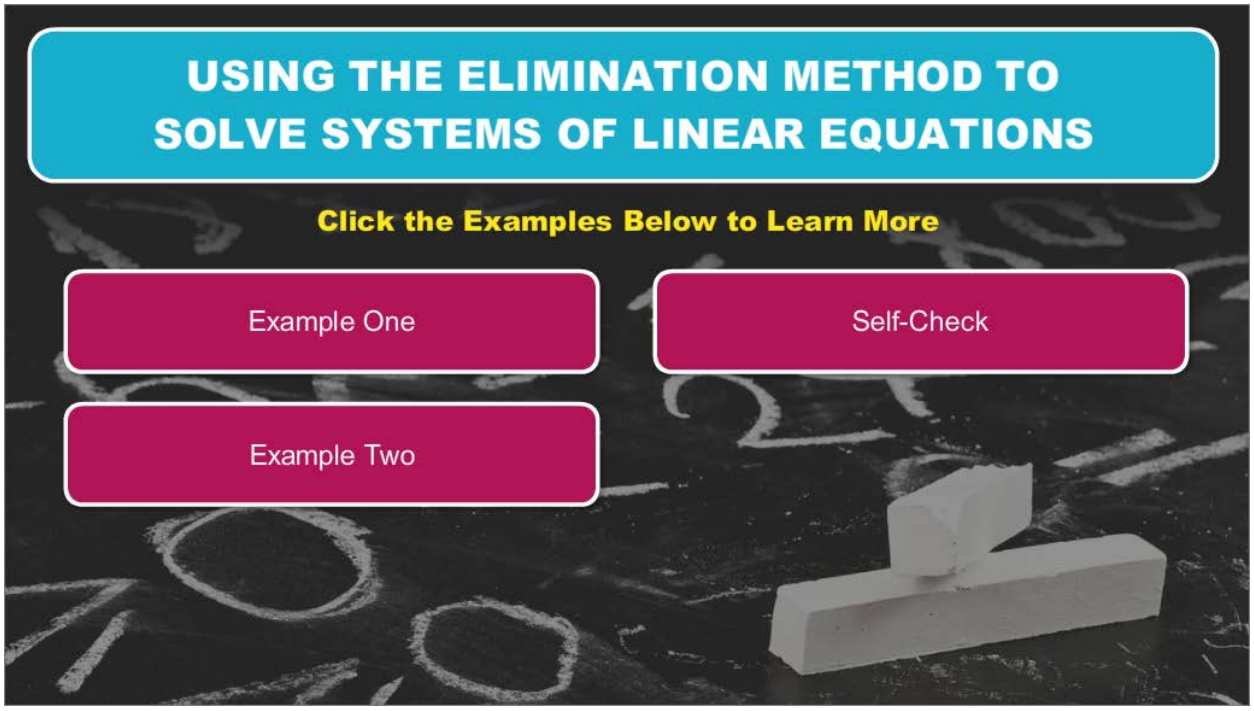
Module 5: Solving Linear Equations

Topic 3 Content: Using the Elimination Method to Solve Systems of Linear Equations

Introduction

Hello and welcome! I'm so glad you could join me for this lesson in Algebra I. In this lesson, you will learn how to use the elimination method to solve a system of linear equations.

Using the Elimination Method to Solve Systems of Linear Equations



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Example 1

Use the elimination method to solve the system of linear equations below.

$$\begin{cases} 3x + y = 4 \\ 2x - y = 21 \end{cases}$$

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$$\begin{array}{rl} 3x + y & = 4 \\ +\ 2x - y & = 21 \\ \hline 5x & = 25 \end{array}$$

When using the elimination method to solve a system of linear equations, the goal is to combine the equations in a manner that will eliminate a pair of variables. Like most problems in math, there is more than one way to arrive at the solution. In the case of the elimination method, you can use multiplication, addition, or subtraction to find the solution.

Consider the given system of equations. Notice that the first equation includes a y-term and the second equation includes a $-y$-term. If you add the equations, you will be able to eliminate the y-terms.

Combine the equations by adding each pair of like terms separately.

$3x + 2x = 5x$

$y + (-y) = 0$. So the y-terms have been eliminated.

$4 + 21 = 25$

Example 1 (continued)

Use the elimination method to solve the system of linear equations below.

$$\begin{cases} 3x + y = 4 \\ 2x - y = 21 \end{cases}$$

$5x = 25$ — The resulting equation only consists of one variable, x.

$\frac{5x}{5} = \frac{25}{5}$ — Divide each term by 5 to solve for x. The solution is $x = 5$.

$x = 5$

$x = 5$ — Now that you know the value of x, substitute it into either equation to find y.

$2x - y = 21$

$2(5) - y = 21$ — You can use the second equation, for example. Substitute 5 for x and solve for y.

$10 - y = 21$ — $2 \cdot 5 = 10$. Bring down the subtraction sign and y, along with the equals sign and 21.

$10 - y = 21$ — Subtract 10 from each side to begin isolating y.

$-10 \qquad -10$

$-y = 11$ — $10 - 10 = 0$. So these terms are cancelled out. Bring down the subtraction sign, y, and the equals sign.

$21 - 10 = 11$.

Example 1 (continued)

Use the elimination method to solve the system of linear equations below.

$$\begin{cases} 3x + y = 4 \\ 2x - y = 21 \end{cases}$$

$$\frac{-y}{-1} = \frac{11}{-1}$$

$$y = -11$$

The last step is to divide term side by -1. The result is $y = -11$.

