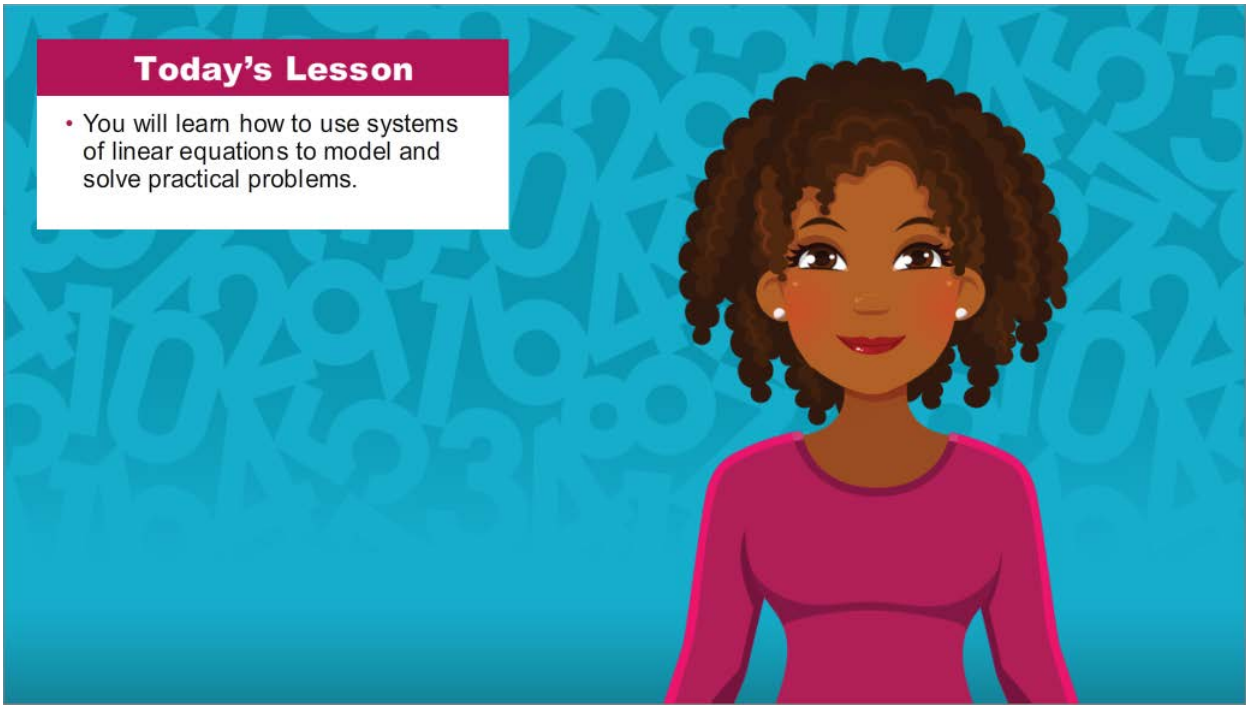
Module 5: Solving Linear Equations

Topic 4 Content: Practical Problems Involving Systems of Linear Equations

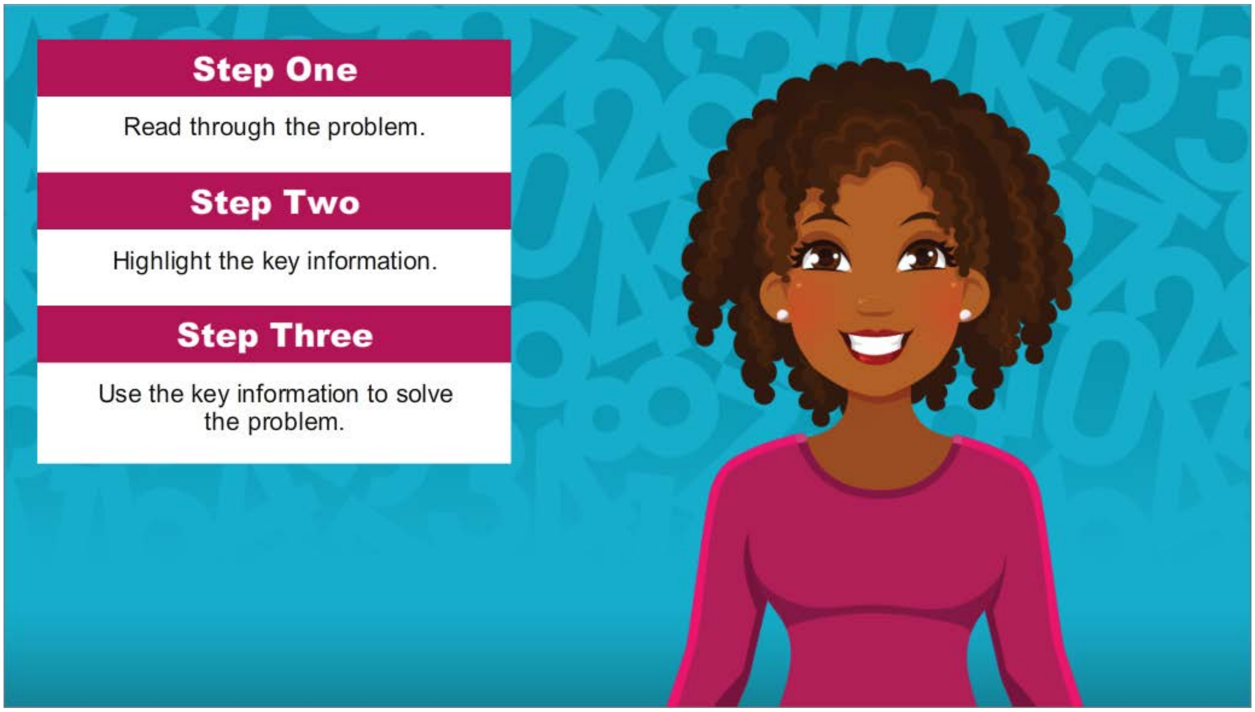
Introduction



Hi there! I'm so glad you could join me for this lesson in Algebra I, where you will learn how to use systems of linear equations to model and solve practical problems.

Module 5: Solving Linear Equations
Topic 4 Content: Practical Problems Involving Systems of Linear Equations

Anticipatory Set



Use the following steps to guide you in the process of using systems of linear equations to solve real-world problems.

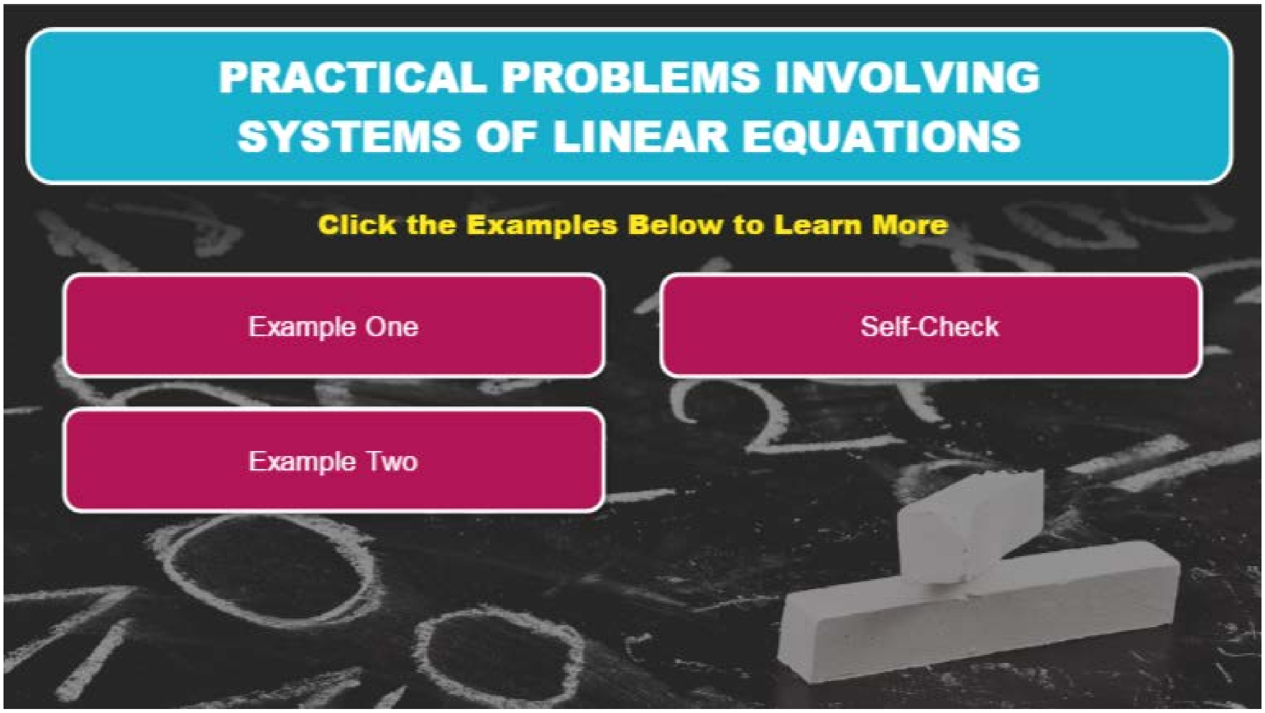
Step 1: Read through the problem.

Step 2: Highlight the key information.

Step 3: Use the key information to solve the problem.

Keep these steps in mind while you work through the following examples.

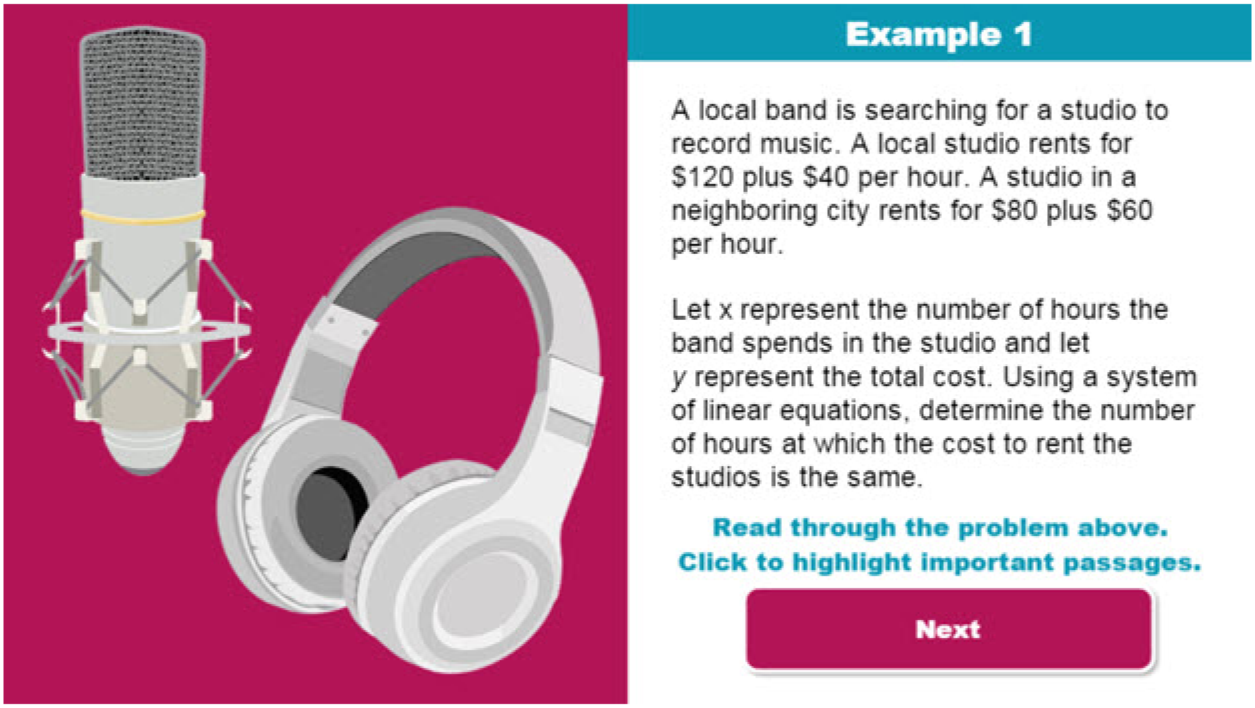
Practical Problems Involving Systems of Linear Equations



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Example 1

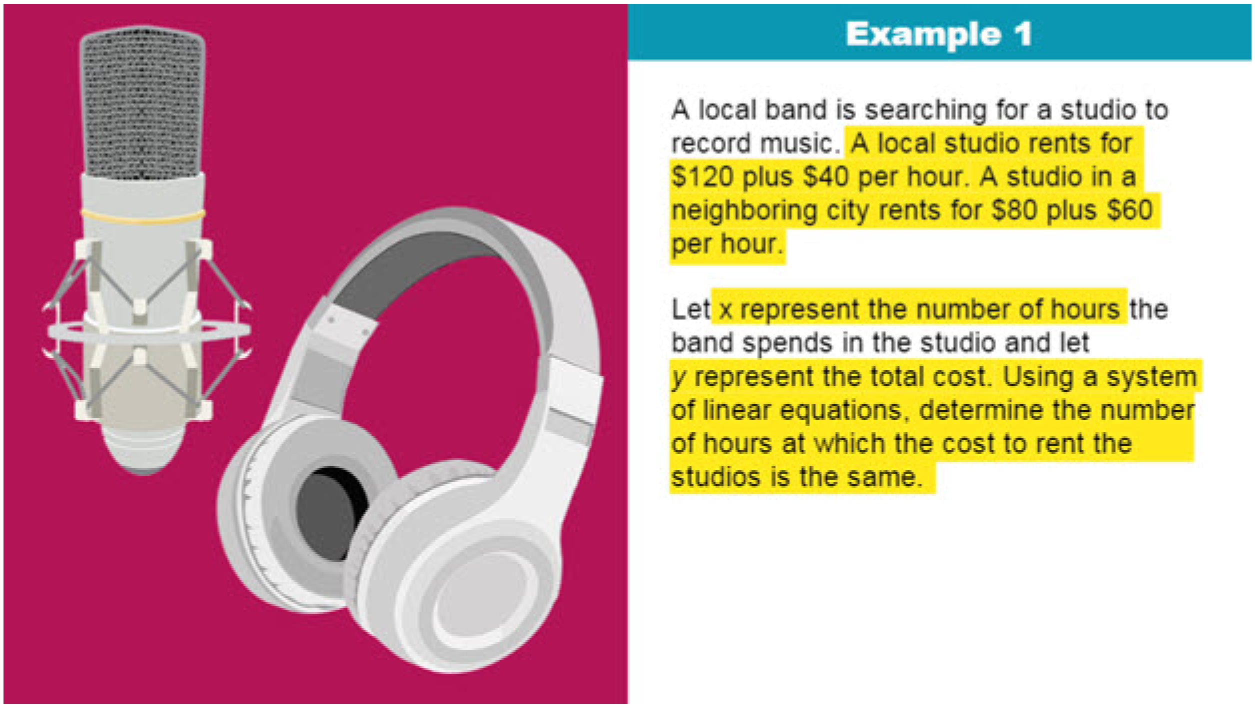


Take a few moments to read Example 1. Highlight the information you think is necessary to solve the problem.

