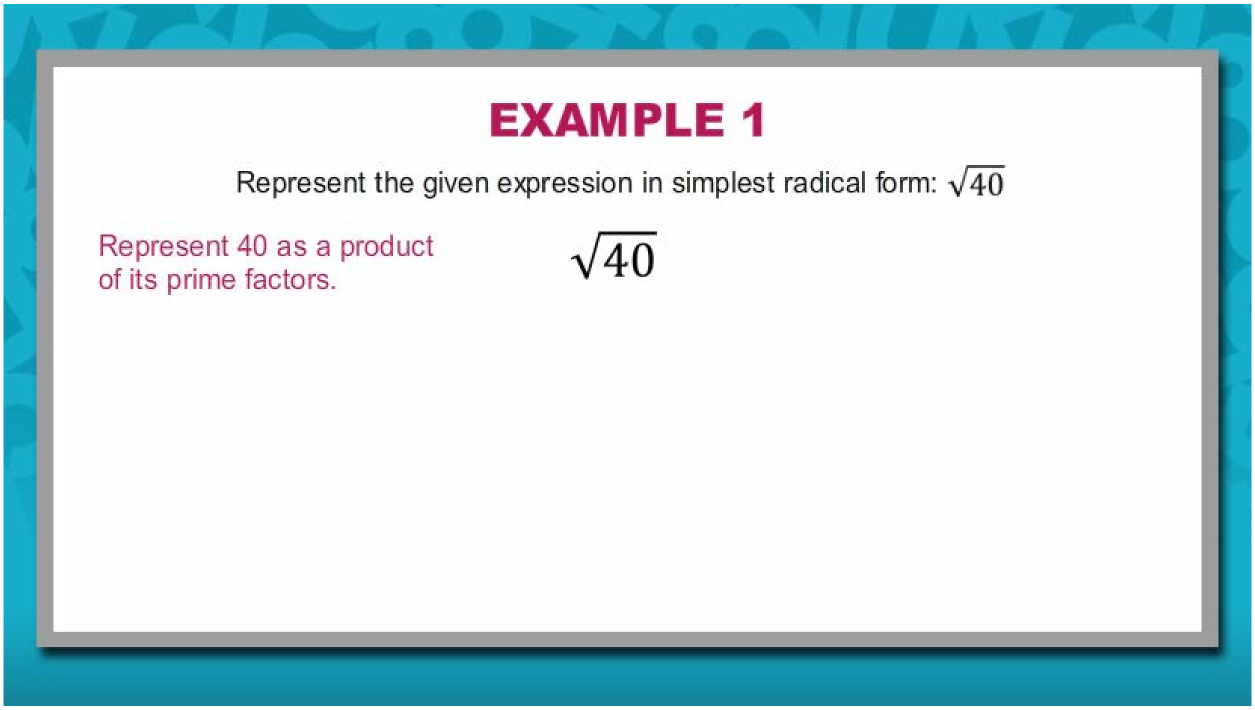
Simplifying Square Roots of Whole Numbers



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Example 1

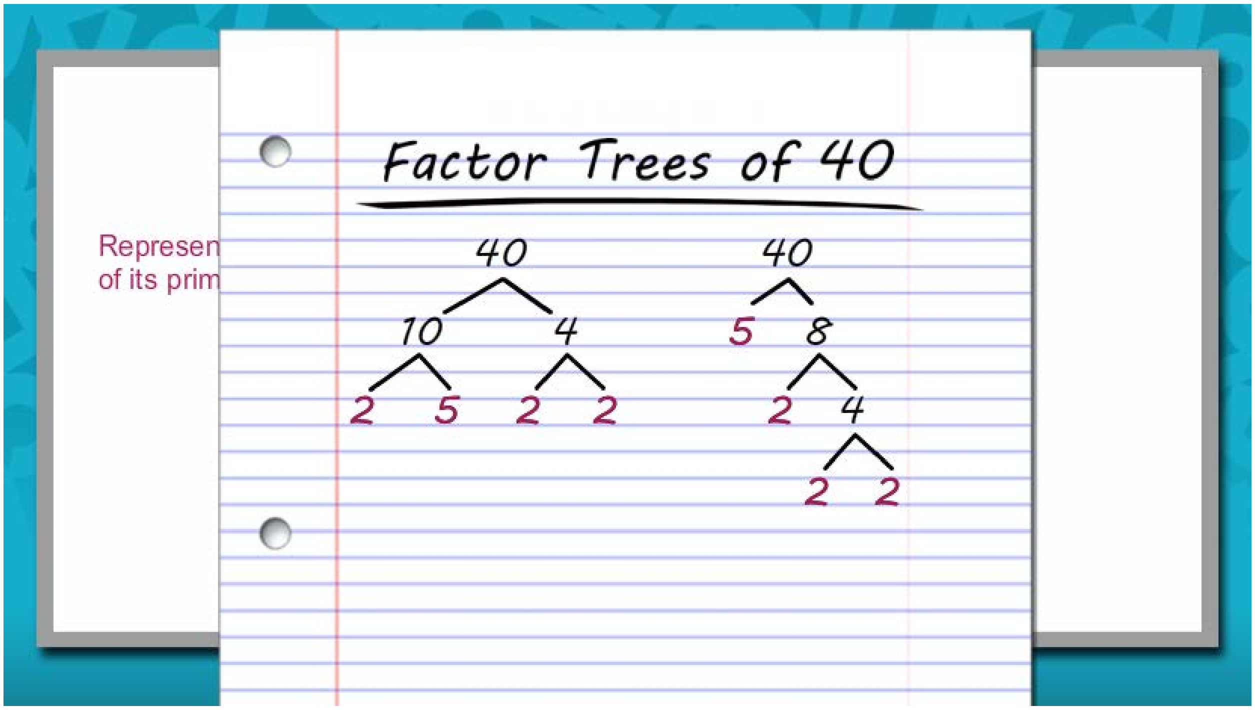


Represent the given expression in simplest radical form.

$$\sqrt{40}$$

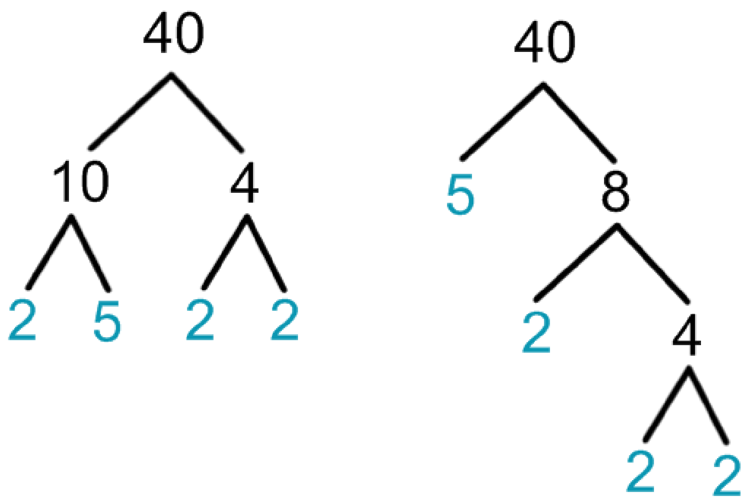
Recall that a square root expression is in simplest form when the radicand has no perfect square factors other than 1. To represent $\sqrt{40}$ in simplest radical form, start by completing the prime factorization of 40. In other words, represent 40 as a product of its prime factors.