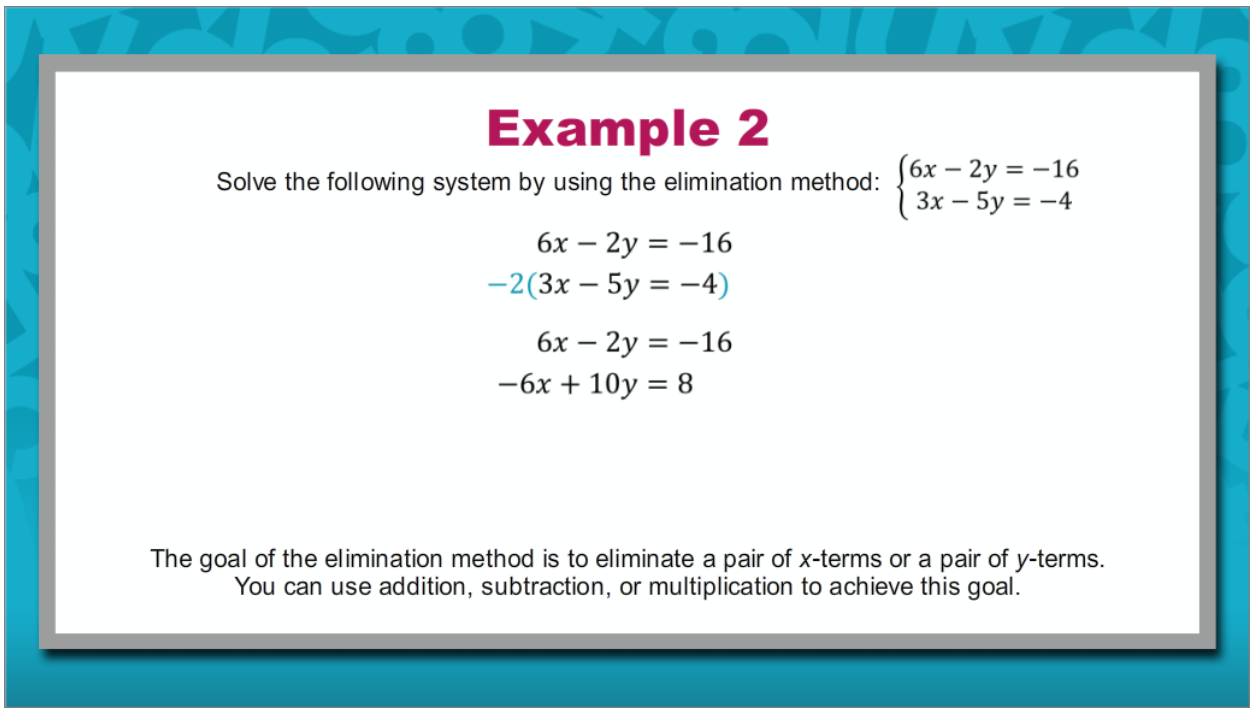
$$\begin{cases} 3x + y = 4 \\ 2x - y = 21 \end{cases}$$

$x = 5$ and $y = -11$

$(5, -11)$

The solution to the system of linear equations is $x = 5$ and $y = -11$. You can represent the solution as the ordered pair $(5, -11)$.

Example 2 (continued)



Solve the system below by using the elimination method.

$$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$$

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Remember that the goal of the elimination method is to eliminate a pair of x-terms or a pair of y-terms. You can use addition, subtraction, or multiplication to achieve this goal.

$6x - 2y = -16$

$-2(3x - 5y = -4)$

Consider the given system of equations. There are no terms that can be eliminated by simply adding the equations. You can, however, eliminate a pair of variables by multiplying one of the equations by a constant term and then adding the equations. For example, multiplying the second equation by -2 and then adding the equations will allow you to eliminate the x-terms.

$6x - 2y = -16$

$-6x + 10y = 8$

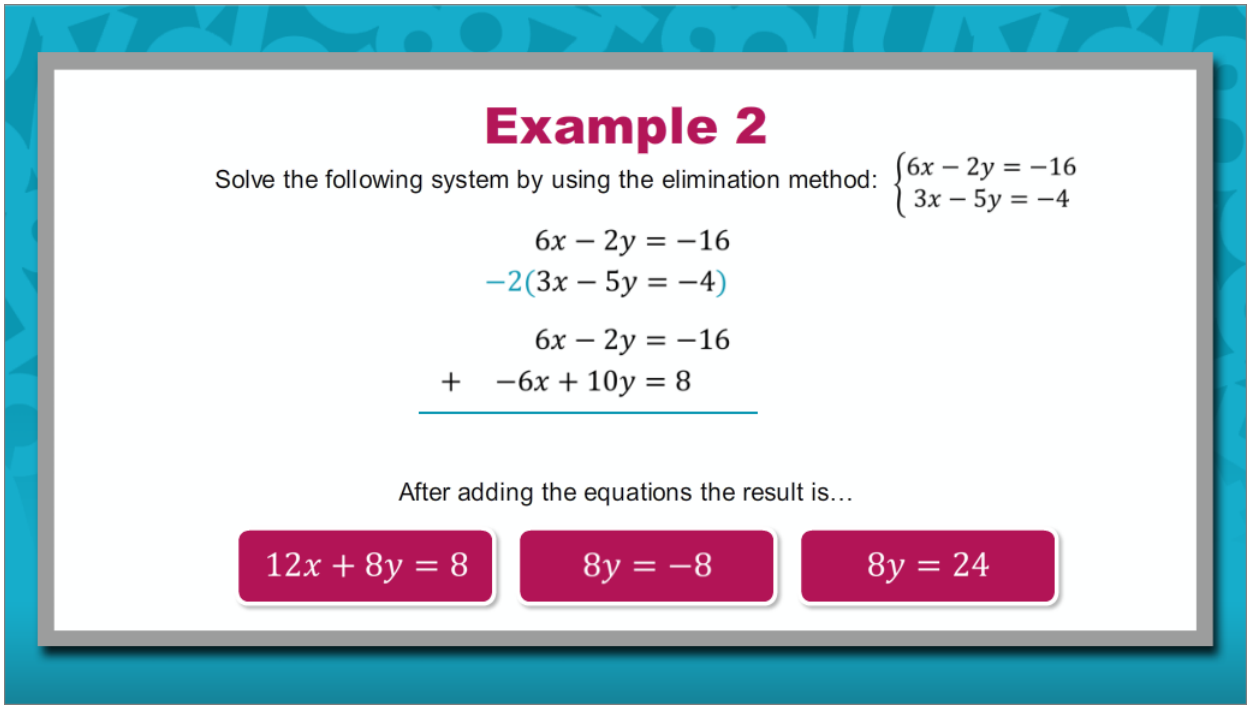
The first equation is unchanged. Multiply each term in the second equation by -2.