A local band is searching for a studio to record music. A local studio rents for $120 *plus* $40 *per hour. A studio in a neighboring city rents for* $80 *plus* $60 *per hour. Let* x *represent the number of hours the band spends in the studio and let* y *represent the total cost. Using a system of linear equations, determine the number of hours at which the cost to rent the studios is the same.*

Read through the problem above. Click to highlight important passages.

Example 1 (continued)

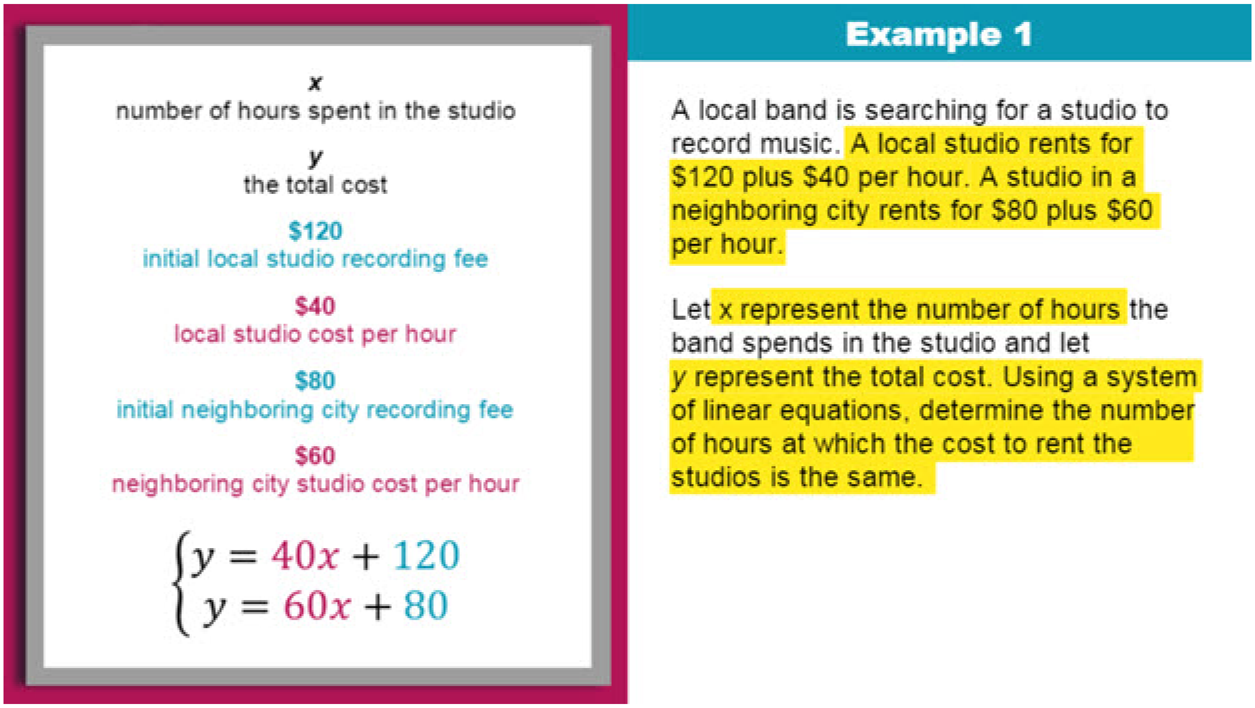


Did you highlight the important facts?

A local band is searching for a studio to record music. A local studio rents for $120 *plus* $40 *per hour. A studio in a neighboring city rents for* $80 *plus* $60 *per hour. Let* x *represent the number of hours the band spends in the studio and let* y *represent the total cost. Using a system of linear equations, determine the number of hours at which the cost to rent the studios is the same.*

- A local studio rents for $120 plus $40 per hour
- A studio in a neighboring city rents for $80 plus $60 per hour
- x represents the number of hours
- y represents the total cost
- Using a system of linear equations, determine the number of hours at which the cost to rent the studios is the same

Example 1 (continued)



Recall that a system of equations is a group of equations. Therefore, you will write more than one equation to model this situation. In this scenario, x represents the number of hours the band spends in the studio and y represents the total cost.

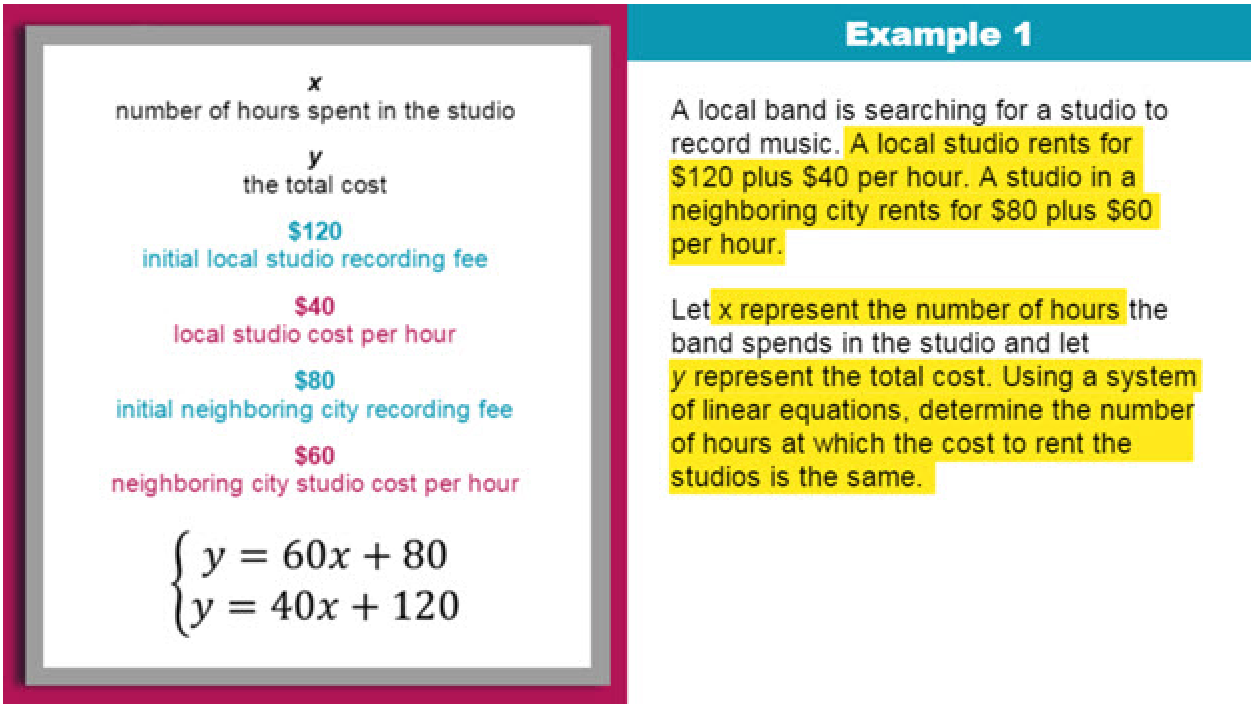
If the band chooses to reserve time at the local studio, the total cost will be $120 plus $40 per hour. Therefore, this part of the scenario can be represented by the equation:

$$y = 40x + 120$$

If the band chooses the studio located in a neighboring city, the total cost will be $80 plus $60 per hour. So, this part of the scenario can be represented by the equation:

$$y = 60x + 80$$

Example 1 (continued)



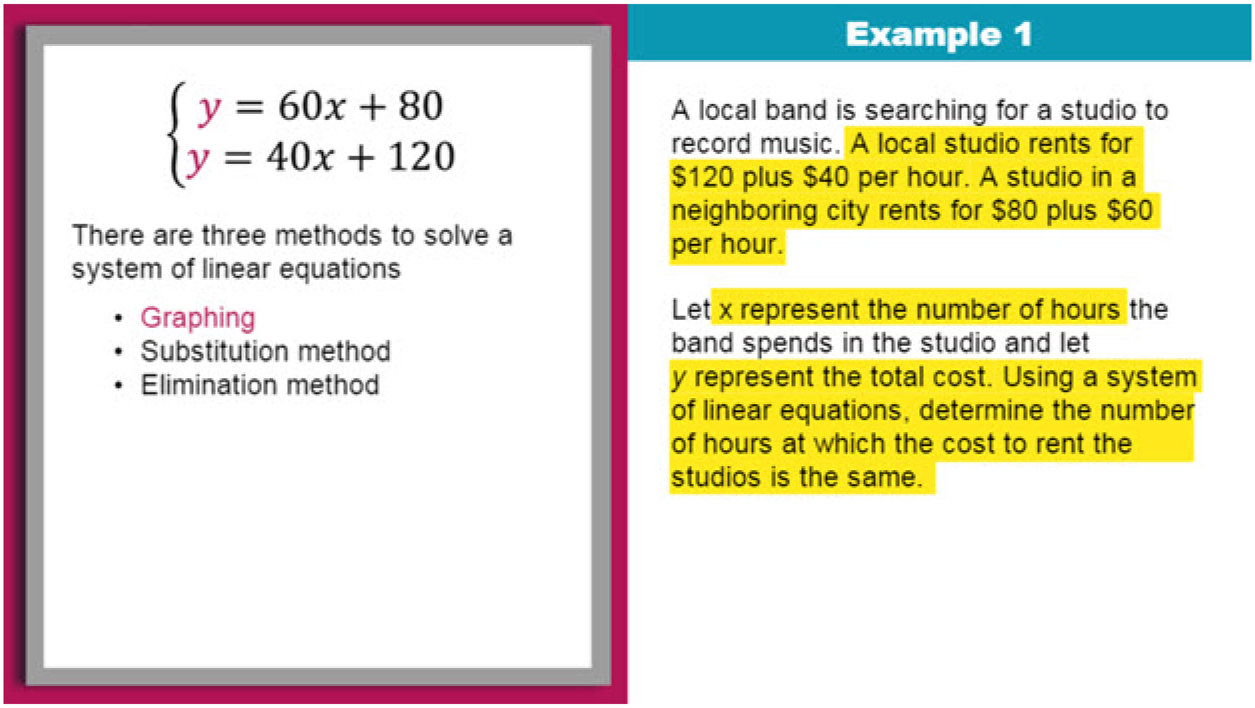
Now, you have written the two linear equations necessary to complete the system:

$$\begin{cases} y = 40x + 120 \\ y = 60x + 80 \end{cases}$$

It is worth it to mention that the equations in a system can be written in any order. If you were to switch the order that the equations were written, the system would still correctly model the situation.

$$\begin{cases} y = 60x + 80 \\ y = 40x + 120 \end{cases}$$

Example 1 (continued)



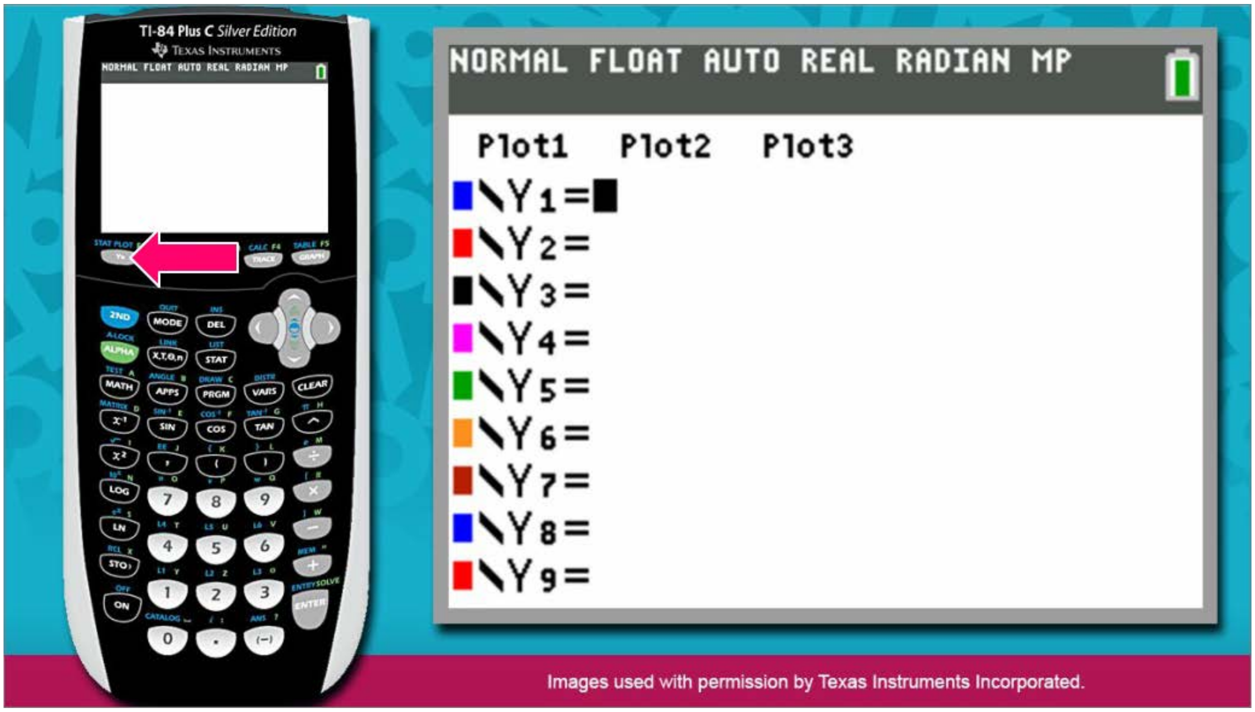
$$\begin{cases} y = 40x + 120 \\ y = 60x + 80 \end{cases}$$

There are three methods to solve a system of linear equations:

- By graphing;
- By the substitution method; and
- By the elimination method.