Example 1 (continued)



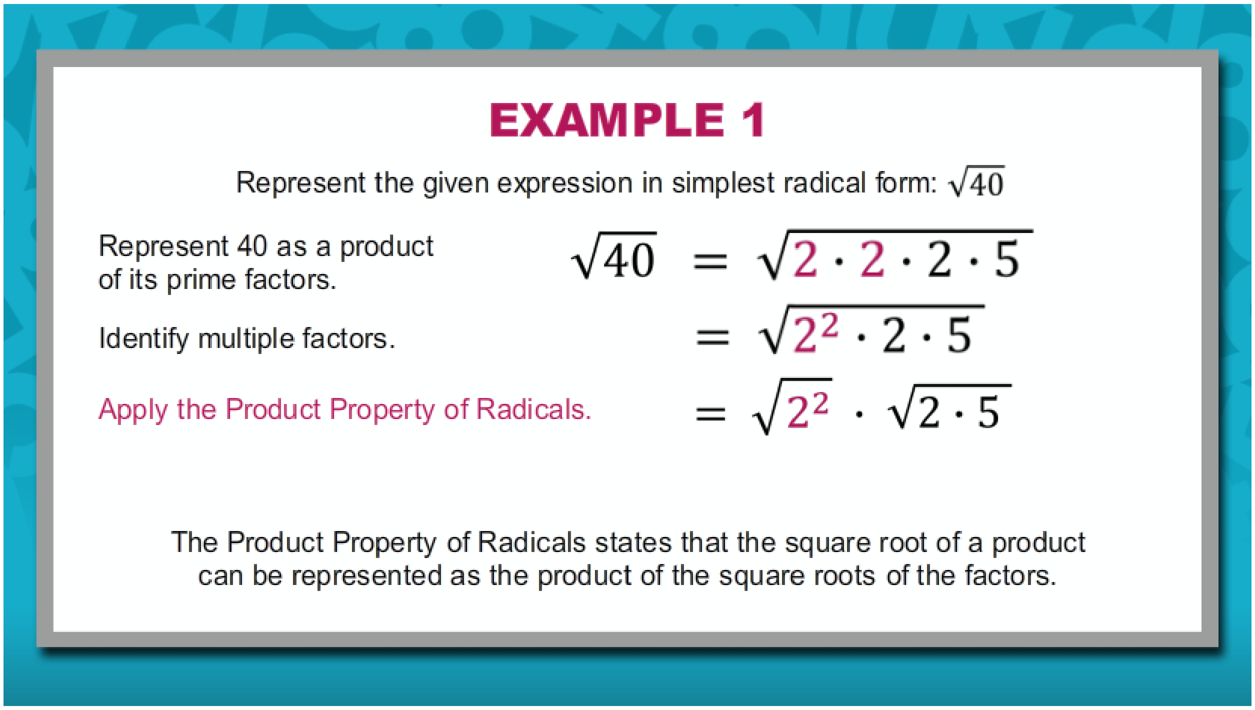
You may find it helpful to use a factor tree. The branches of your factor tree will depend on the initial factors of 40 that you choose to start with. The prime factors, however, will be the same.

Two possible factor trees of 40:



The prime factorization of 40 is $2 \cdot 2 \cdot 2 \cdot 5$.

Example 1 (continued)



Now that you know the prime factorization of 40, you can represent $\sqrt{40}$ as $\sqrt{2 \cdot 2 \cdot 2 \cdot 5}$.

$$\sqrt{40} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5}$$

The next step is to identify any multiple factors. Because you are simplifying a square root expression, the goal is to identify multiple factors that can be rewritten as a square term. The factors $2 \cdot 2$ can be rewritten as 2^2.

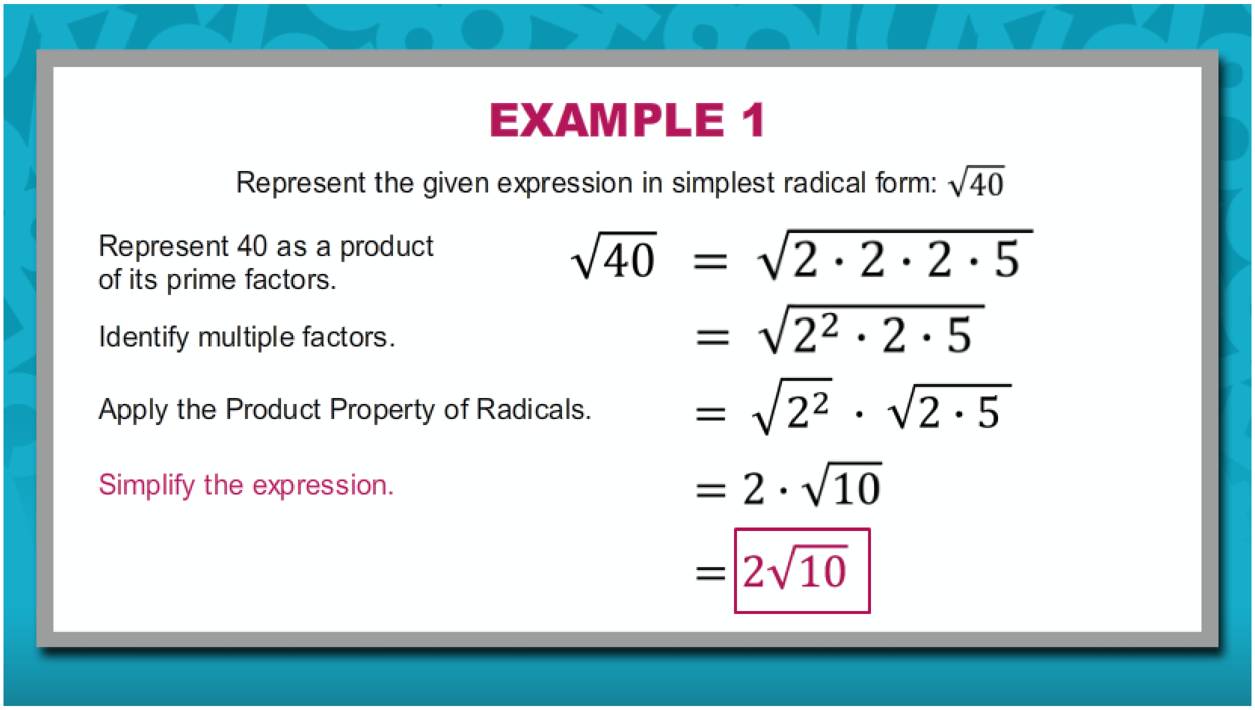
$$\sqrt{40} = \sqrt{2^2 \cdot 2 \cdot 5}$$

Recall that the Product Property of Radicals states that the square root of a product can be represented as the product of the square roots of the factors.

So $\sqrt{40}$ can be represented as $\sqrt{2^2} \cdot \sqrt{2 \cdot 5}$.

$$\sqrt{40} = \sqrt{2^2} \cdot \sqrt{2 \cdot 5}$$

Example 1 (continued)



Now simplify the expression. Begin with the first square root factor. Recall that squaring a number and taking the square root of a number are inverse operations. This means that $\sqrt{2^2} = 2$.

$$\sqrt{40} = 2 \cdot \sqrt{2 \cdot 5}$$

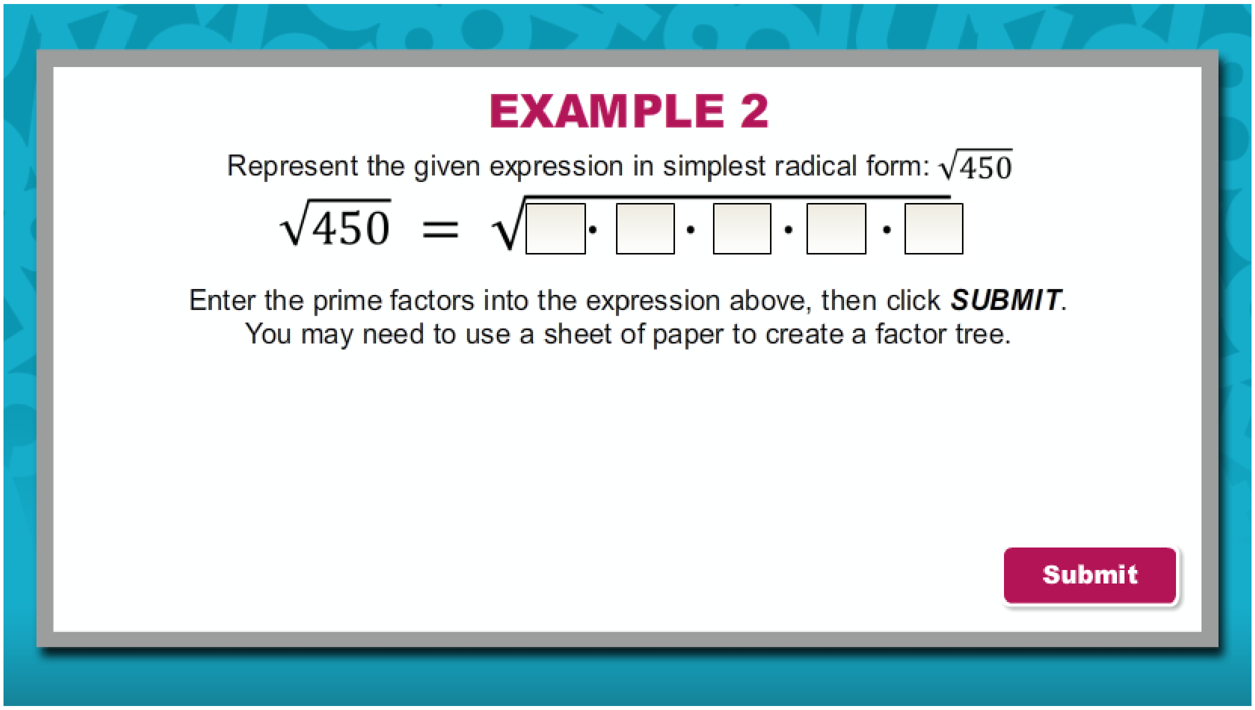
Next, simplify the radicand of the second square root factor by multiplying 2 and 5, which equals 10.