$-2 \cdot 3x = -6x$

$-2 \cdot -5y = 10y$

$-2 \cdot -4 = 8$

Example 2 (continued)



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$$\begin{cases} 6x-2y=-16 \\ 3x-5y=-4 \end{cases}$$

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$$6x-2y=-16$$

$$-2(3x-5y=-4)$$

$$6x-2y=-16$$

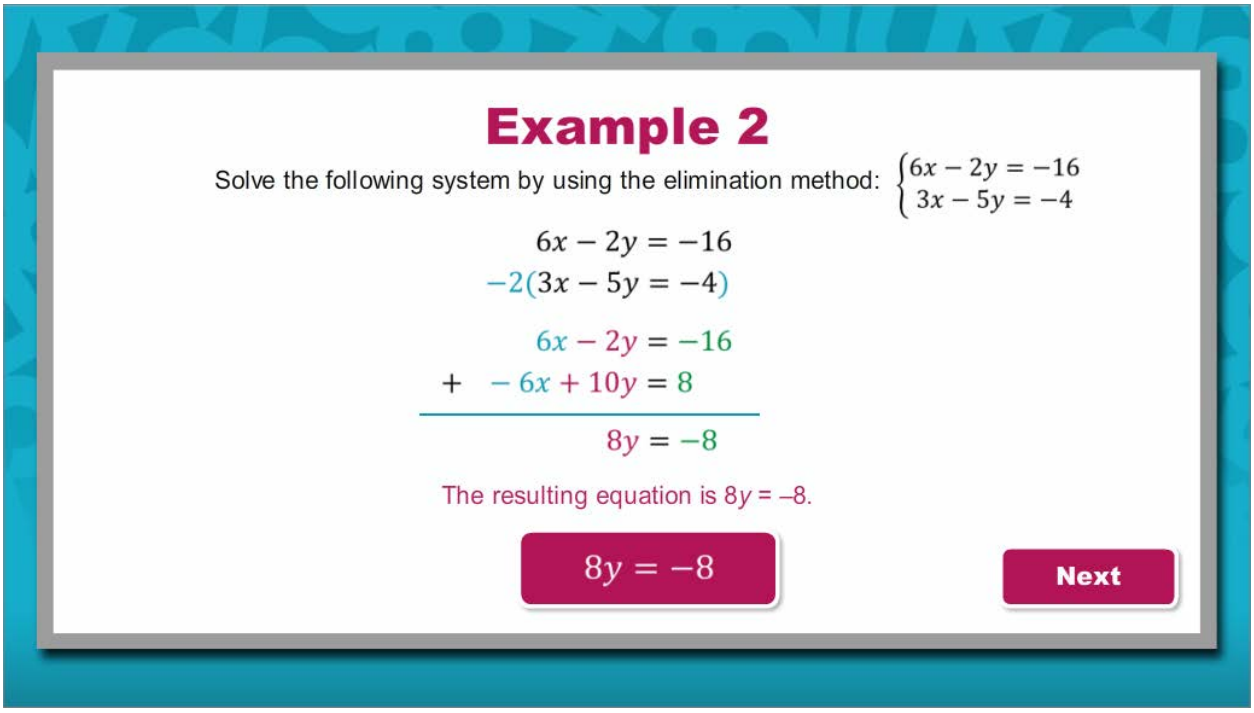
$$+\quad -6x+10y=8$$

Now add the equations.

After adding the equations, the result is…

A) $12x+8y=8$
B) $8y=-8$
C) $8y=24$

Example 2 (continued)



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$6x - 2y = -16$

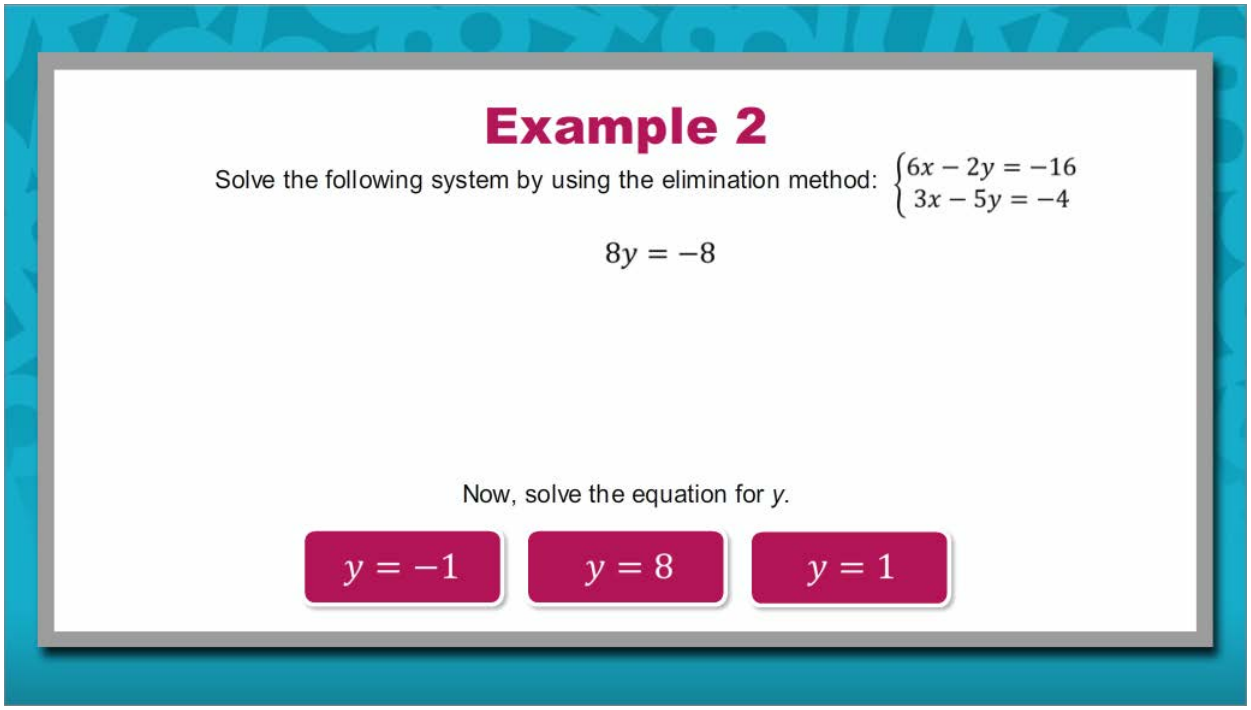
$-2(3x - 5y = -4)$

$6x - 2y = -16$

$+ \quad -6x + 10y = 8$

$8y = -8$

Example 2 (continued)