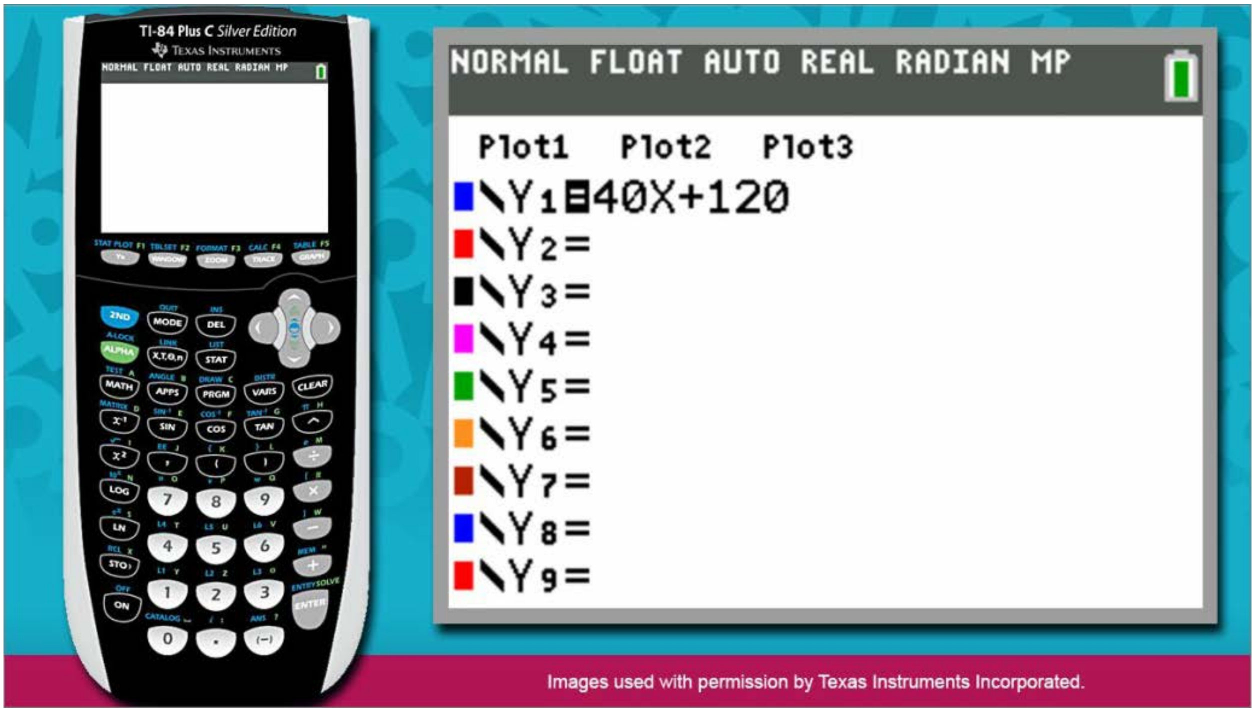
You may choose any of these methods to determine the solution to the system of linear equations. For this example, because both equations include relatively large numbers and are already solved for y, the solution will be determined by using the graphing calculator to graph the system.

Example 1 (continued)



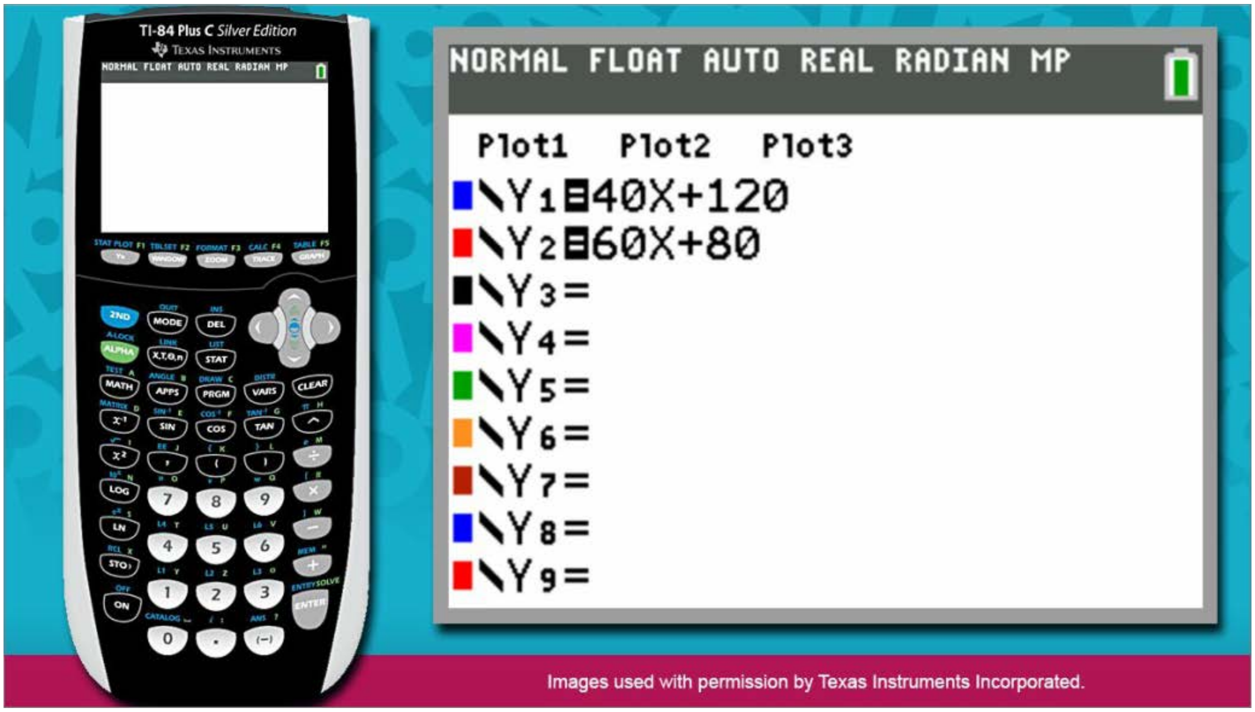
Begin by clearing the calculator's memory. Press the Y= key, located in the top left corner of the keys.

Example 1 (continued)



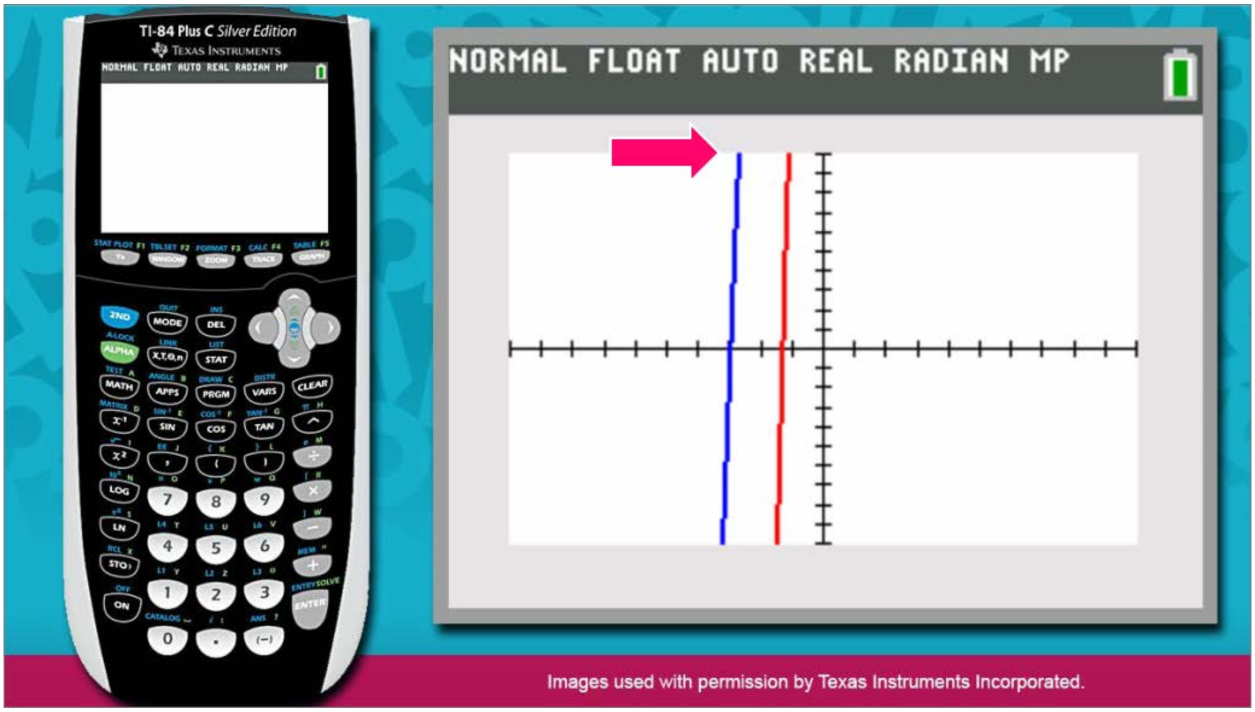
Enter the right side of the first equation to the right of Y1.

Example 1 (continued)



Press the down arrow key. Then enter the right side of the second equation to the right of Y2.

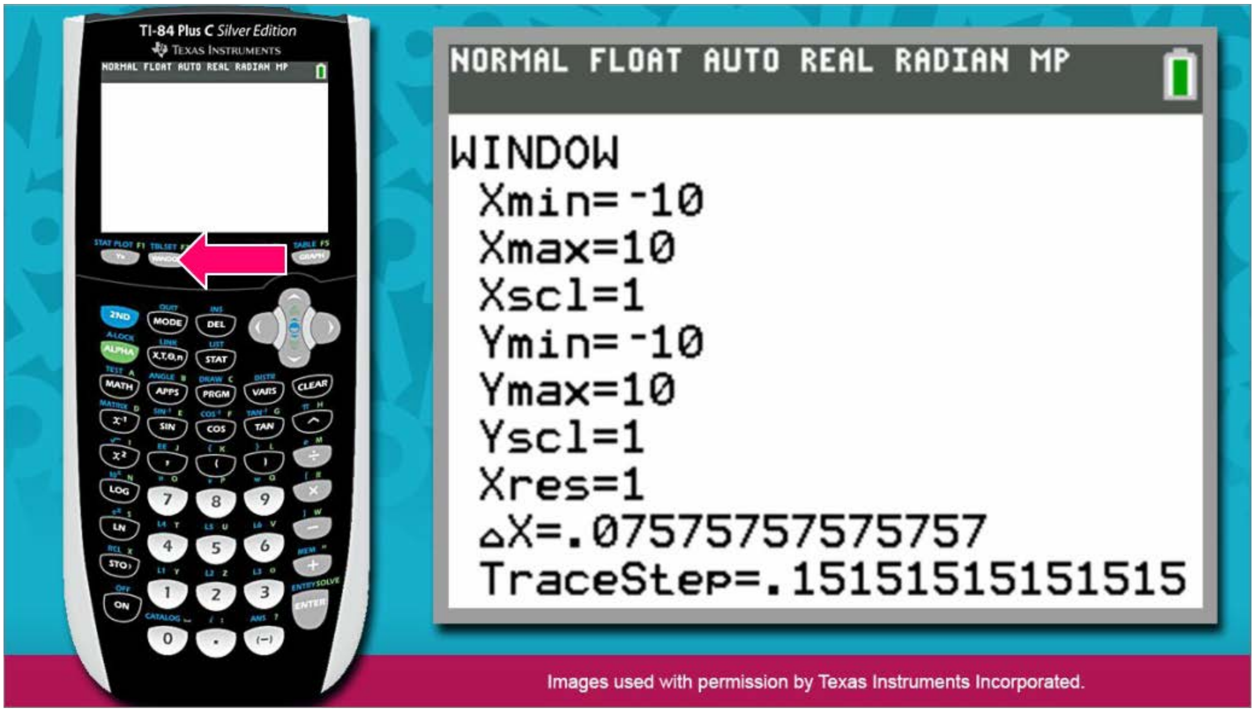
Example 1 (continued)



Now it's time to graph the system. Press the GRAPH key, located in the top right corner of the keys.

Notice that you are only able to view a small portion of the lines. Also, the distance between the lines appears to narrow near the top of the window. It is possible that the lines intersect at a point that you are currently unable to view. In order to view a larger portion of the graph of this system, you will need to adjust the scale along the x- and y-axes. This is the time for you to consider what values of x and y make sense for this real-world scenario.

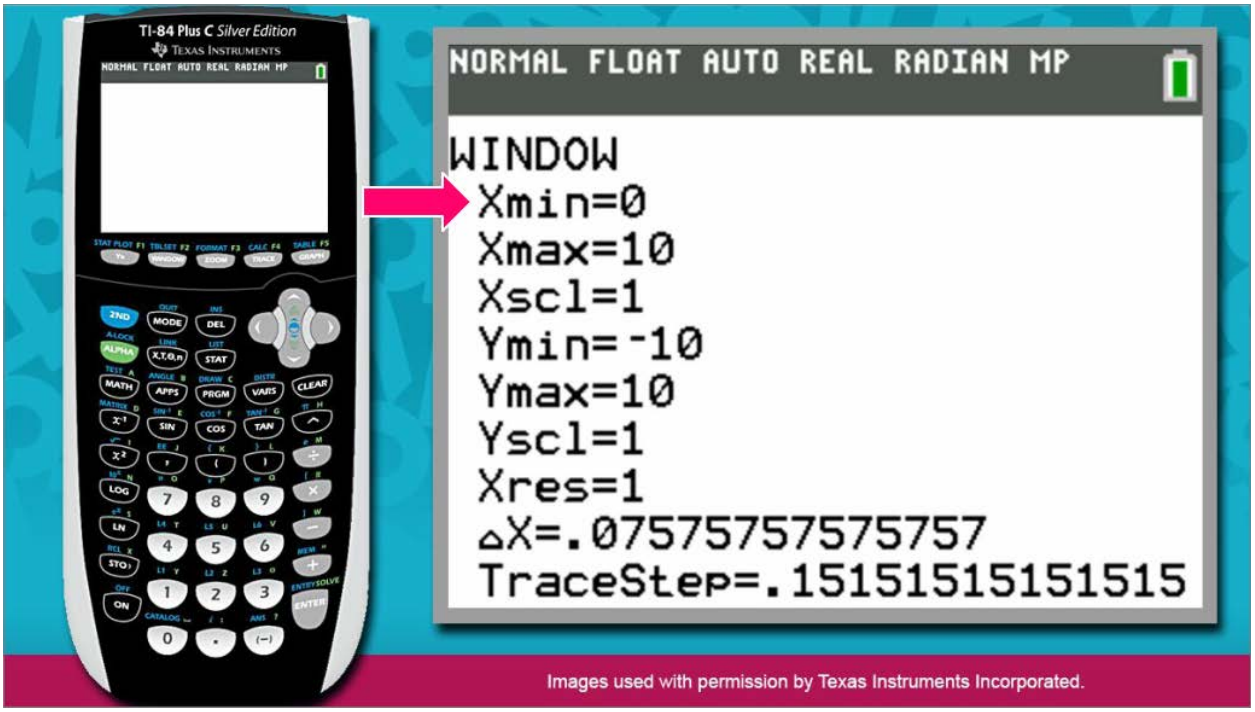
Example 1 (continued)



Press WINDOW, the key located to the right of the Y= key.