$$\sqrt{40} = 2 \ \cdot \sqrt{10}$$

The final step is to simplify the resulting expression. Your work is complete. The simplest radical form of $\sqrt{40}$ is $2\sqrt{10}$.

$$\sqrt{40} = 2\,\sqrt{10}$$

Example 2

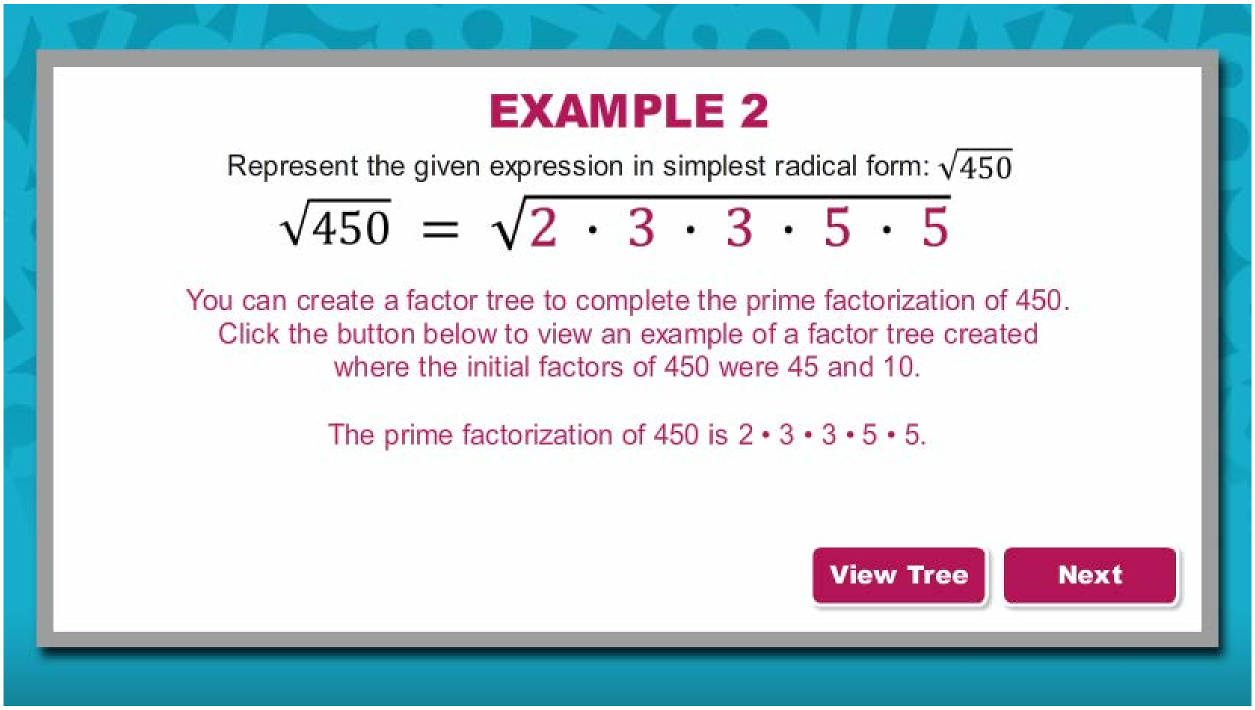


Represent the given expression in simplest radical form.

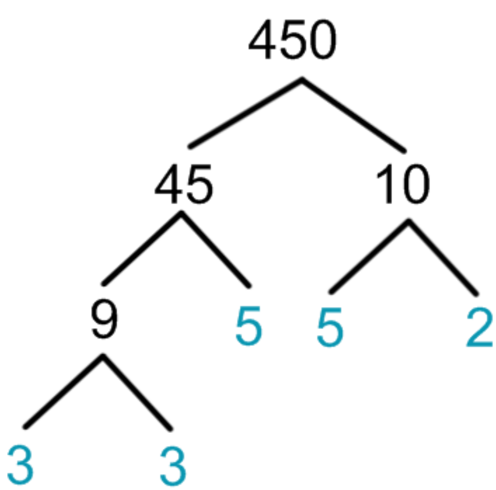
$$\sqrt{450}$$

To simplify the expression, begin by representing the radicand as a product of its prime factors. What are the prime factors of the expression above? You may need to use a sheet of paper to create a factor tree.

Example 2 (continued)



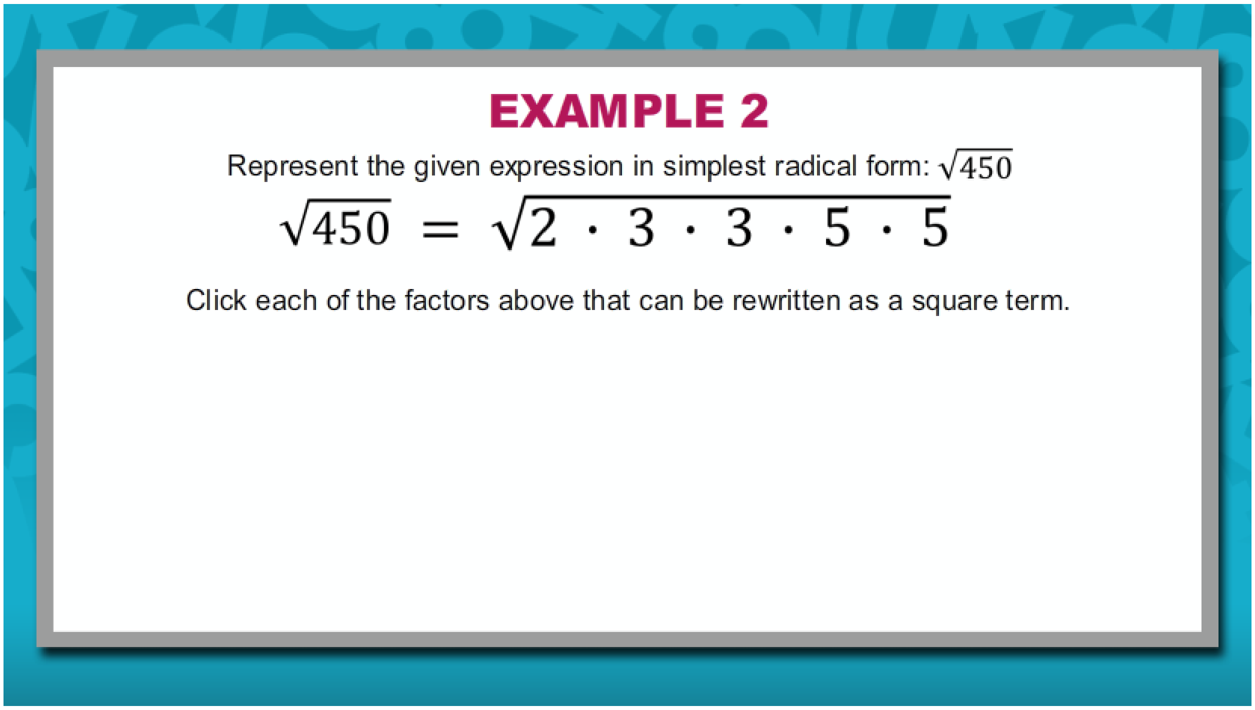
You can create a factor tree to complete the prime factorization of 450. Take a look at the example of a factor tree created when the initial factors of 450 were 45 and 10.