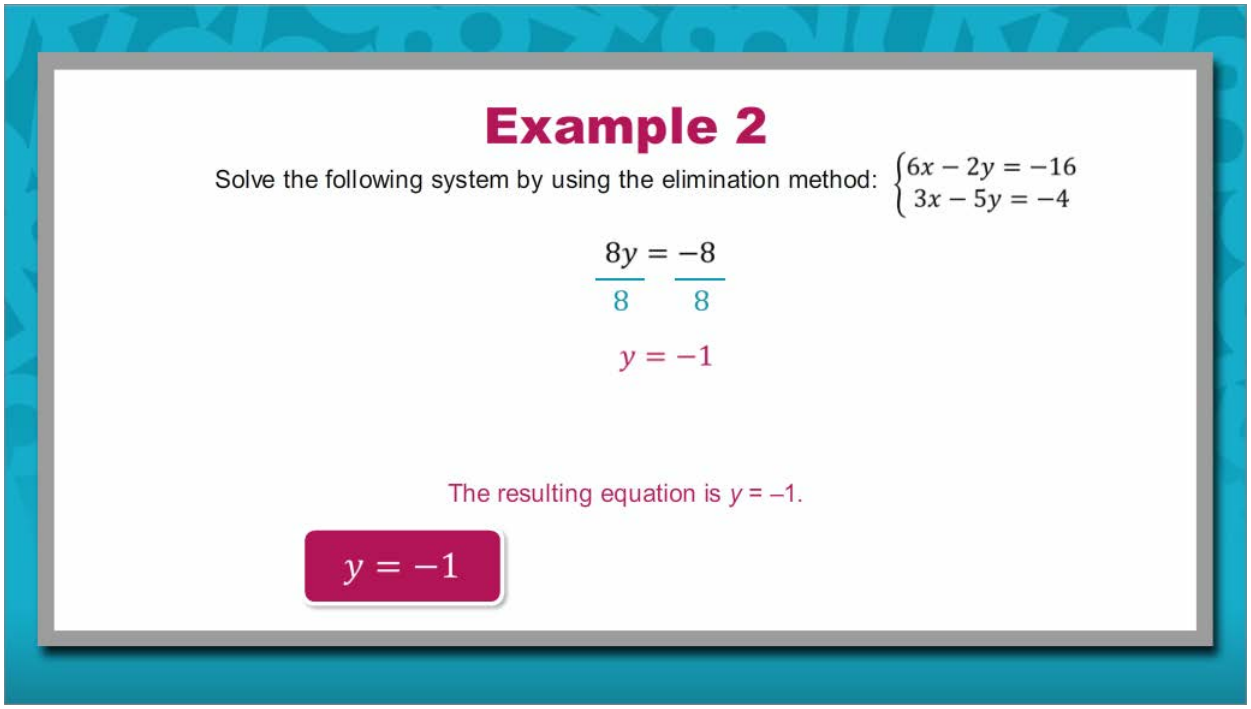
Solve the system below by using the elimination method.

$$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$$

$8y = -8$ Now, solve the resulting equation for y.

A) $y = -1$
B) $y = 8$
C) $y = 1$

Example 2 (continued)



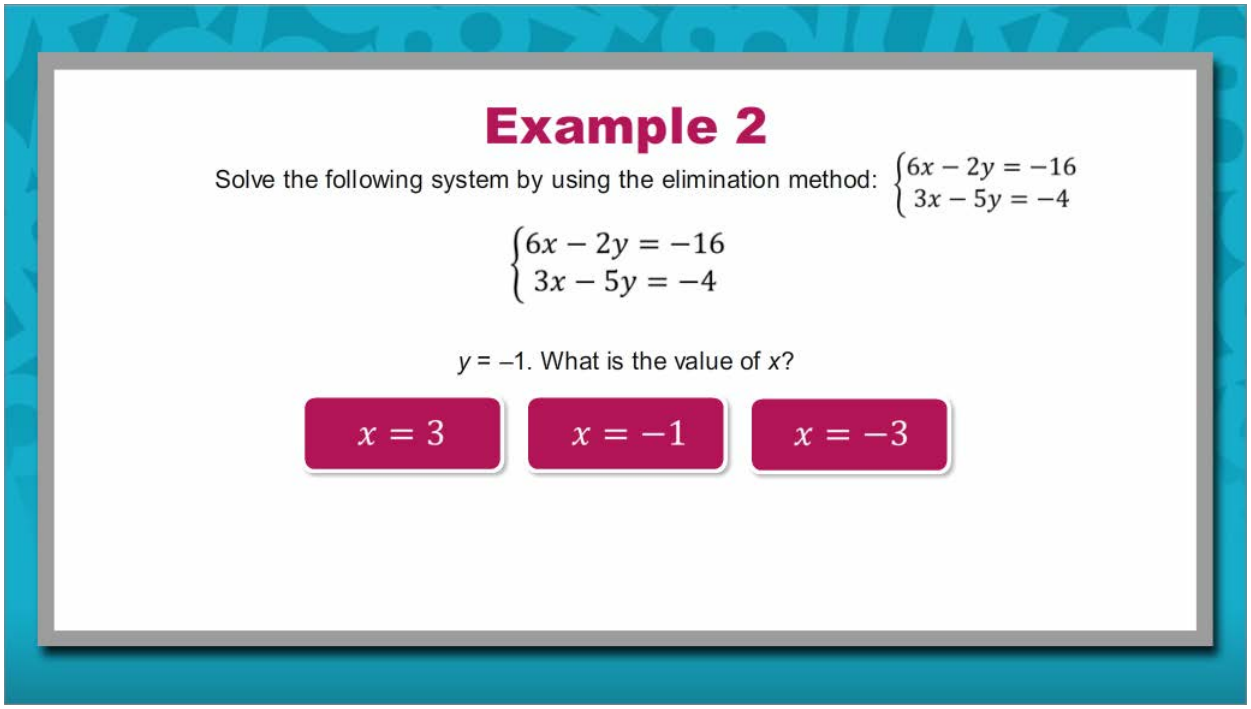
Solve the system below by using the elimination method.

$$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$$

$\frac{8y}{8} = \frac{-8}{8}$ The resulting equation is $y = -1$.