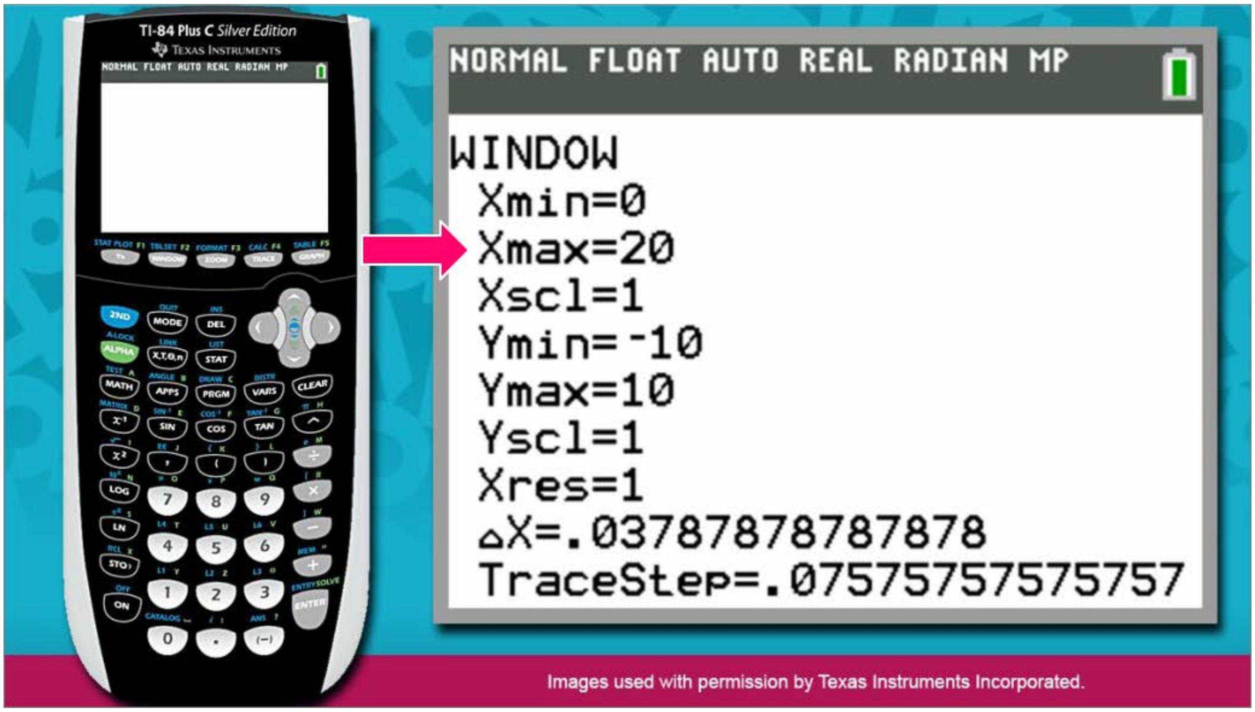
Notice a list of values appear. Here, you can adjust the scale of the x- and y-axes.

Example 1 (continued)



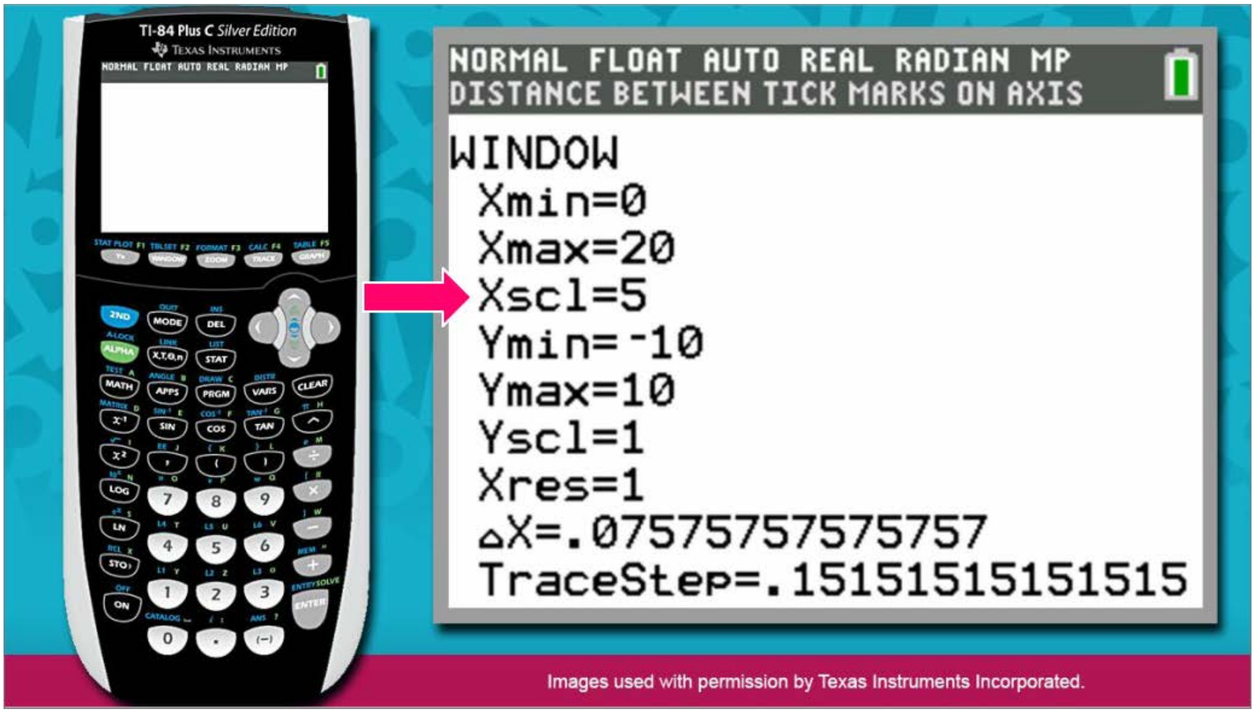
Because x represents the number of hours the band spends in the studio, negative x-values will not make sense. It is impossible to spend a negative number of hours in the studio. The smallest amount of time that the band can spend in the studio is 0 hours. So set the minimum x-value to 0.

Example 1 (continued)



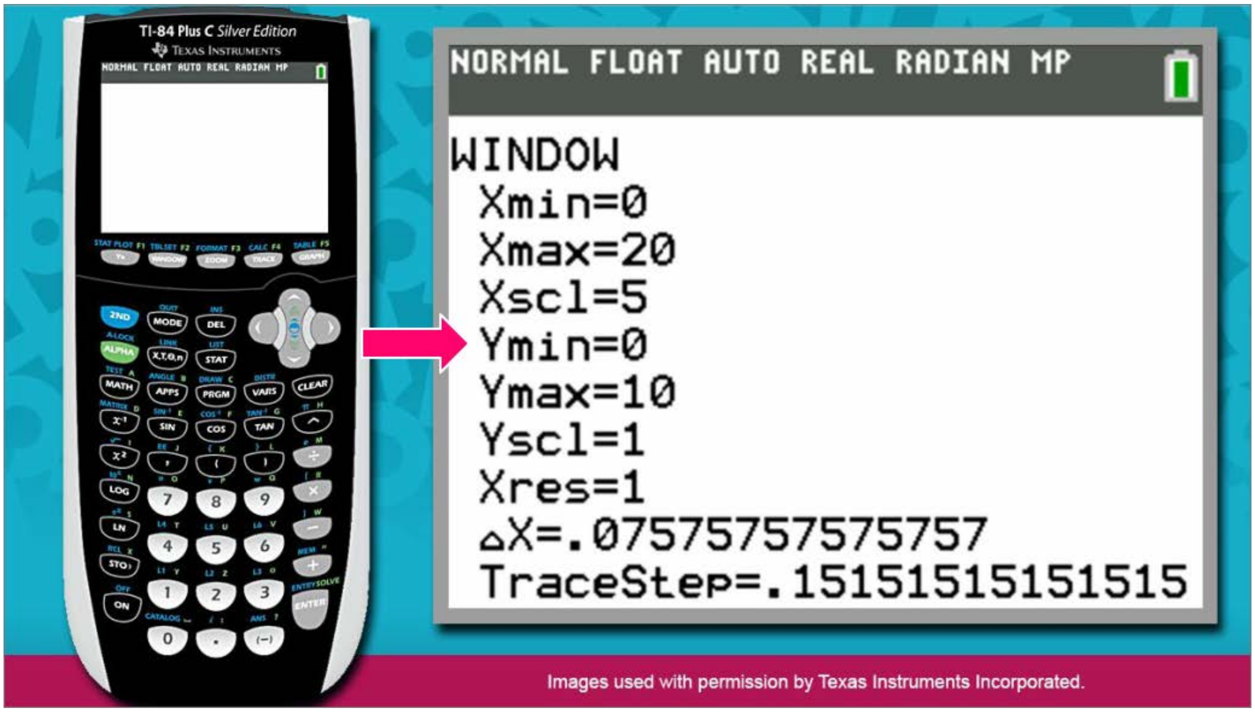
In theory, the band could spend an unlimited number of hours in the recording studio. So it impossible to say for certain what the maximum x-value should be. You will have to use your best judgment to decide on this value. There are 24 hours in a day. The band members may not spend an entire day in the studio. So begin by setting the maximum x-value to 20. If you notice that this value is too small or too large, you can change it later.

Example 1 (continued)



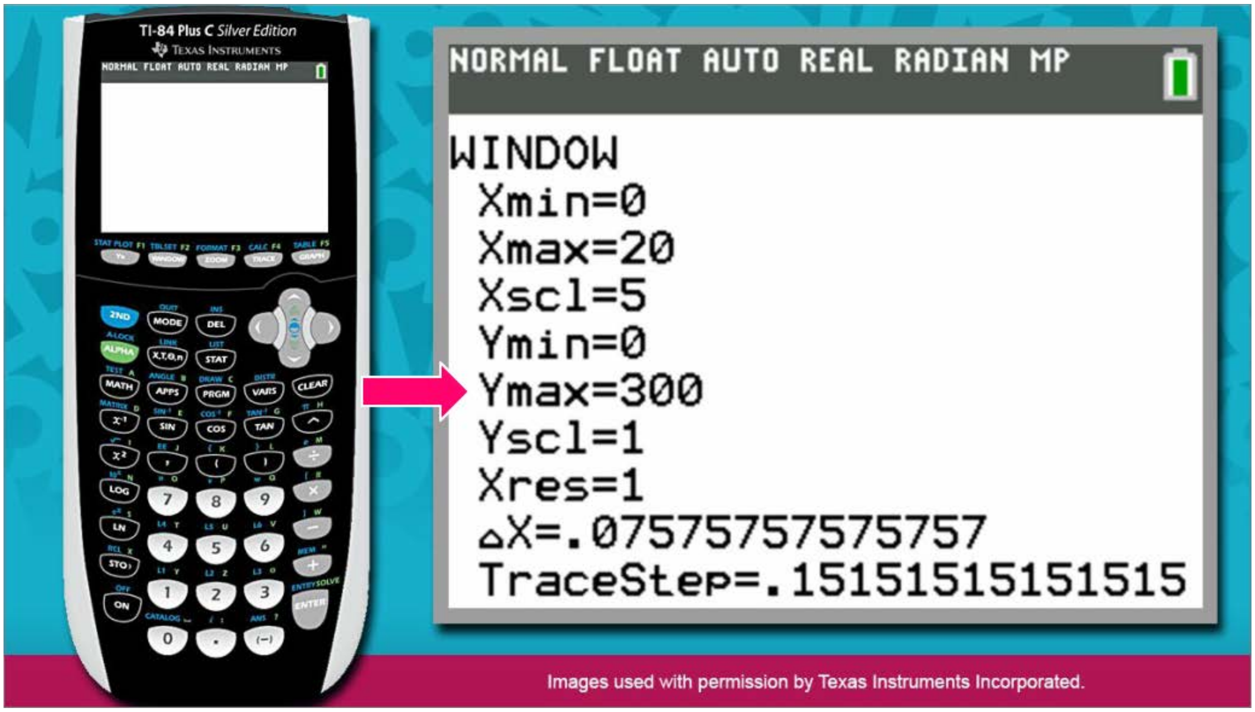
Because the minimum x-value is set to 0 and the maximum x-value is set to 20, it is appropriate to increase the x-scale, so that you will have a good view of the system once it is graphed. Change the x-scale from 1 to 5. Now, there will be 5 units between consecutive tick marks along the x-axis.