The prime factorization of 450 is $2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$.

$$\sqrt{450} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5}$$

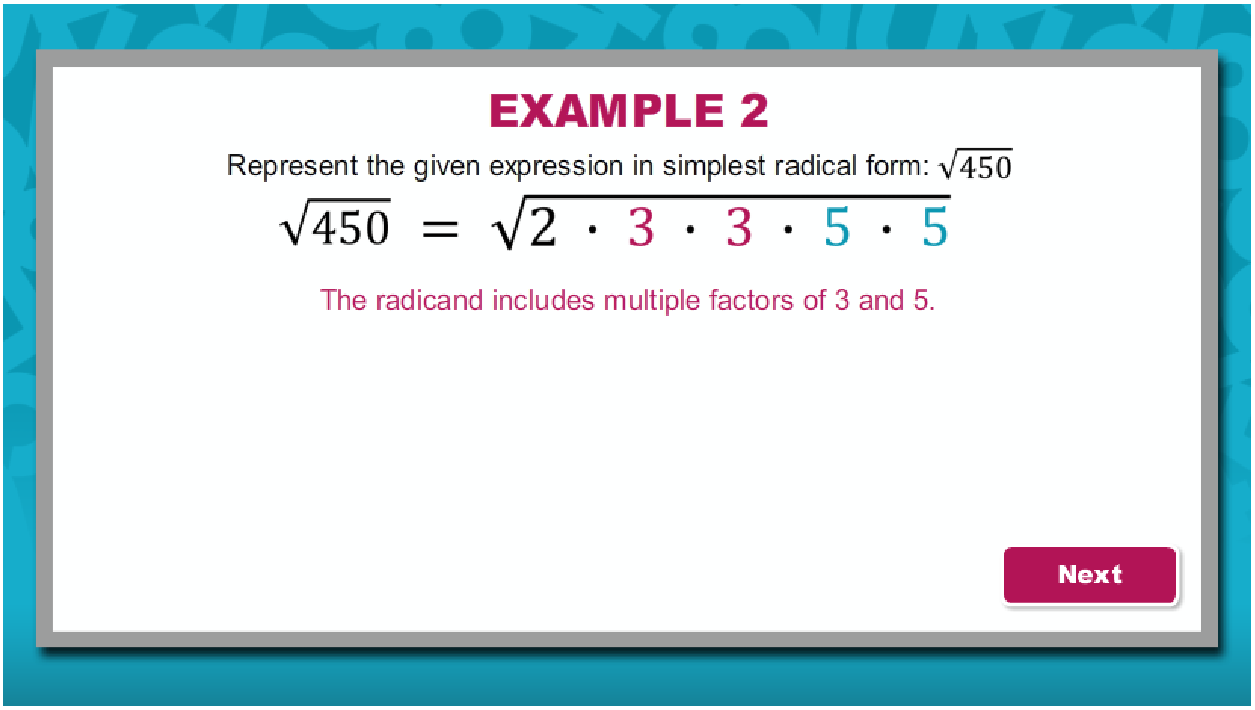
Example 2 (continued)



Now identify multiple factors that can be rewritten as a square term.

$$\sqrt{450} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5}$$

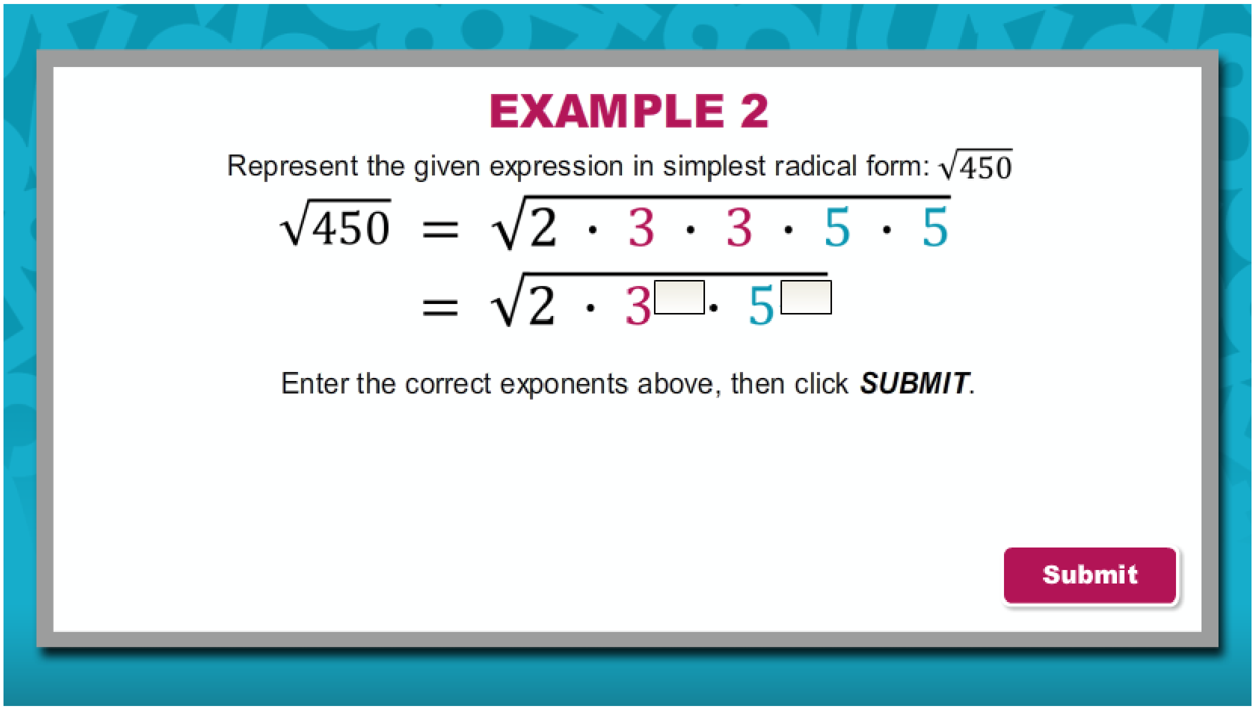
Example 2 (continued)



The radicand includes multiple factors of 3 and 5.

$$\sqrt{450} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5}$$

Example 2 (continued)

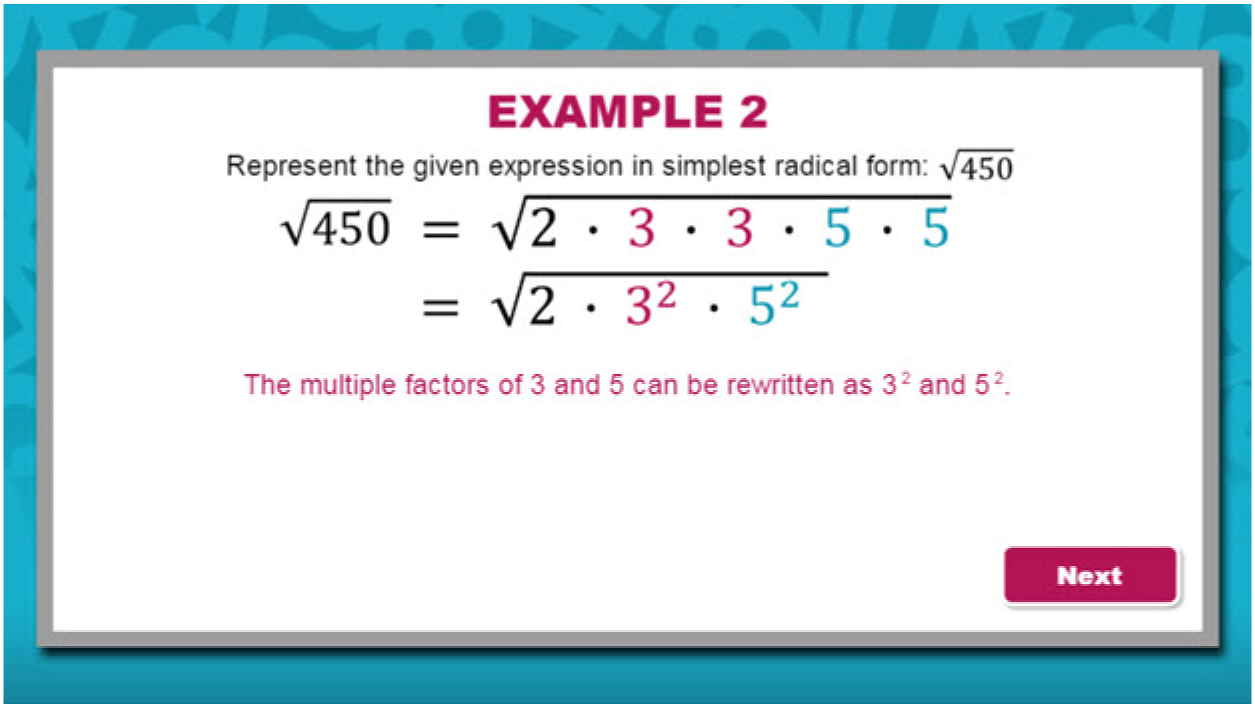


Next, represent the multiple factors as square terms.

$$\begin{aligned} \sqrt{450} &= \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} \\ &= \sqrt{2 \cdot 3^{?} \cdot 5^{?}} \end{aligned}$$

What exponents should replace the "?" symbols above?