$y = -1$

Example 2 (continued)



Solve the system below by using the elimination method.

$$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$$

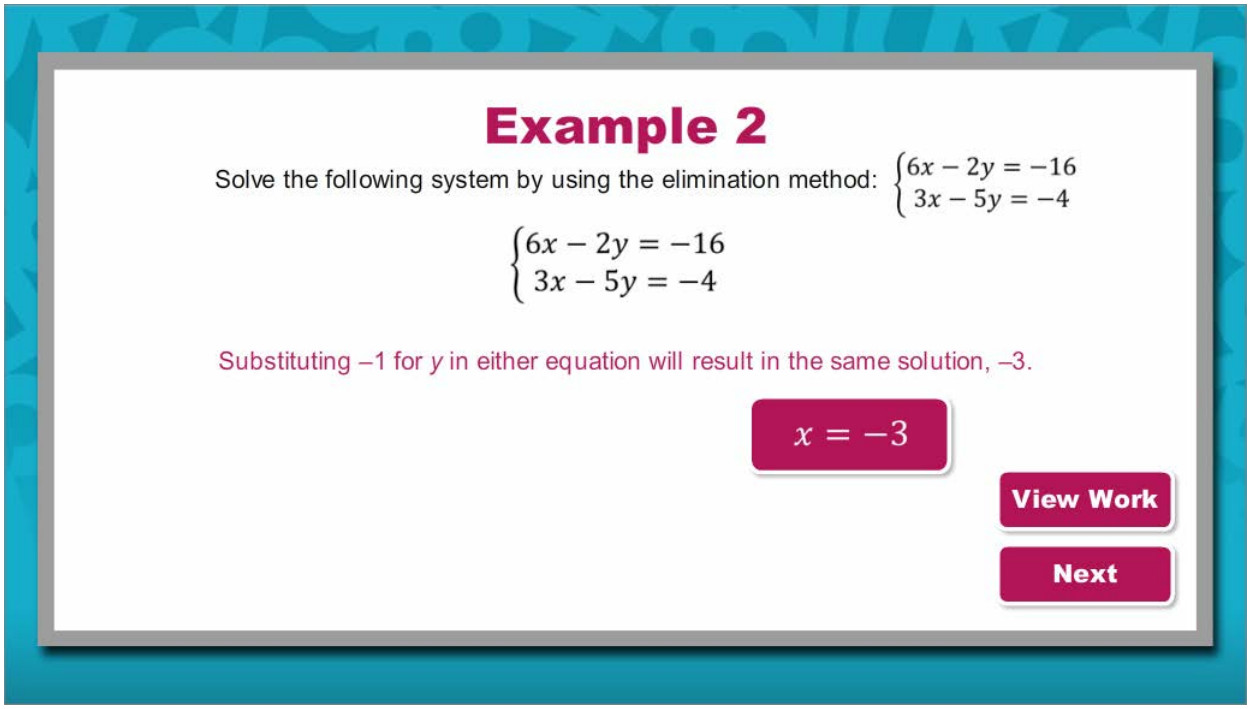
$y = -1$

Now that you know that $y = -1$, substitute the value into one of the two original equations and solve for x.

What is the value of x?

A) $x = 3$
B) $x = -1$
C) $x = -3$

Example 2 (continued)



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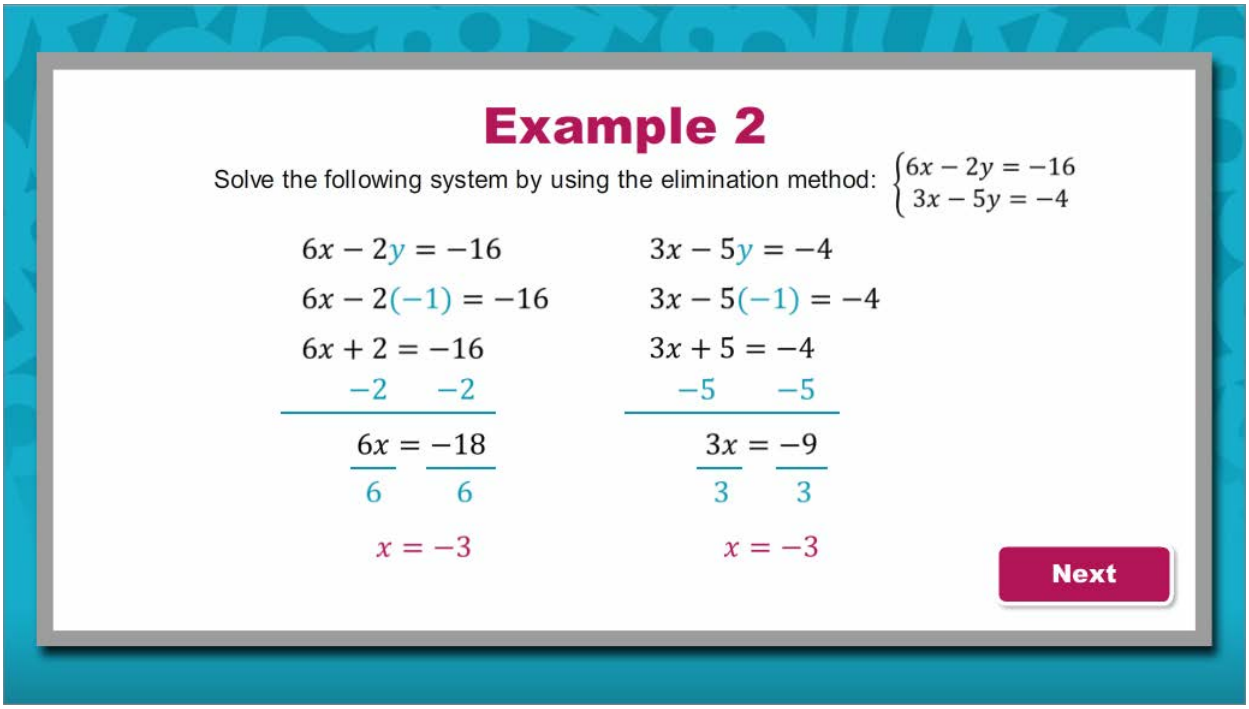
$$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$$

$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$ Substituting -1 for y in either equation, will result in the same solution, -3 .