Example 1 (continued)



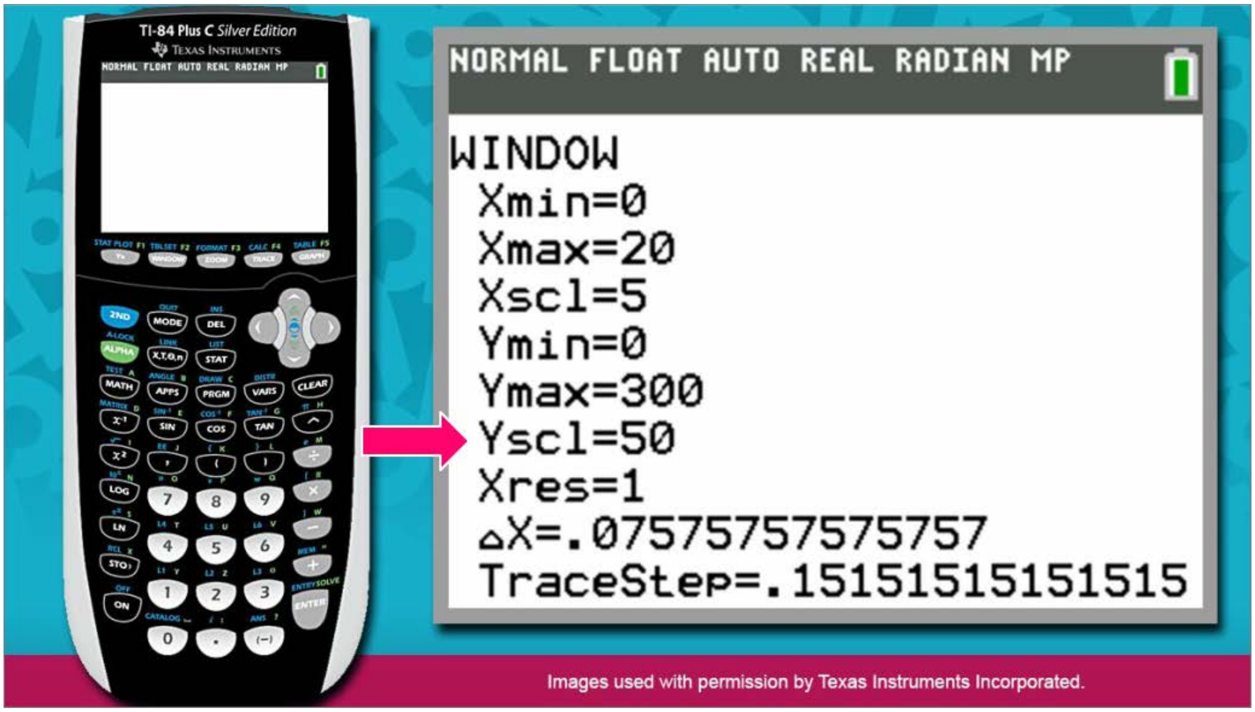
Now move on to adjust the values on the y-axis. Because y represents the total cost of renting the studio, negative y-values will not make sense. It is impossible for the total cost to be a negative amount. The smallest possible amount is $0. So set the minimum y-value to 0.

Example 1 (continued)



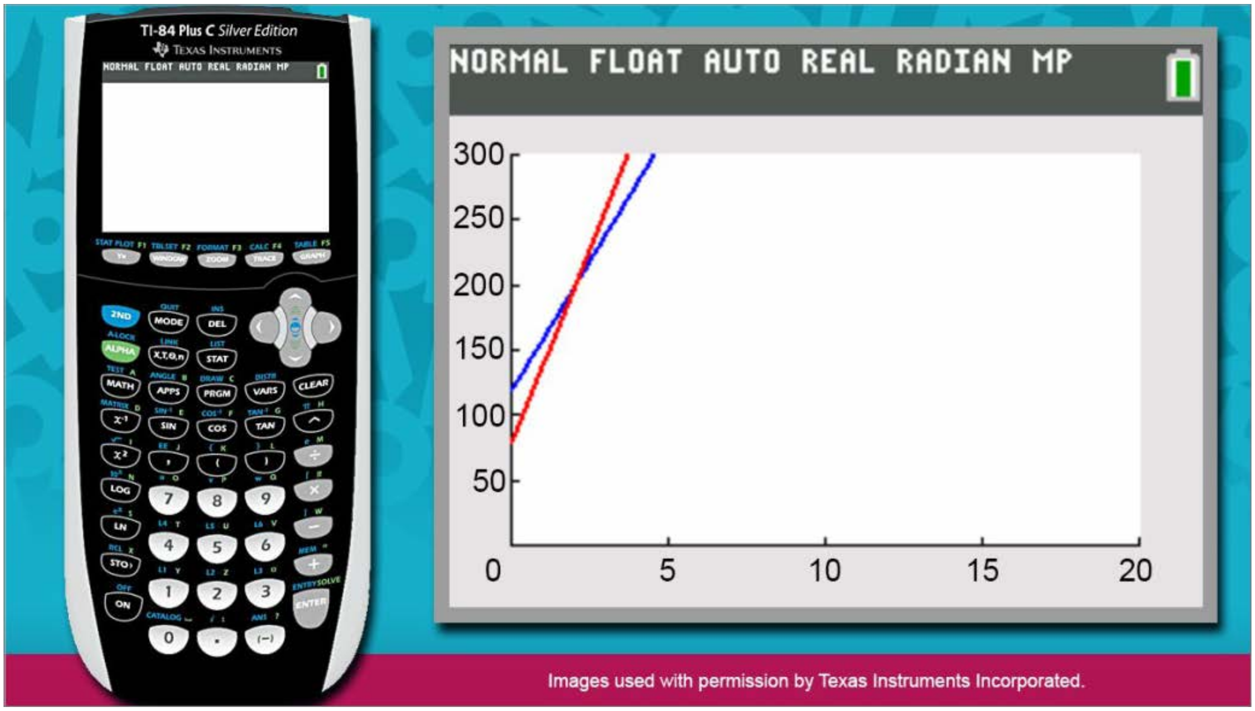
The total cost of renting the studio depends on the number of hours the band spends in the studio. So the maximum cost is unlimited. You will have to use your best judgment to determine a reasonable maximum y-value to input into the calculator. You know that the local studio rents for \$120 plus \$40 per hour and the studio in the neighboring city rents for \$80 plus \$60 per hour. It is safe to say that the band members could pay at least \$200 for their time in either studio. So set the maximum y-value to a value more than 200; say 300. If you notice that this value is too small or too large, you can change it later.

Example 1 (continued)



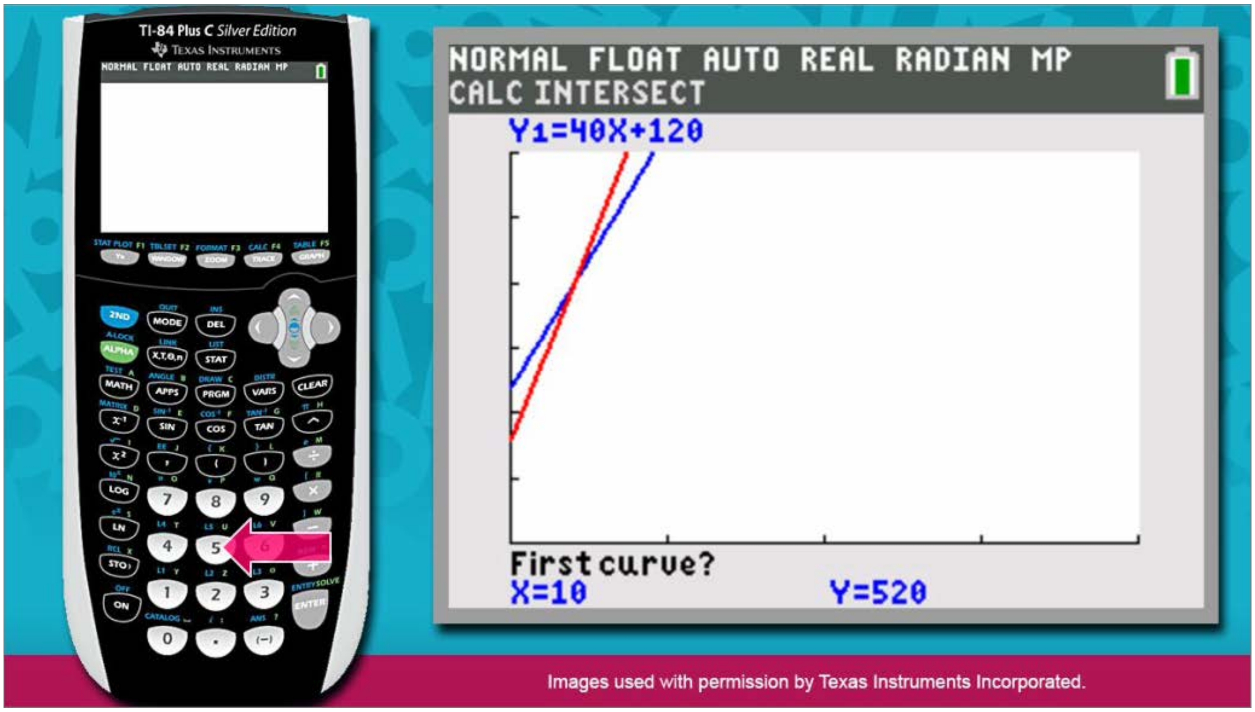
Because the minimum y-value is set to 0 and the maximum y-value is set to 300, it is appropriate to increase the y-scale so that you will have a good view of the system once it is graphed. Change the y-scale from 1 to 50. Now, the y-axis will count off by 50s. Leave the remaining values as they are.

Example 1 (continued)



Notice that the calculator only shows the portion of the system included in the first quadrant. The x-axis counts off by fives, beginning at 0 and ending at 20. The y-axis counts off by 50s, beginning at 0 and ending at 300.

Example 1 (continued)

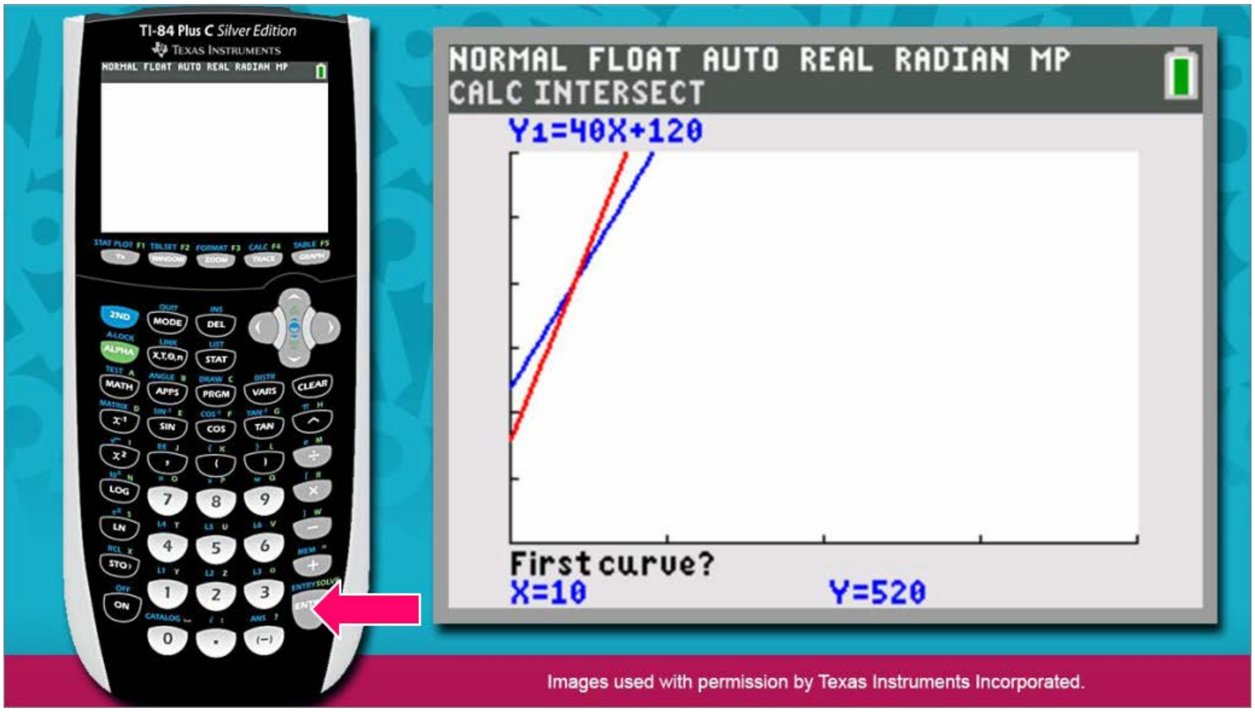


Now, determine the coordinates of the point of intersection.

Press 2^{nd} and then press TRACE to access the menu of CALCULATE functions.

Then press 5 to access the intersect function.

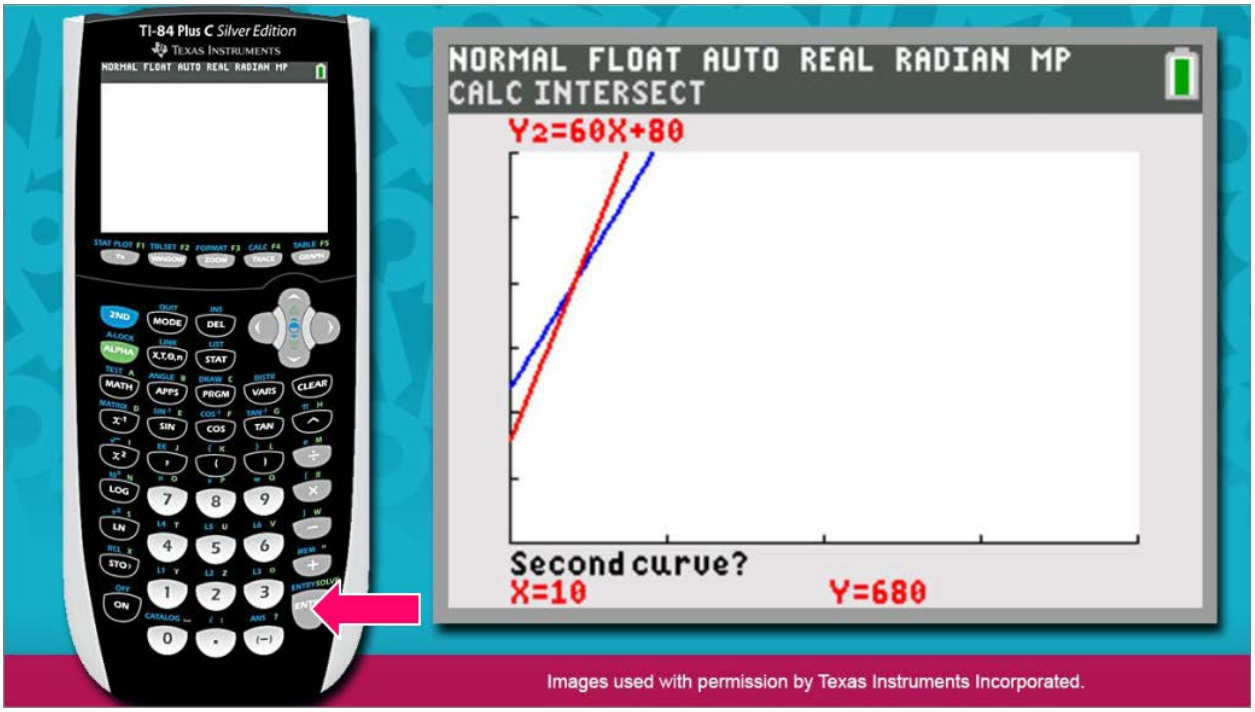
Example 1 (continued)



At this time, you will notice that you do not see the cursor blinking on the first line. It is at a point that is not viewable in the window. You can press the left and right arrow keys to move the cursor to a point that you are able to see, so that you can verify that it is blinking on the first line. Or, you can make this confirmation by verifying that the equation that appears directly above the graph is the first equation included in the system.