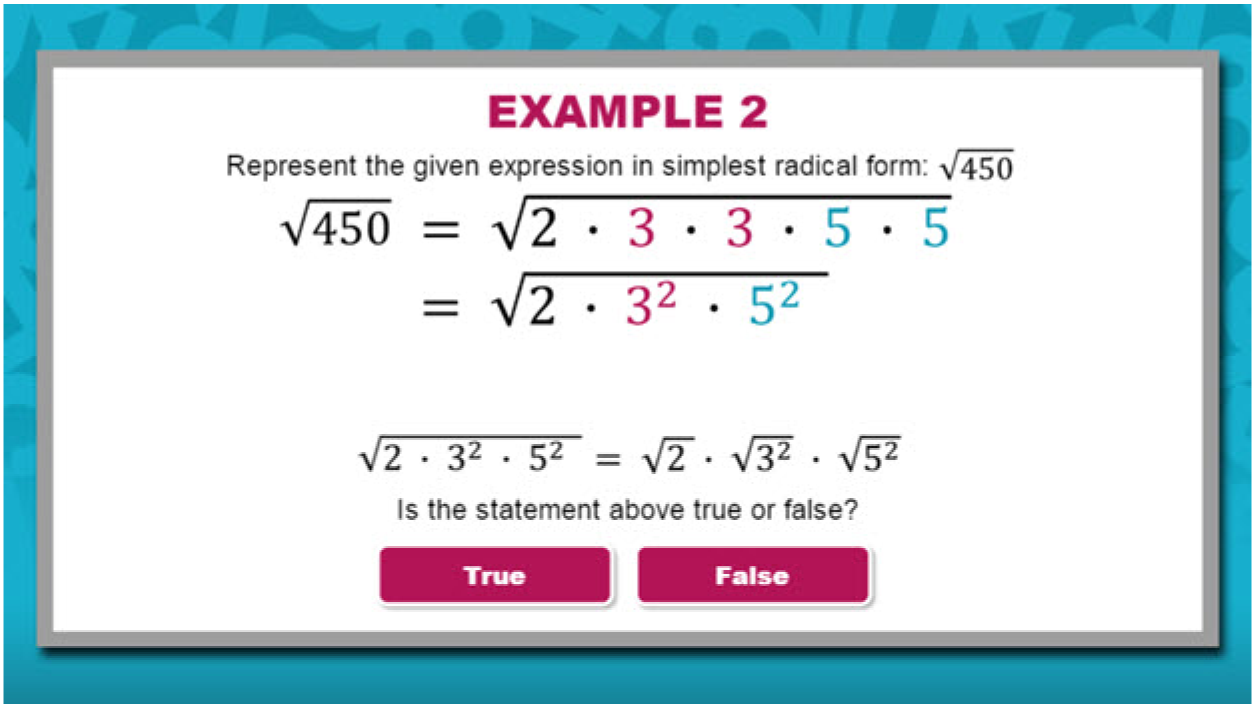
Example 2 (continued)



The multiple factors of 3 and 5 can be rewritten as 3^2 and 5^2.

$$\begin{aligned}\sqrt{450} &= \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} \\ &= \sqrt{2 \cdot 3^2 \cdot 5^2}\end{aligned}$$

Example 2 (continued)



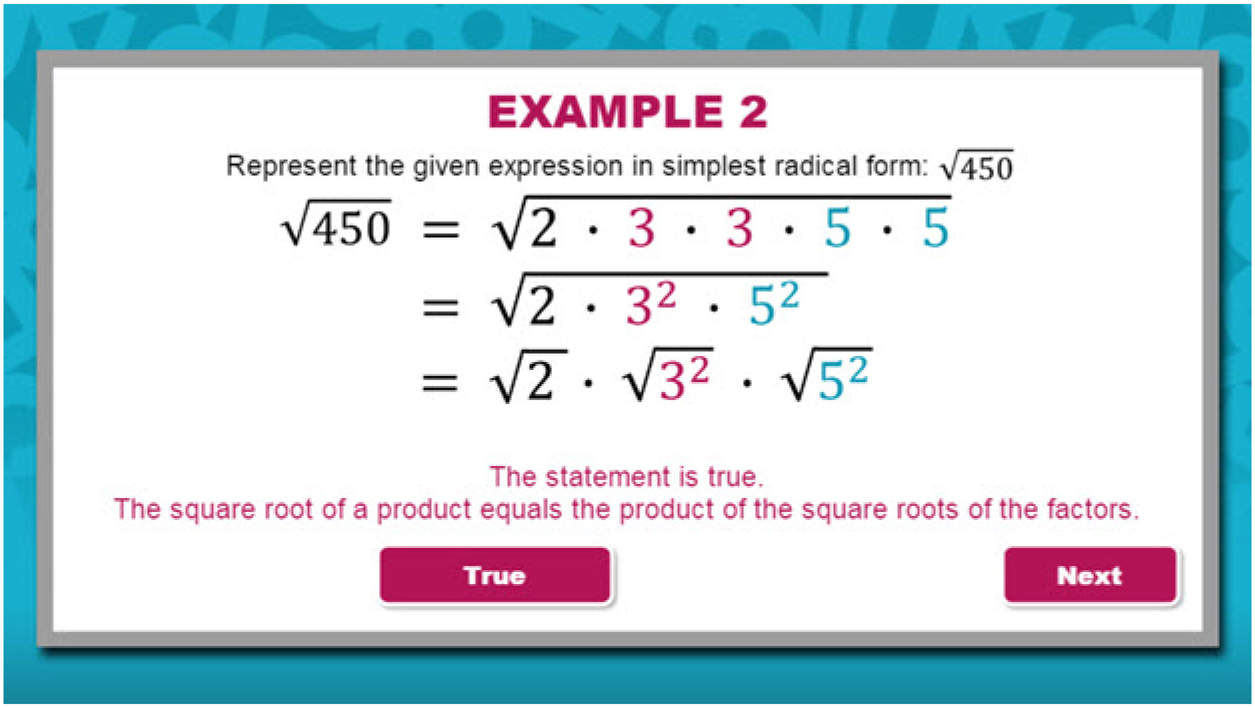
Now apply the Product Property of Radicals to represent the square root of the product as the product of the square roots of the factors.

$$\sqrt{2 \cdot 3^2 \cdot 5^2} = \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5^2}$$

Is the statement above true or false?