Example 2 (continued)



Solve the system below by using the elimination method.

$$\begin{cases} 6x-2y=-16 \\ 3x-5y=-4 \end{cases}$$

$6x-2y=-16$

$6x-2(-1)=-16$

$6x+2=-16$

$\frac{6x+2}{-2}=\frac{-16}{-2}$

$\frac{6x}{6}=\frac{-18}{6}$

$x=-3$

$3x-5y=-4$

$3x-5(-1)=-4$

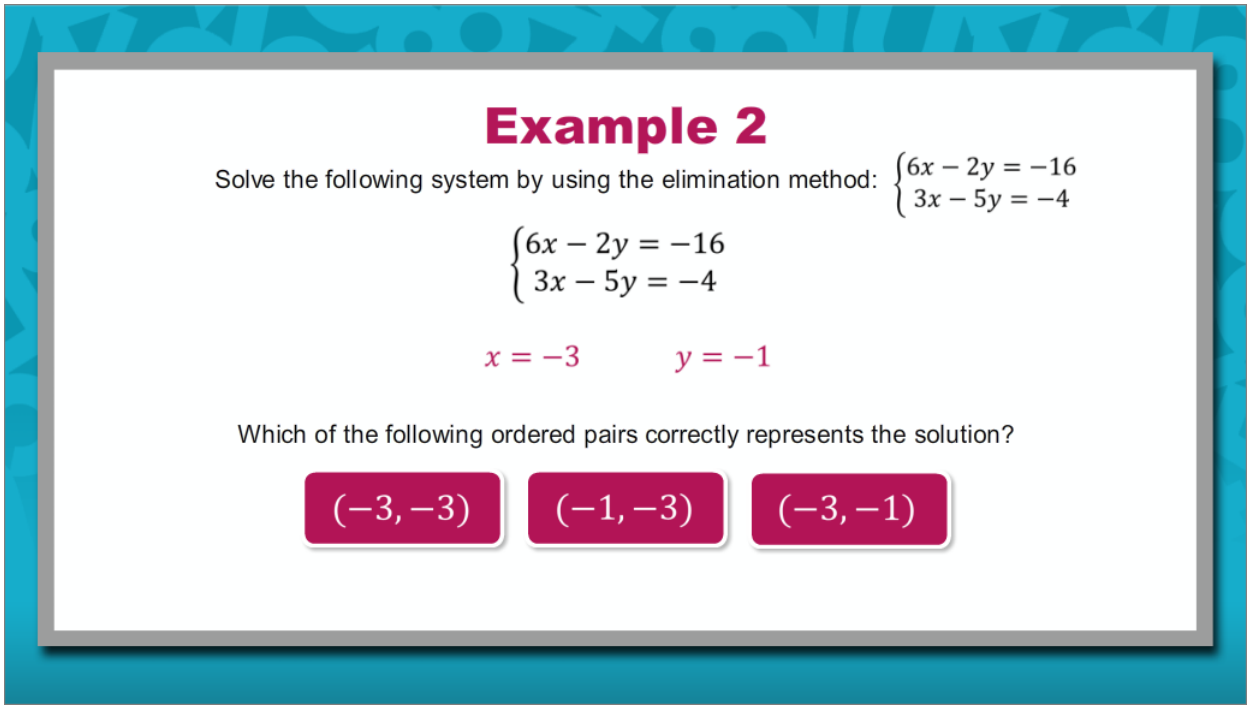
$3x+5=-4$

$\frac{3x+5}{-5}=\frac{-4}{-5}$

$\frac{3x}{3}=\frac{-9}{3}$

$x=-3$

Example 2 (continued)



Solve the system below by using the elimination method.

$$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$$

$\begin{cases} 6x - 2y = -16 \\ 3x - 5y = -4 \end{cases}$

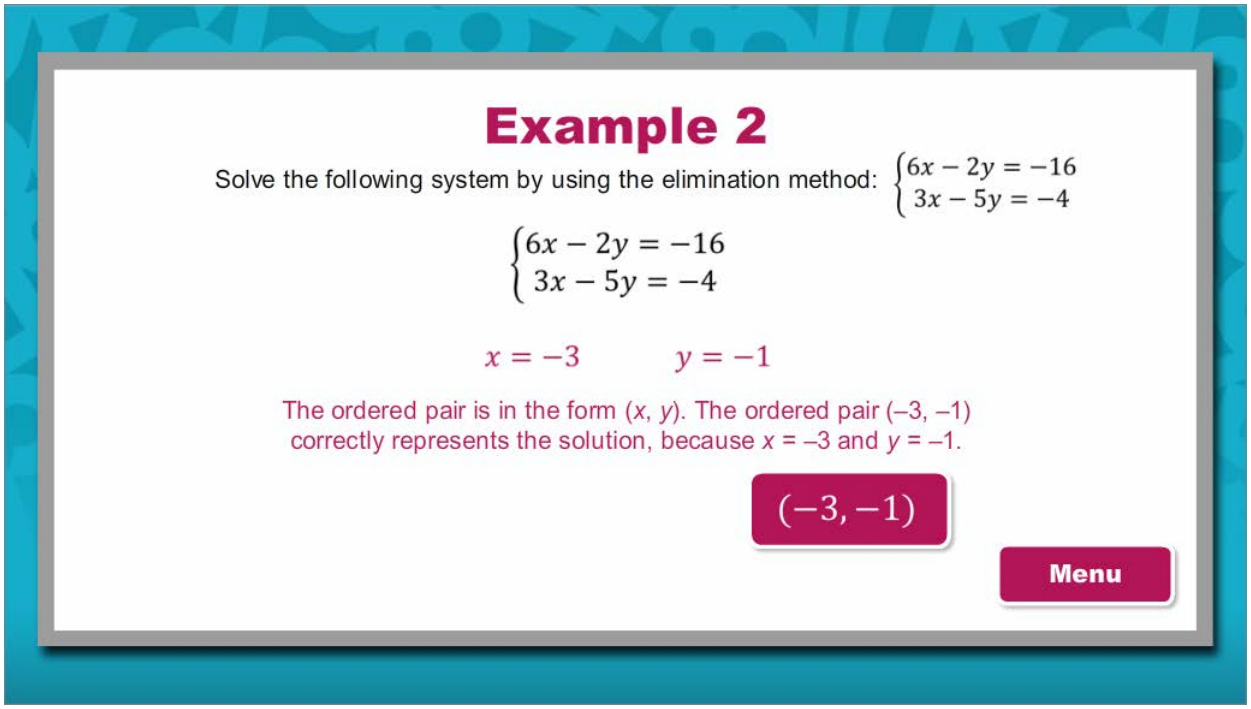
$x = -3$ $y = -1$

The solution to the system of linear equations is $x = -3$ and $y = -1$.