Because this is indeed the first equation included in the system, press ENTER to confirm.

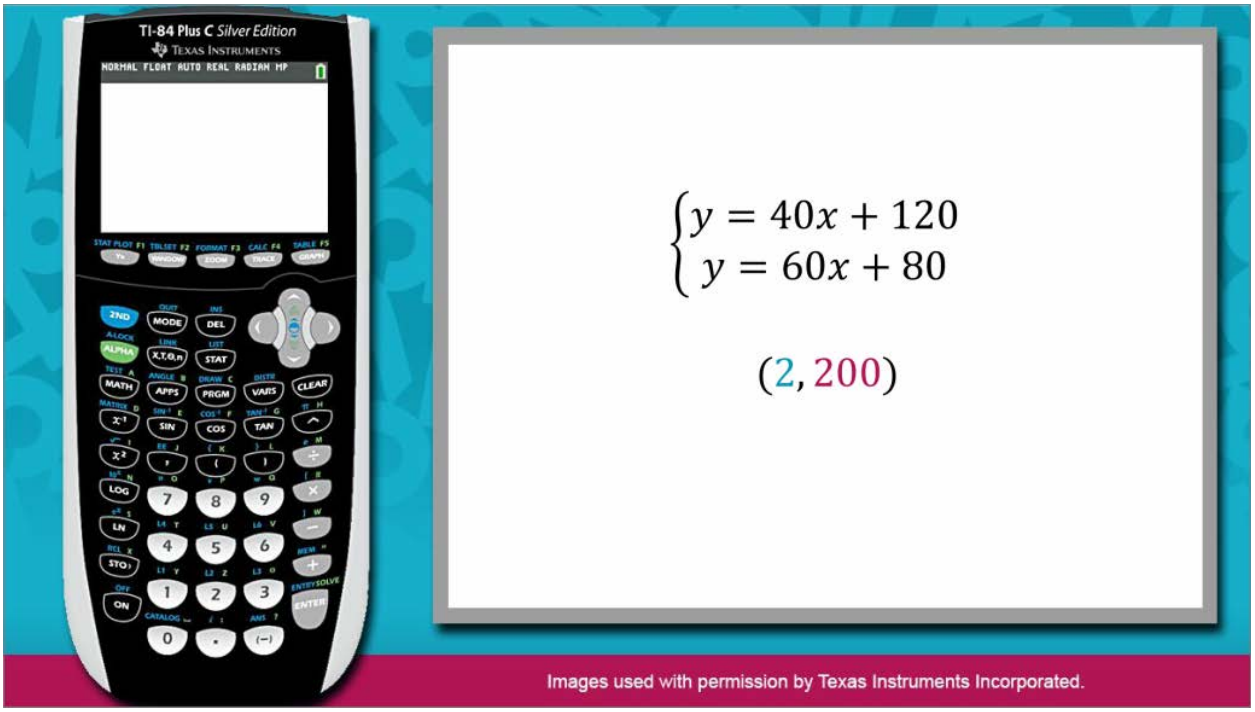
Example 1 (continued)



Notice, again, that you cannot see the cursor blinking on the second line. You can, however, confirm that it is located on the second line included in the system by verifying that the equation that now appears directly above the graph is the second equation included in the system.

Because this is indeed the second equation included in the system, press ENTER to confirm.

Example 1 (continued)

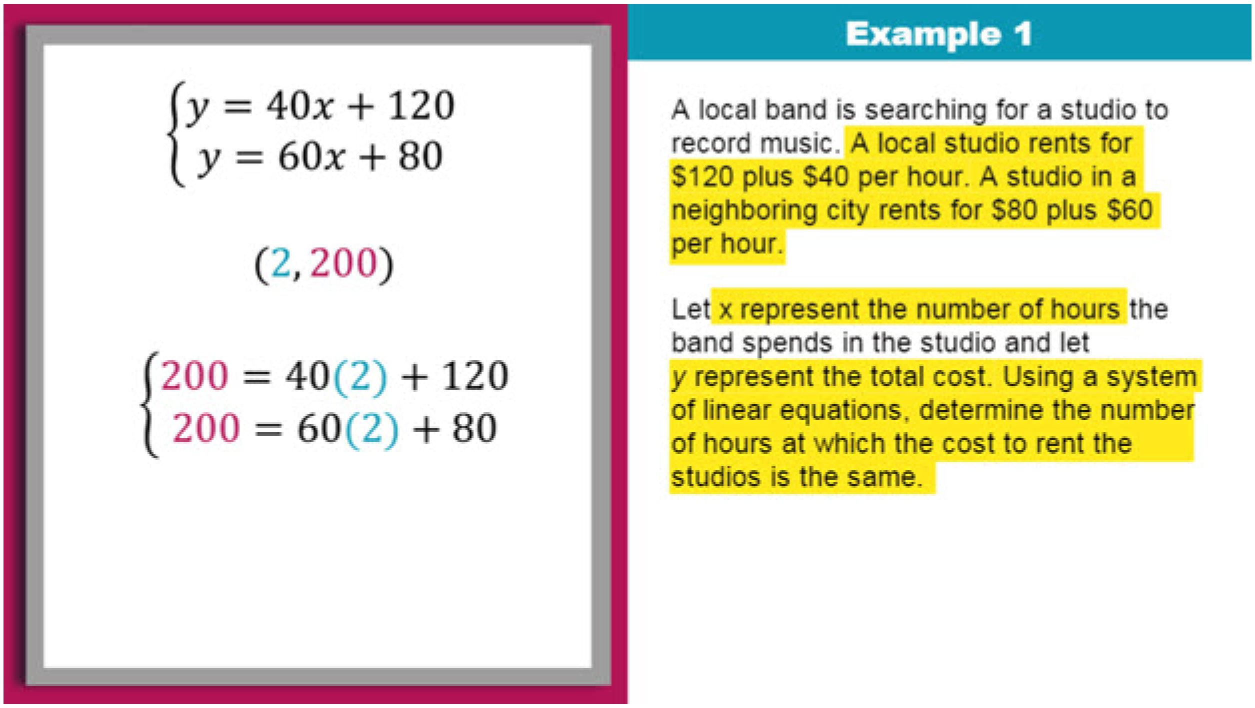


Press ENTER again to confirm that you are ready for the calculator to determine the coordinates of the point of intersection.

Notice that the cursor now appears at the point of intersection and the coordinates of the point are provided below: $x = 2$ and $y = 200$.

The solution to the system can be represented by the ordered pair $(2, 200)$.

Example 1 (continued)



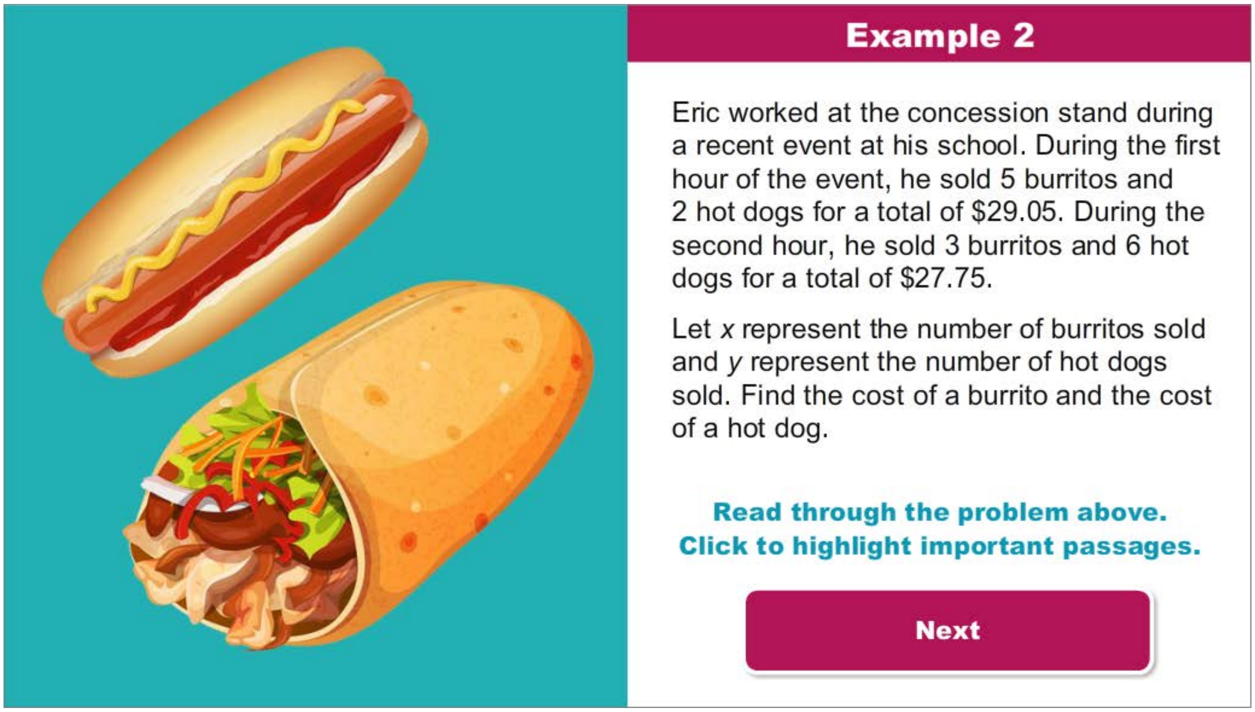
$$\begin{cases} y = 40x + 120 \\ y = 60x + 80 \end{cases}$$

$$(2, 200)$$

$$\begin{cases} 200 = 40(2) + 120 \\ 200 = 60(2) + 80 \end{cases}$$

The solution to a system of equations satisfies the equations of a system simultaneously. In this situation, the solution $(2, 200)$ informs you that regardless of which studio the band members choose to rent, if they spend 2 hours in a recording session they would pay a total of $200.

Example 2



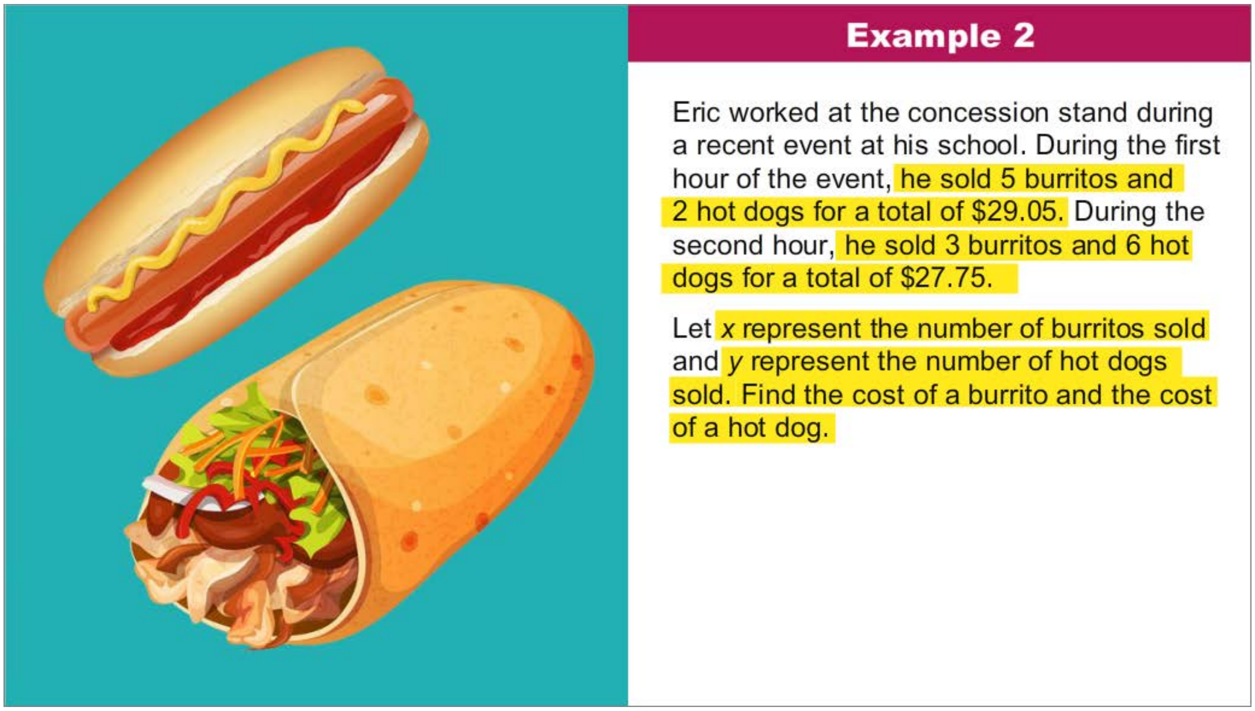
Take a few moments to read Example 2. Highlight the information you think is necessary to solve the problem.

Eric worked at the concession stand during a recent event at his school. During the first hour of the event, he sold 5 *burritos and* 2 *hot dogs for a total of* $\$29.05$*. During the second hour, he sold* 3 *burritos and* 6 *hot dogs for a total of* $\$27.75$*.*

Let x *represent the number of burritos sold and* y *represent the number of hot dogs sold. Find the cost of a burrito and the cost of a hot dog.*

Read through the problem above. Click to highlight important passages.