A) True
B) False

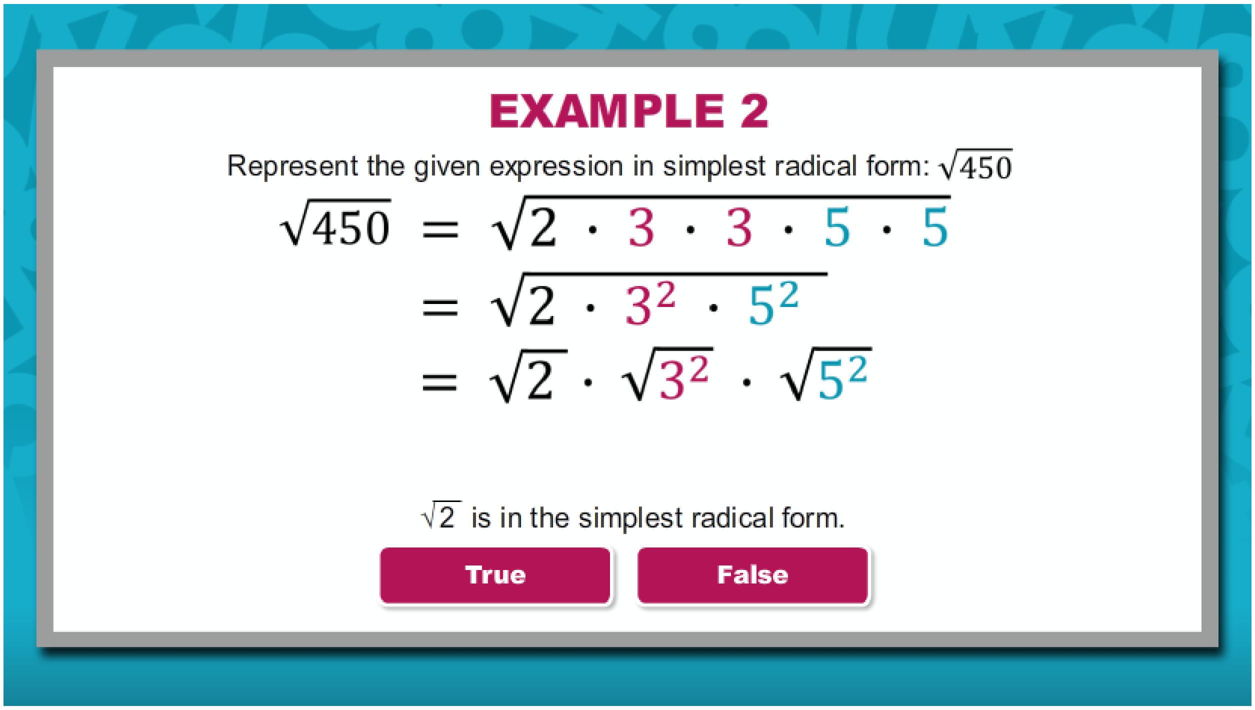
Example 2 (continued)



The statement is true. The square root of a product equals the product of the square roots of the factors.

$$\sqrt{2 \cdot 3^2 \cdot 5^2} = \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5^2}$$

Example 2 (continued)



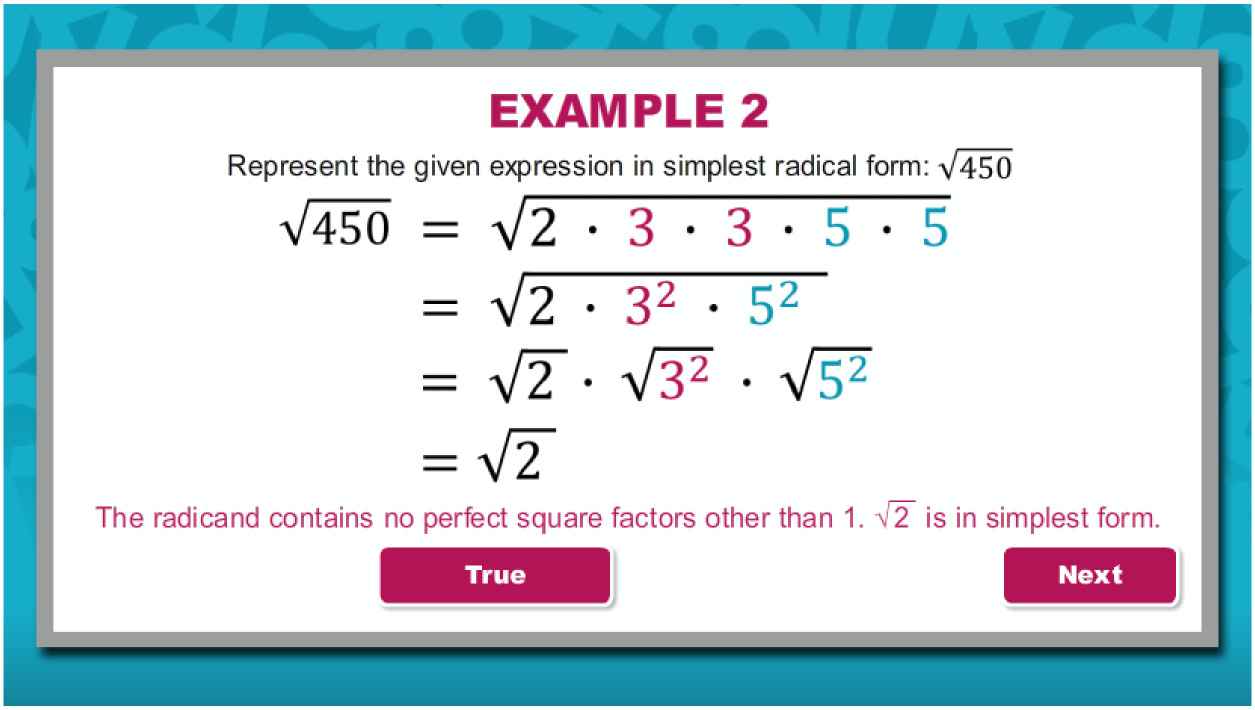
Next, simplify the expression $\sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5^2}$.

The radicand $\sqrt{2}$ is in simplest radical form.

Is the statement above true or false?

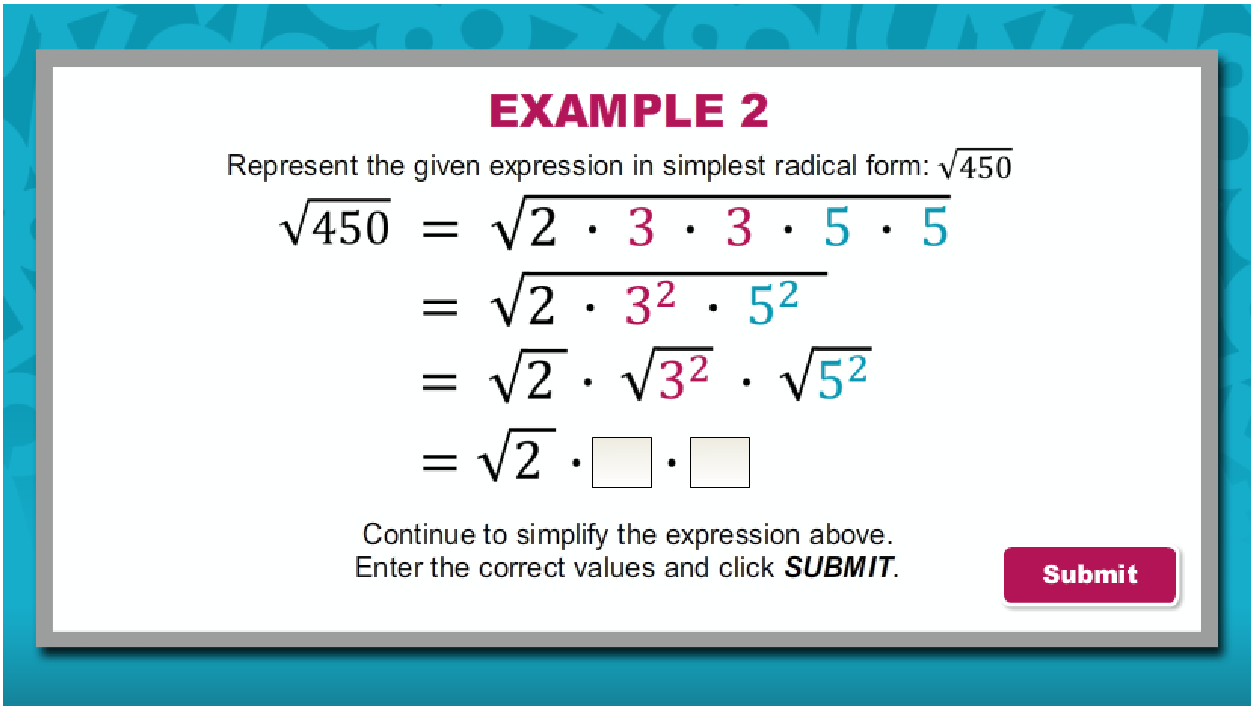
A) True
B) False

Example 2 (continued)



The statement is true. The radicand contains no perfect square factors other than 1. Therefore, $\sqrt{2}$ is in simplest radical form.