Which of the following ordered pairs correctly represents the solution?

A) $(-3, -3)$
B) $(-1, -3)$
C) $(-3, -1)$

Example 2 (continued)



Solve the system below by using the elimination method.

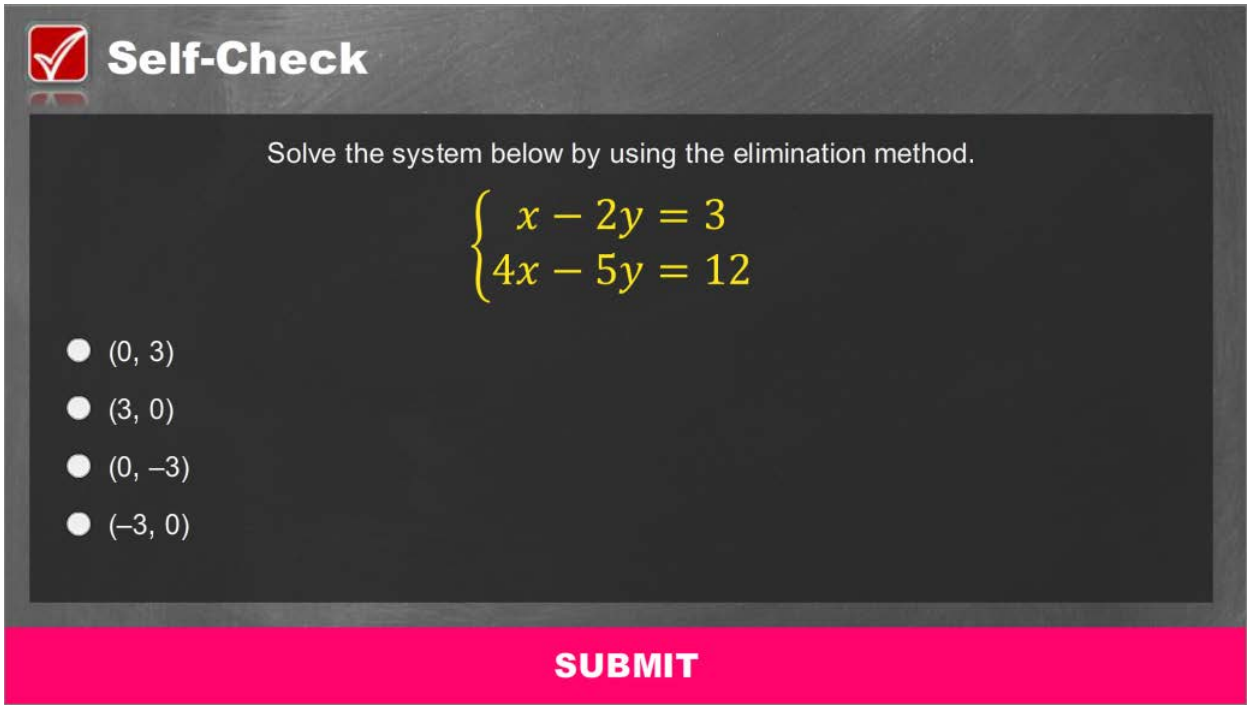
$$\begin{cases}6x-2y=-16\\3x-5y=-4\end{cases}$$

$\begin{cases}6x-2y=-16\\3x-5y=-4\end{cases}$

$x=-3$ $y=-1$

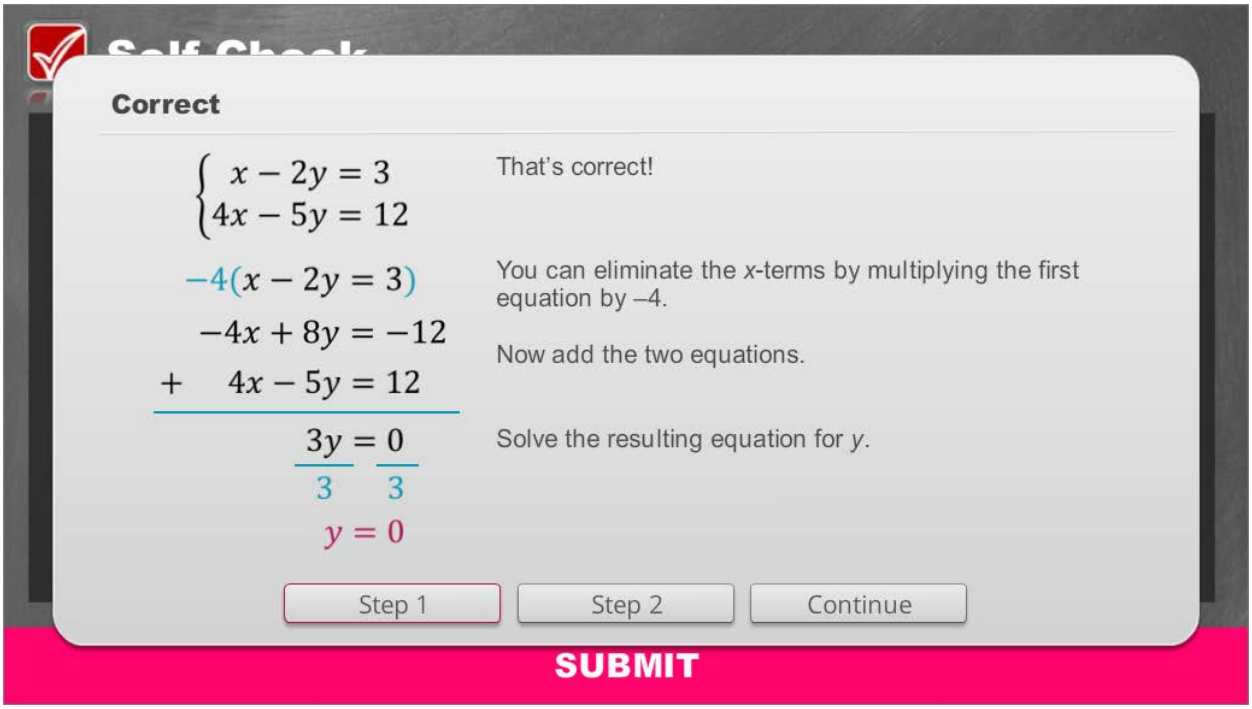
The ordered pair is in the form (x,y). The ordered pair $(-3,-1)$ correctly represents the solution, because $x=-3$ and $y=-1$.