Example 2 (continued)



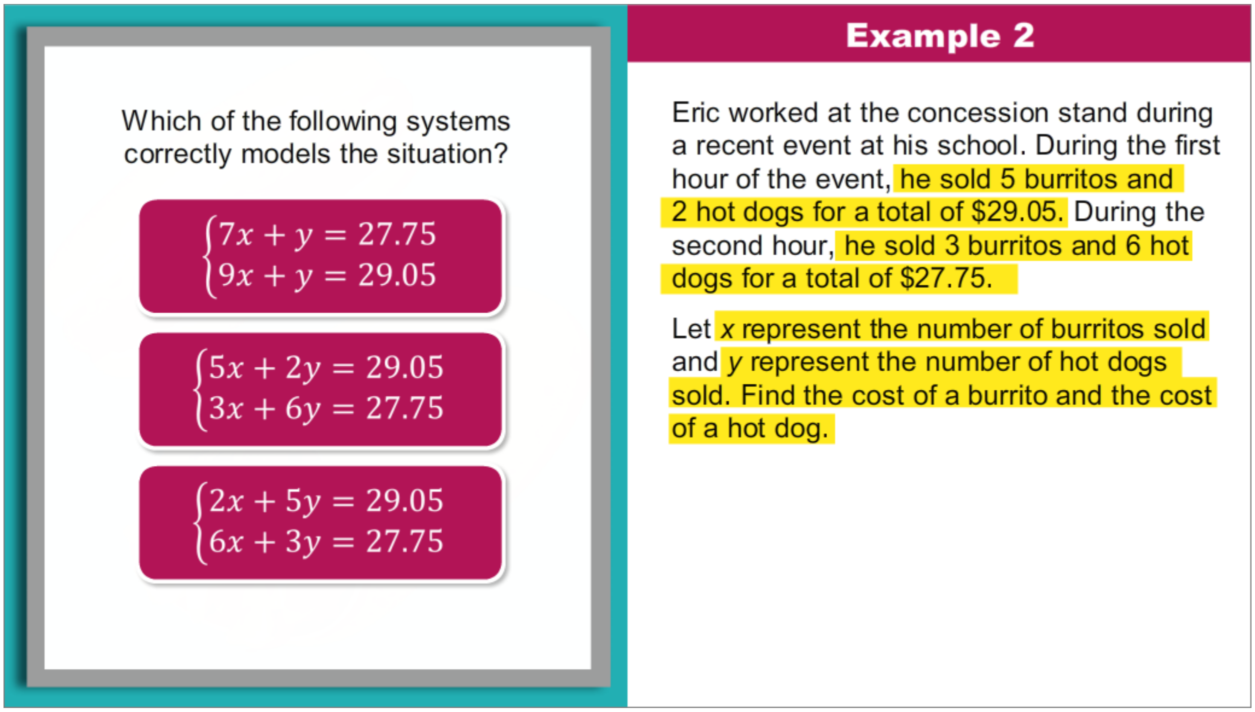
Did you highlight the important facts?

Eric worked at the concession stand during a recent event at his school. During the first hour of the event, he sold 5 *burritos and* 2 *hot dogs for a total of* $29.05. *During the second hour, he sold* 3 *burritos and* 6 *hot dogs for a total of* $27.75.

Let x *represent the number of burritos sold* and y *represent the number of hot dogs sold. Find the cost of a burrito and the cost of a hot dog.*

- Eric sold 5 burritos and 2 hot dogs for a total of $29.05
- He also sold 3 burritos and 6 hot dogs for a total of $27.75
- x represents the number of burritos sold
- y represents the number of hot dogs sold
- Find the cost of a burrito and the cost of a hot dog

Example 2 (continued)



Eric worked at the concession stand during a recent event at his school. During the first hour of the event, he sold 5 *burritos and* 2 *hot dogs for a total of* \$29.05*. During the second hour, he sold* 3 *burritos and* 6 *hot dogs for a total of* \$27.75*.*

Let x represent the number of burritos sold and y represent the number of hot dogs sold. Find the cost of a burrito and the cost of a hot dog.

Now that you have highlighted the key information, use it to help you solve the problem. Begin by writing a system of equations to model the situation.

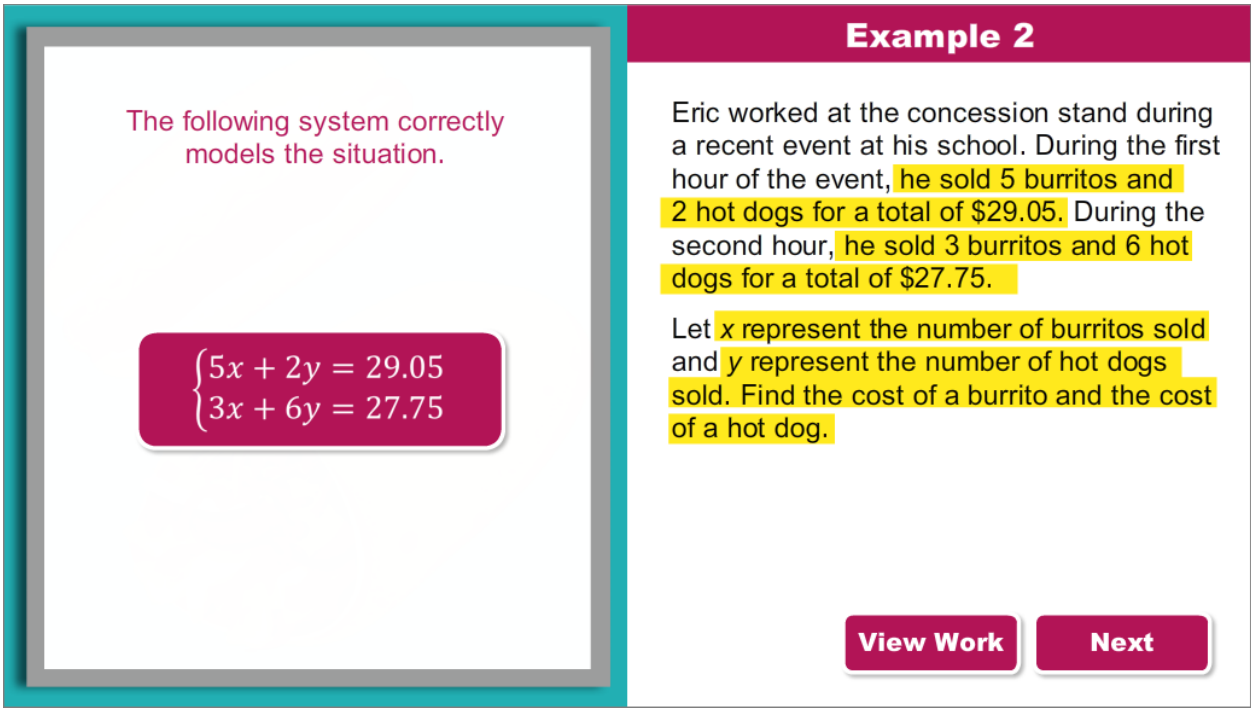
Which of the following systems correctly models the situation?

A) $\begin{cases} 7x + y = 27.75 \\ 9x + y = 29.05 \end{cases}$

B) $\begin{cases} 5x + 2y = 29.05 \\ 3x + 6y = 27.75 \end{cases}$

C) $\begin{cases} 2x + 5y = 29.05 \\ 6x + 3y = 27.75 \end{cases}$

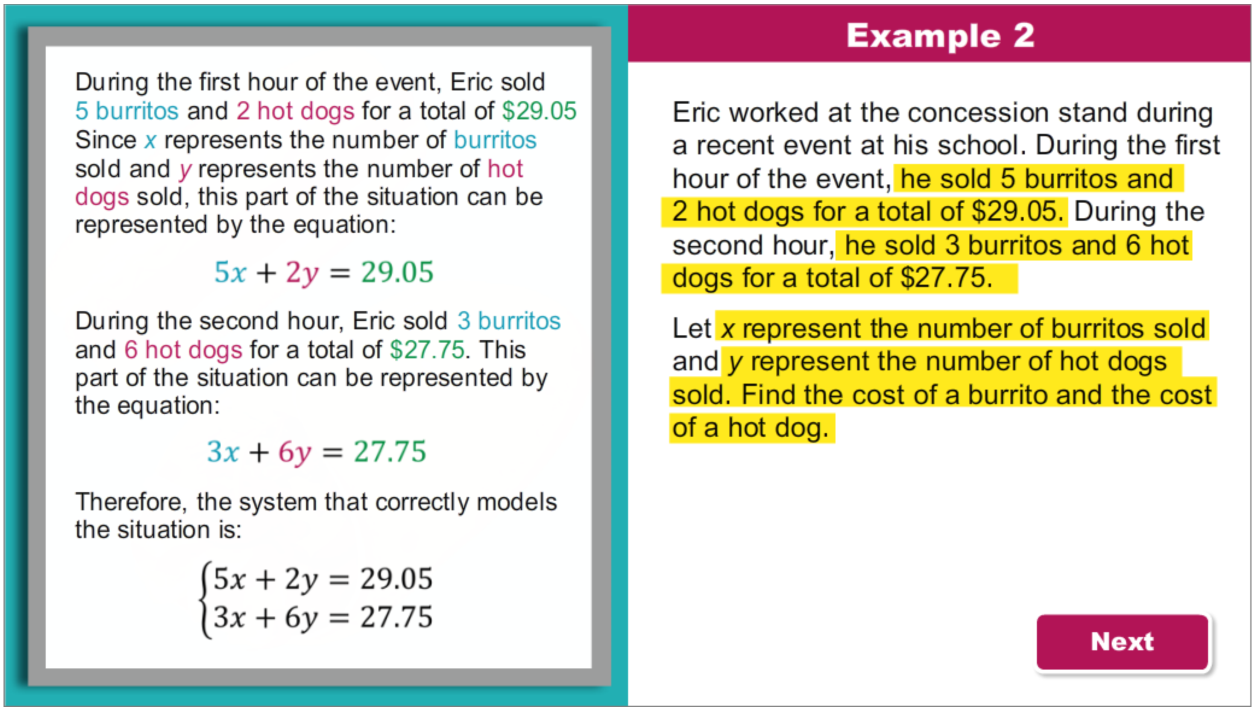
Example 2 (continued)



The following system correctly models the situation.

$$\begin{cases} 5x + 2y = 29.05 \\ 3x + 6y = 27.75 \end{cases}$$

Example 2 (continued)



During the first hour of the event, Eric sold 5 burritos and 2 hot dogs for a total of $29.05. Since x represents the number of burritos sold and y represents the number of hot dogs sold, this part of the situation can be represented by the equation:

$$5x + 2y = 29.05$$

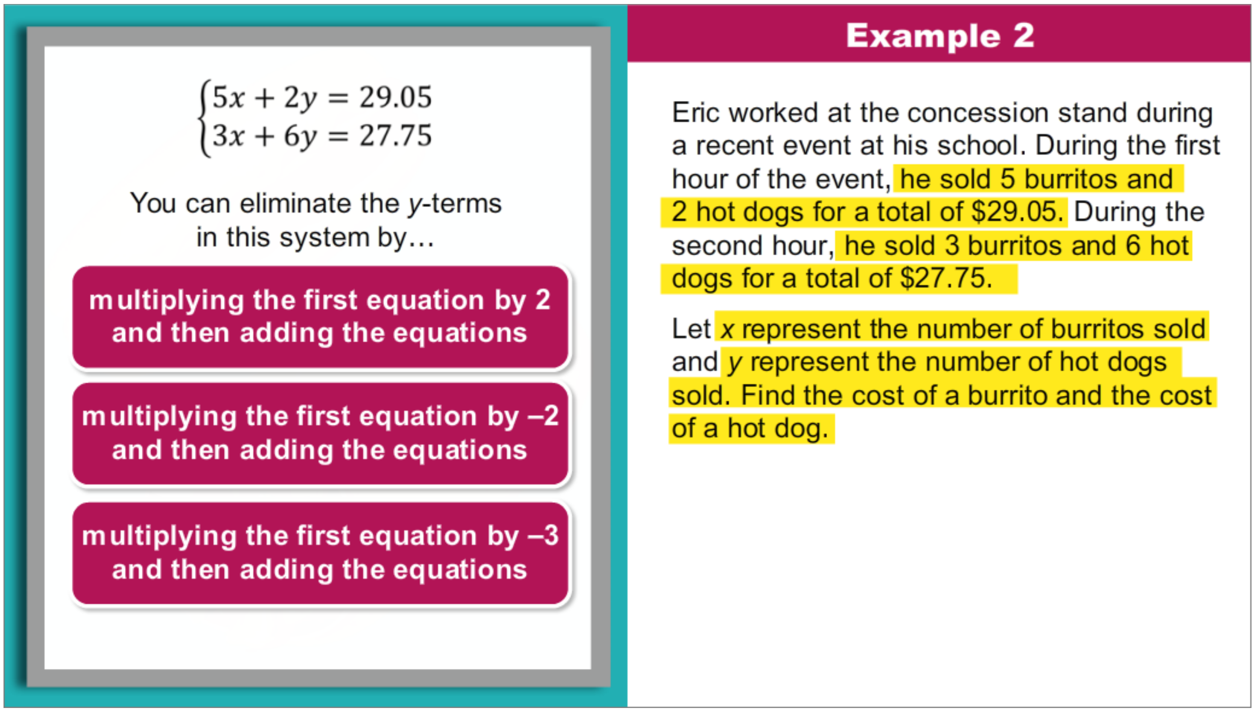
During the second hour, Eric sold 3 burritos and 6 hot dogs for a total of $27.75. This part of the situation can be represented by the equation:

$$3x + 6y = 27.75$$

Therefore, the system that correctly models the situation is:

$$\begin{cases} 5x + 2y = 29.05 \\ 3x + 6y = 27.75 \end{cases}$$

Example 2 (continued)



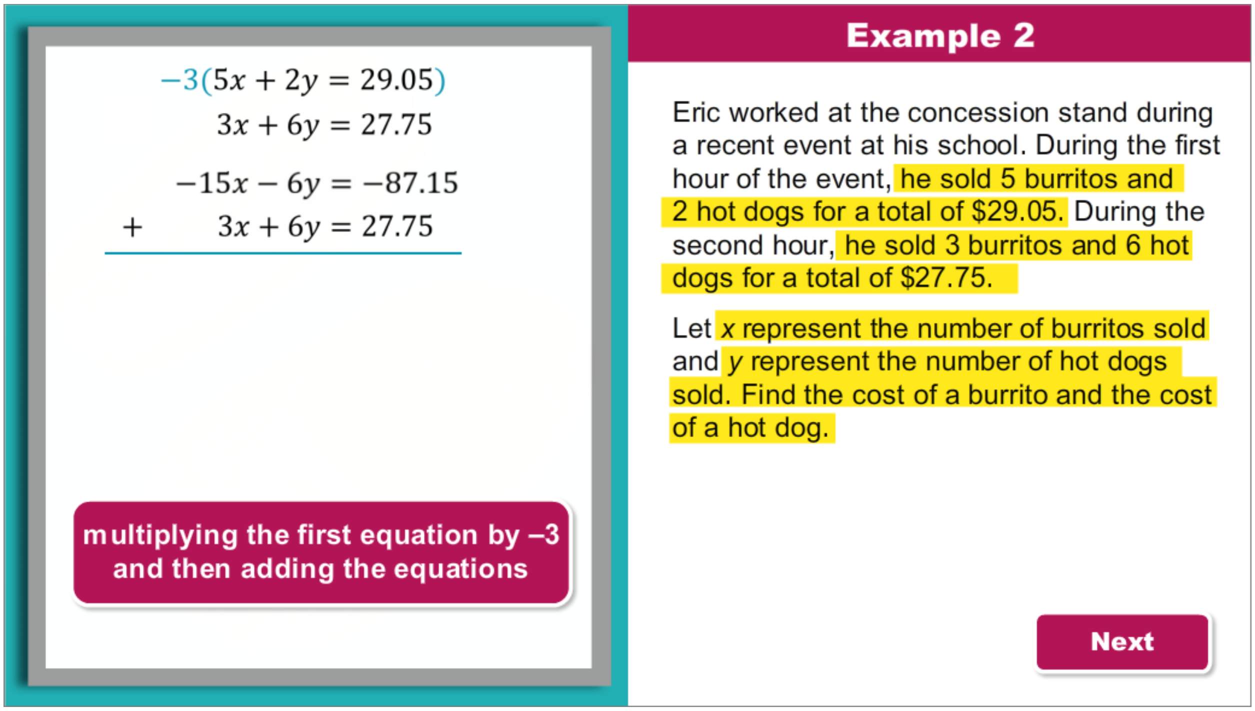
Remember that you can solve a system of linear equations by using the graphing method, the substitution method, or the elimination method. For this example, the elimination method may be the most effective way to solve the system.