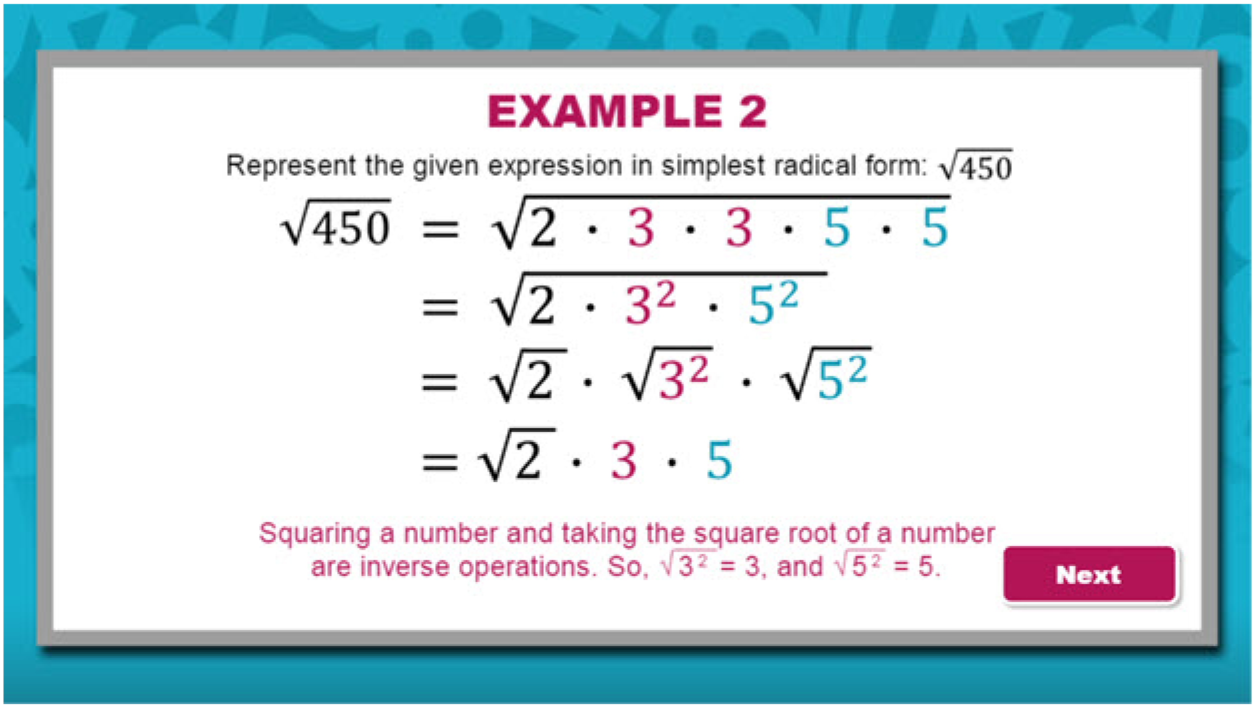
Example 2 (continued)



Continue to simplify $\sqrt{3^2}$ and $\sqrt{5^2}$.

What do $\sqrt{3^2}$ and $\sqrt{5^2}$ simplify to?

Example 2 (continued)



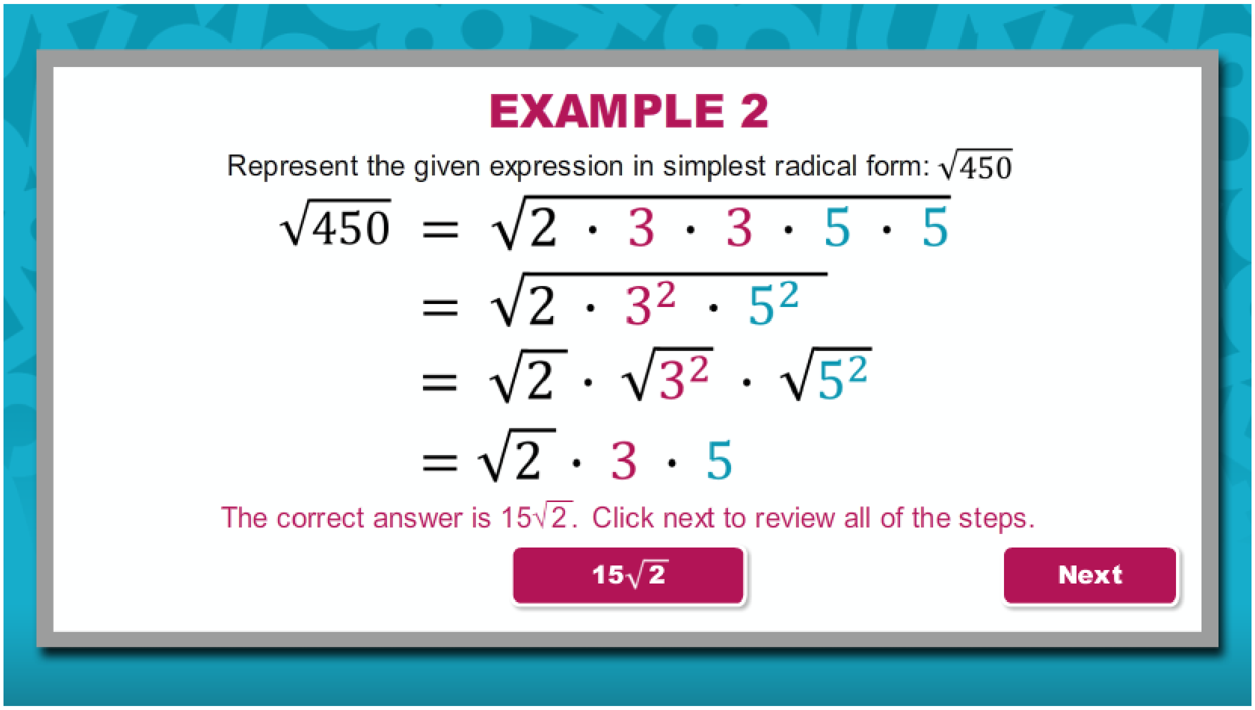
Squaring a number and taking the square root of a number are inverse operations. So, $\sqrt{3^2} = 3$ and $\sqrt{5^2} = 5$.

$$\begin{aligned}
\sqrt{450} &= \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} \\
&= \sqrt{2 \cdot 3^2 \cdot 5^2} \\
&= \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5^2} \\
&= \sqrt{2} \cdot 3 \cdot 5
\end{aligned}$$

In simplest radical form, $\sqrt{450}$ is equivalent to which of the following?

A) $8\sqrt{2}$
B) $15\sqrt{2}$
C) 21.2

Example 2 (continued)