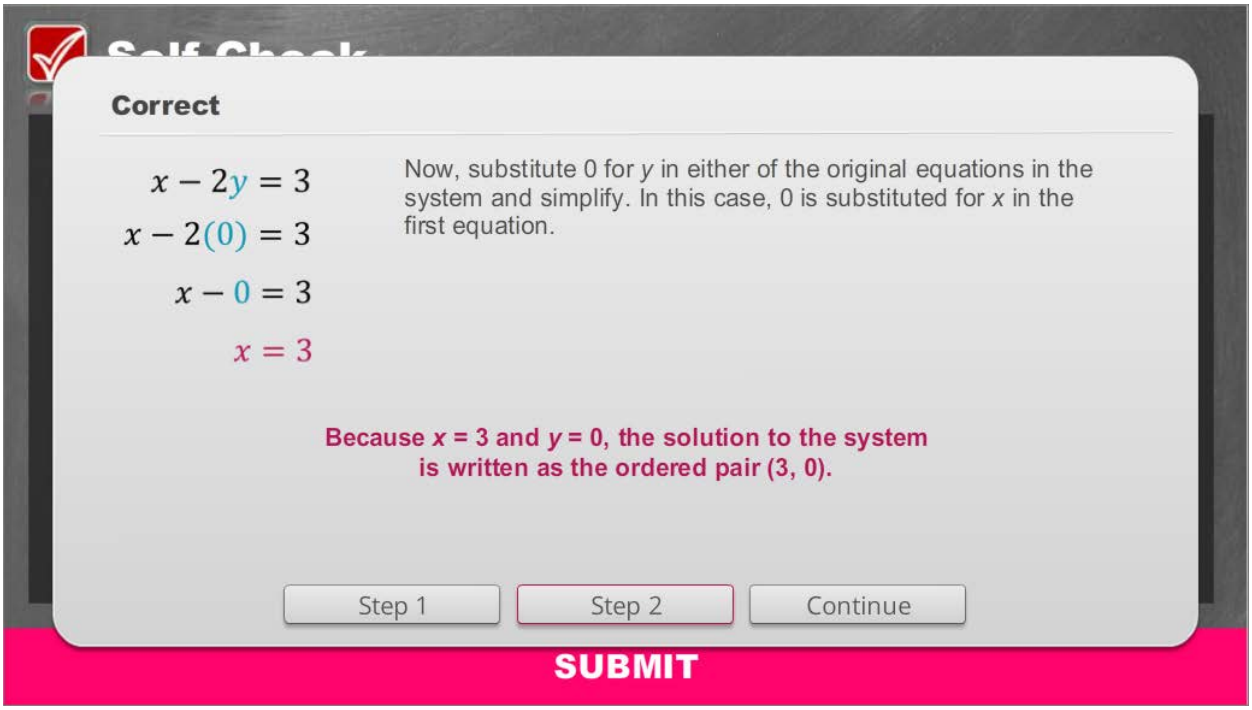
Self-Check



Solve the problem in the image above to check your understanding of the content.

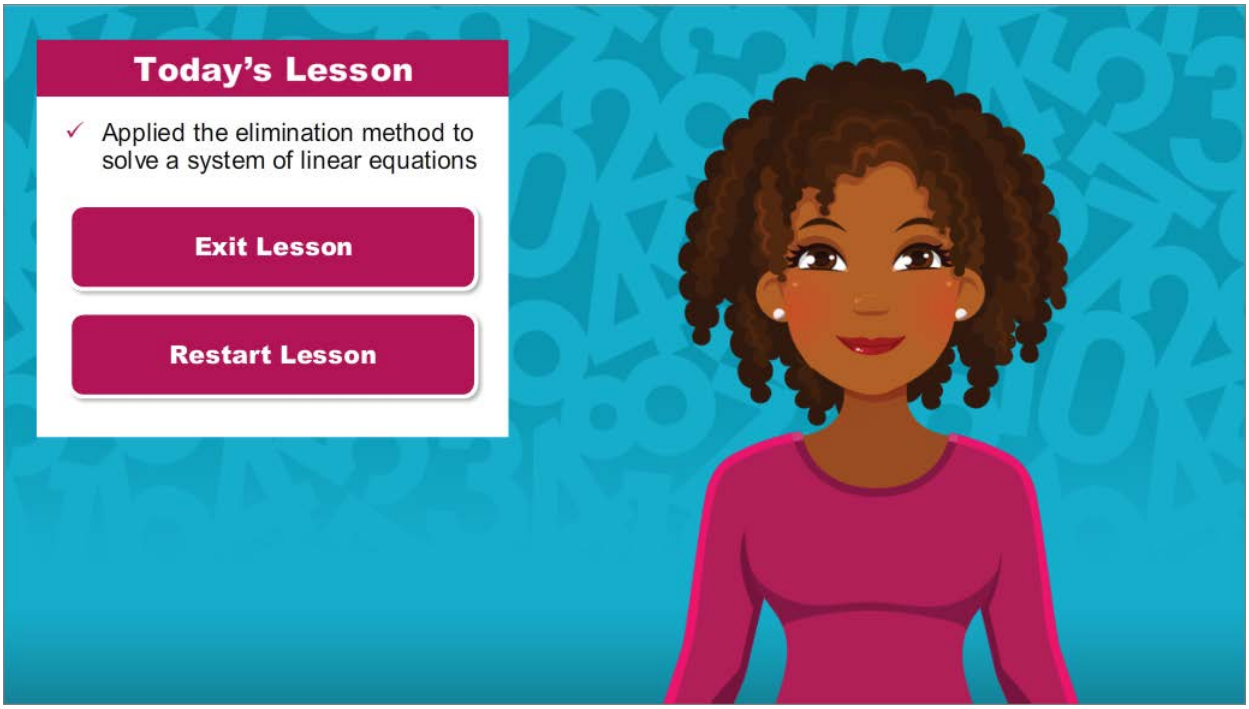
Self-Check: Answer





For your reference, the images above shows the correct solution to the self-check problem.

Conclusion



You have reached the conclusion of this lesson, where you learned how to use the elimination method to solve a system of linear equations.