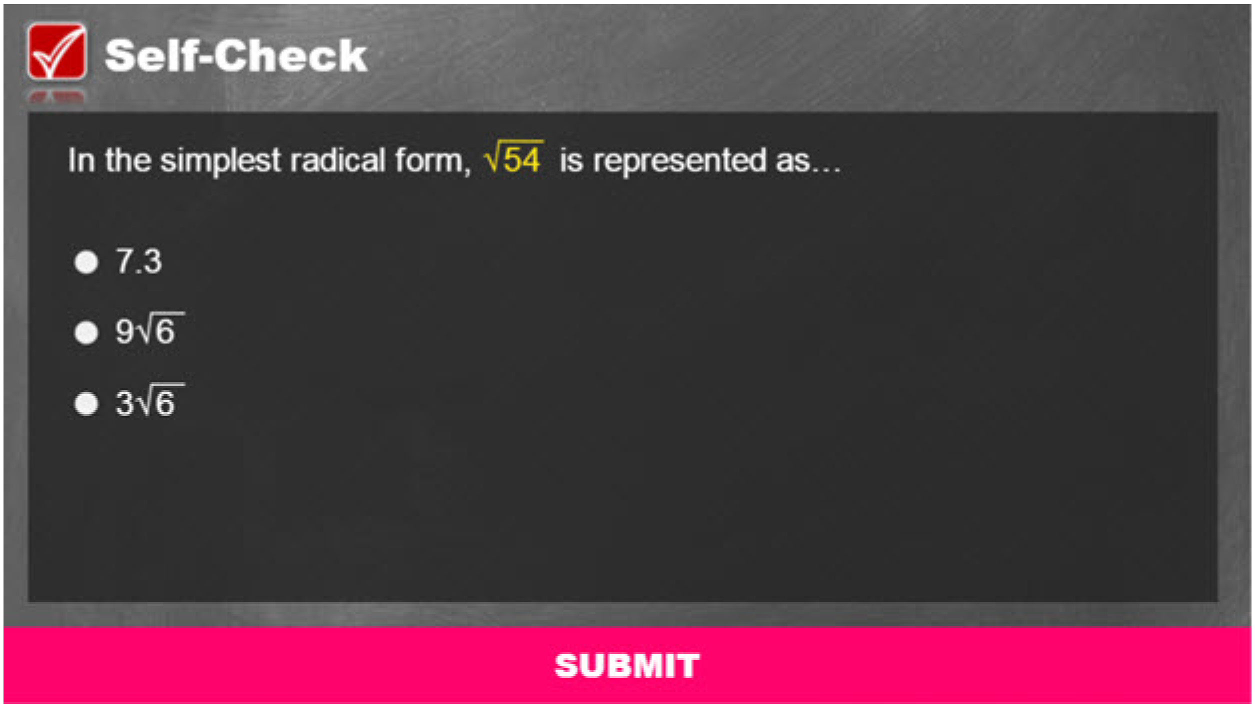


The correct answer is $15\sqrt{2}$. Review all the steps for representing the simplest radical form of $\sqrt{450}$.

$\sqrt{450} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5}$	Complete the prime factorization of 450
$= \sqrt{2 \cdot 3^2 \cdot 5^2}$	Represent multiple factors as square terms
$= \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5^2}$	Apply the Product Property of Radicals
$= \sqrt{2} \cdot 3 \cdot 5$	Simplify the perfect squares
$= \sqrt{2} \cdot 15$	Find the product of 5 and 3
$= 15\sqrt{2}$	Simplify the expression

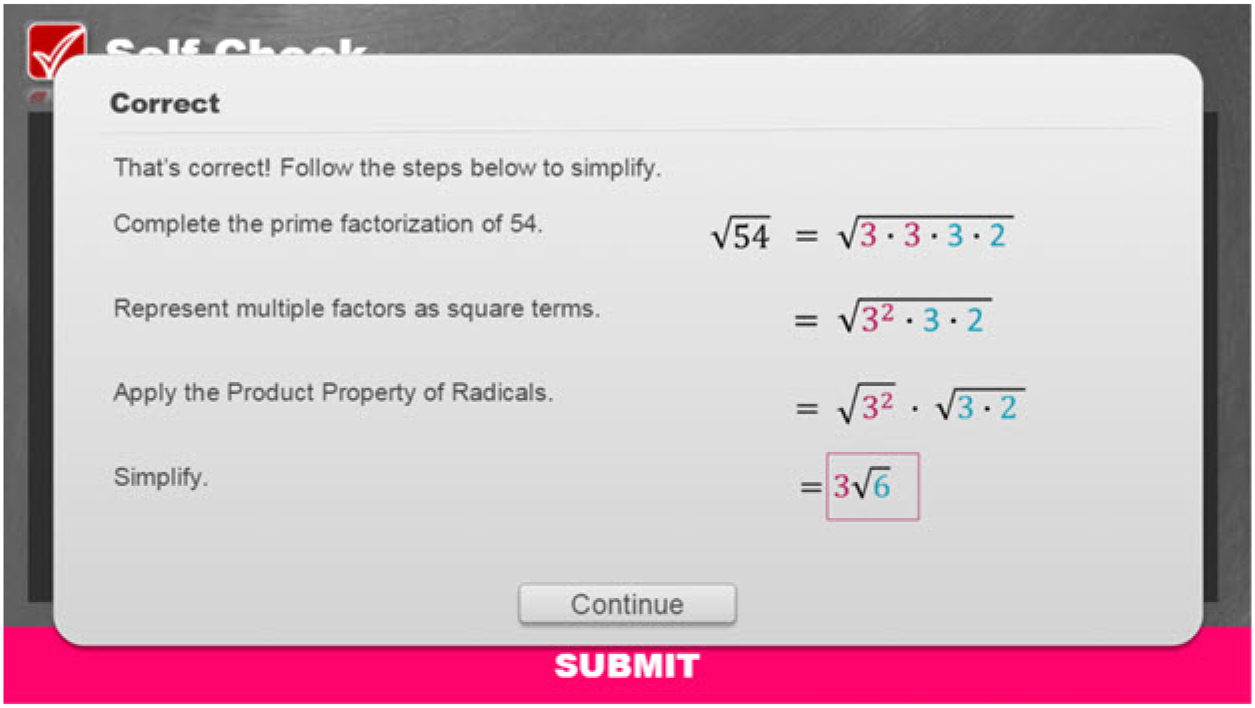
Your work is complete. The simplest radical form of $\sqrt{450}$ is $15\sqrt{2}$.

Self-Check



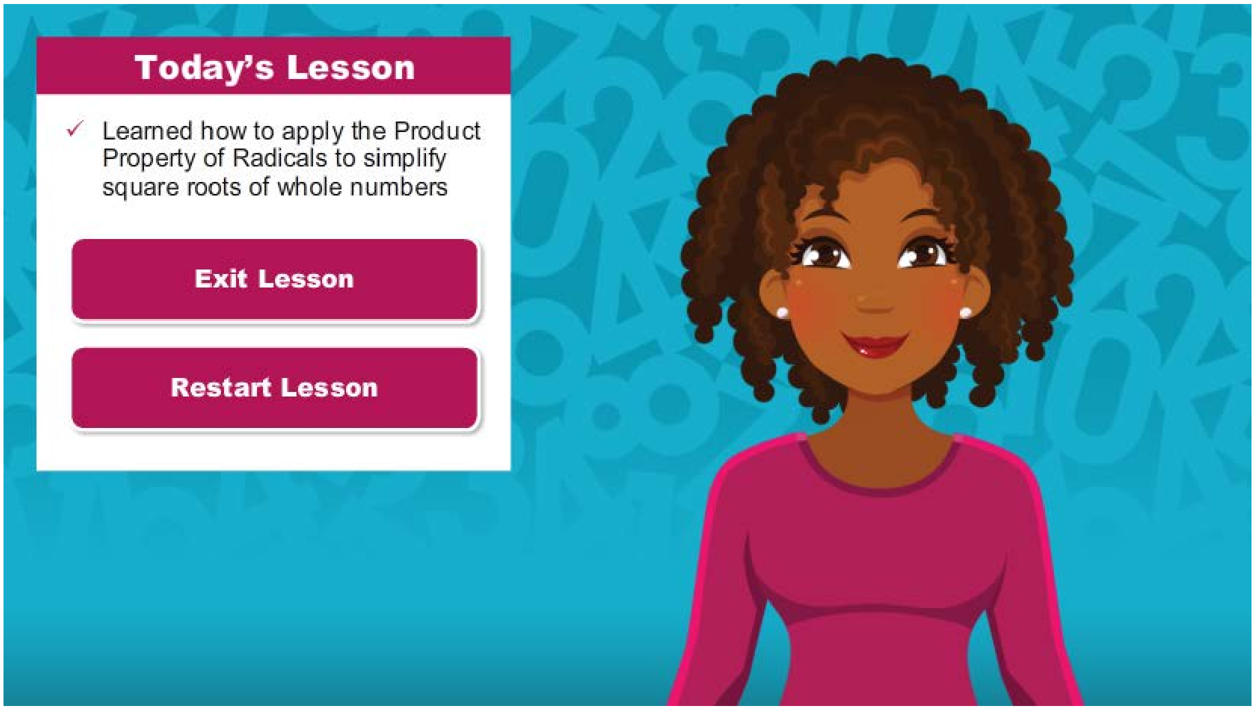
Solve the problem in the image above to check your understanding of the content.

Self-Check: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



You have reached the conclusion of this lesson where you learned how to apply the Product Property of Radicals to simplify square roots of whole numbers.