$$\begin{cases} 5x + 2y = 29.05 \\ 3x + 6y = 27.75 \end{cases}$$

You can eliminate the y-terms in this system by…

A) multiplying the first equation by 2 and then adding the equations
B) multiplying the first equation by -2 and then adding the equations
C) multiplying the first equation by -3 and then adding the equations

Example 2 (continued)

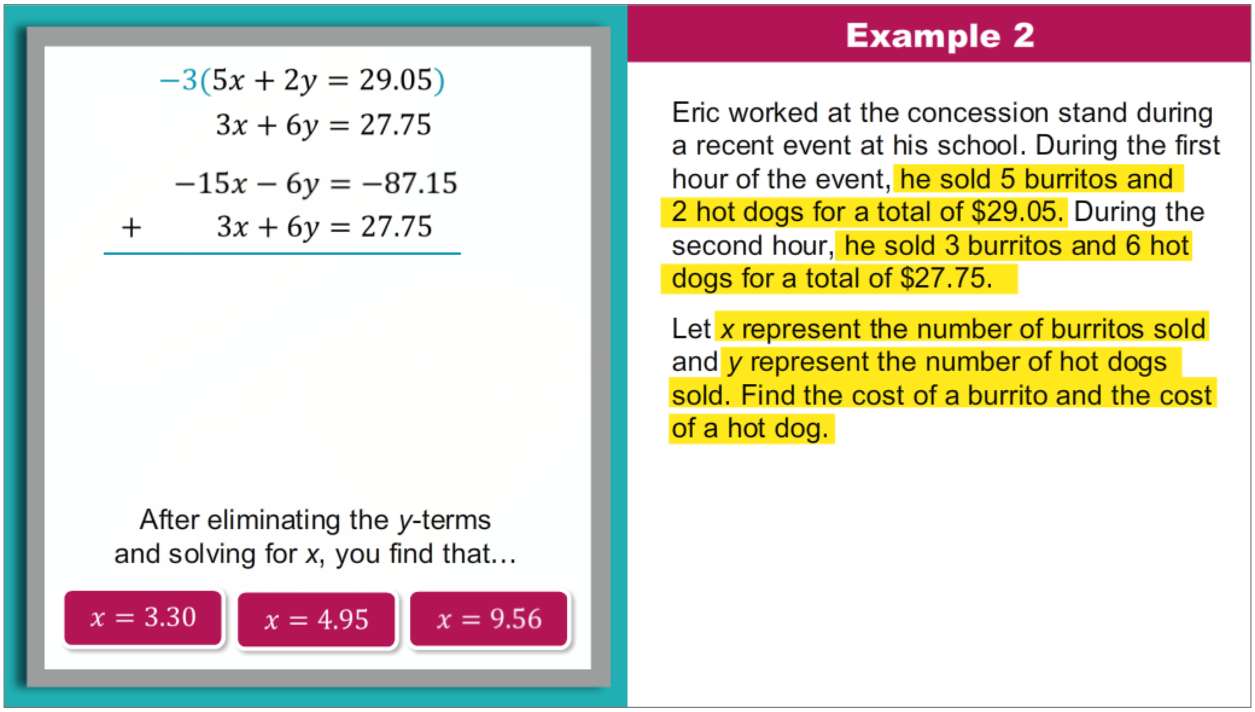


$$-3(5x + 2y = 29.05)$$
$$-15x - 6y = -87.15$$

$$\begin{array}{rcrcr} -15x & - & 6y & = & -87.15 \\ + \quad 3x & + & 6y & = & 27.75 \\ \hline \end{array}$$

The y-terms can be eliminated by multiplying the first equation by -3 and then adding the equations.

Example 2 (continued)



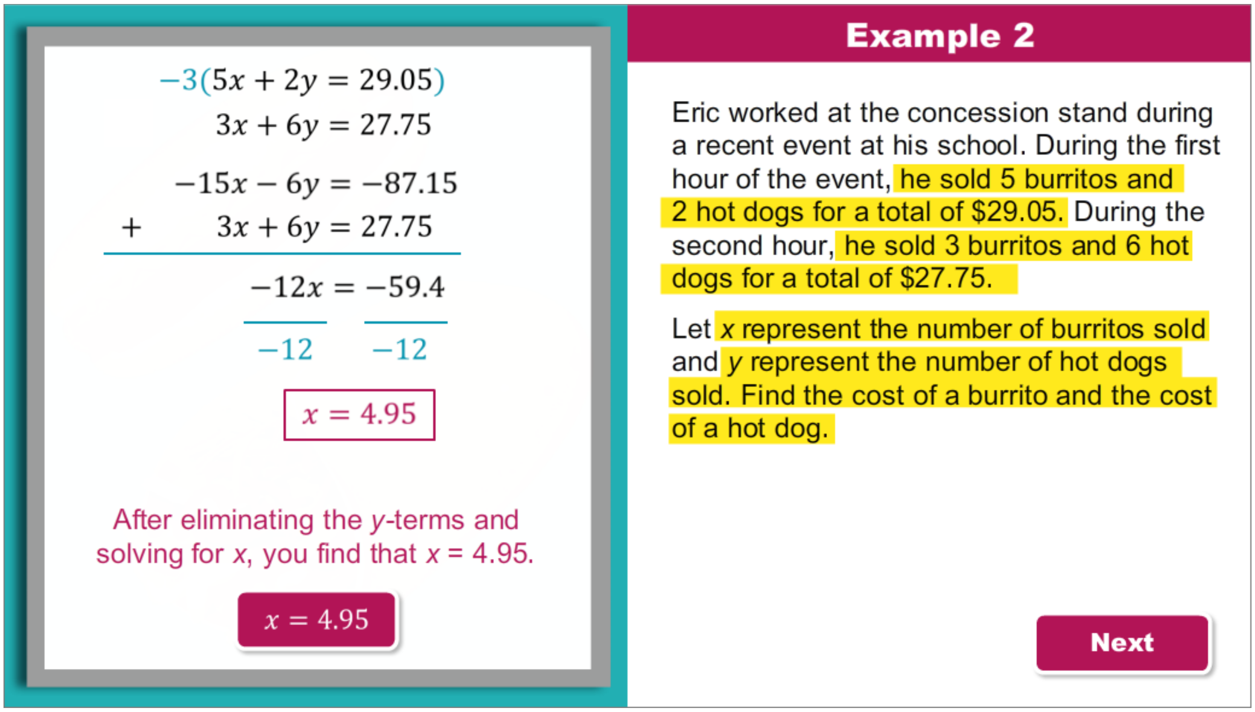
$$-3(5x + 2y = 29.05)$$
$$-15x - 6y = -87.15$$

$$\begin{array}{rrcrcr} & -15x & - & 6y & = & -87.15 \\ + & 3x & + & 6y & = & 27.75 \\ \hline \end{array}$$

After eliminating the y-terms and solving for x, you find that...

A) $x = 3.30$
B) $x = 4.95$
C) $x = 9.56$

Example 2 (continued)

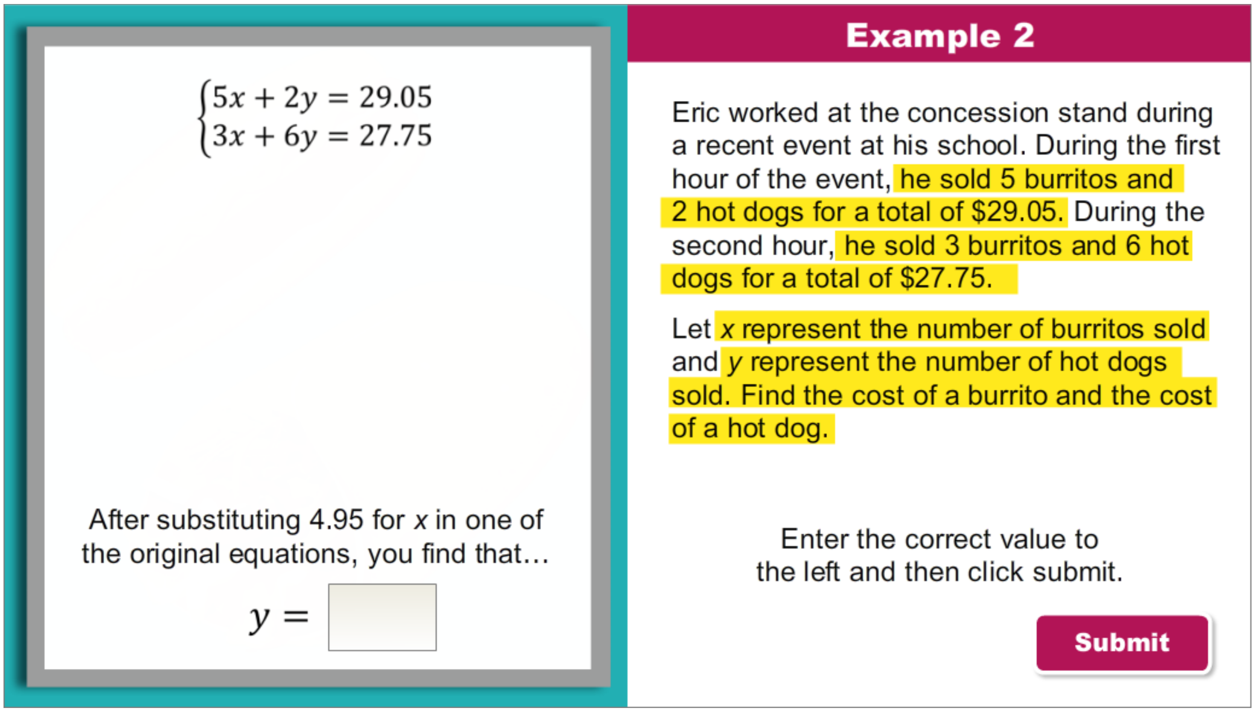


$$-3(5x + 2y = 29.05)$$
$$-15x - 6y = -87.15$$

$$\begin{array}{rcrcr} -15x & - & 6y & = & -87.15 \\ + \quad 3x & + & 6y & = & 27.75 \\ \hline & & -12x & = & -59.4 \\ & & \overline{-12} & & \overline{-12} \\ & & x & = & 4.95 \end{array}$$

After eliminating the y-terms and solving for x, you find that $x = 495$.

Example 2 (continued)



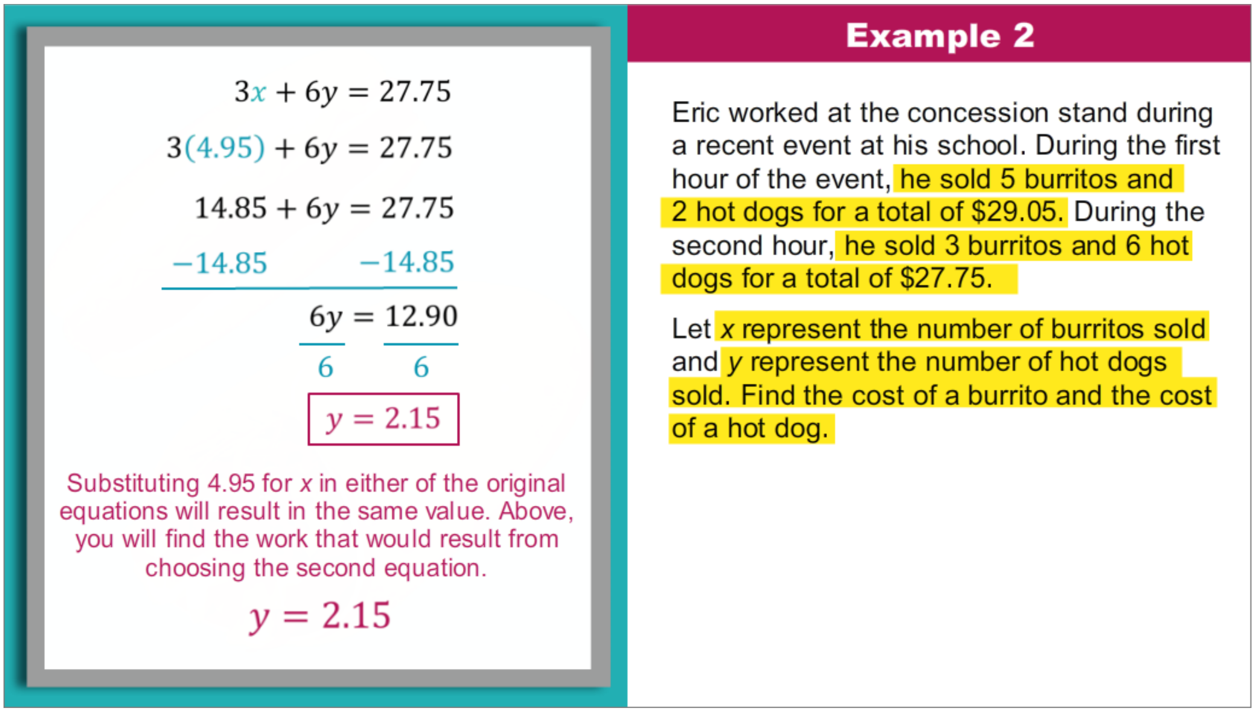
$$\begin{cases} 5x + 2y = 29.05 \\ 3x + 6y = 27.75 \end{cases}$$

Now that you know that $x = 4.95$, substitute this value into one of the original equations of the system to find y.

After substituting 4.95 for x in one the original equations you find that $y =$ __?__.

Enter the correct value to the left and then click submit.

Example 2 (continued)

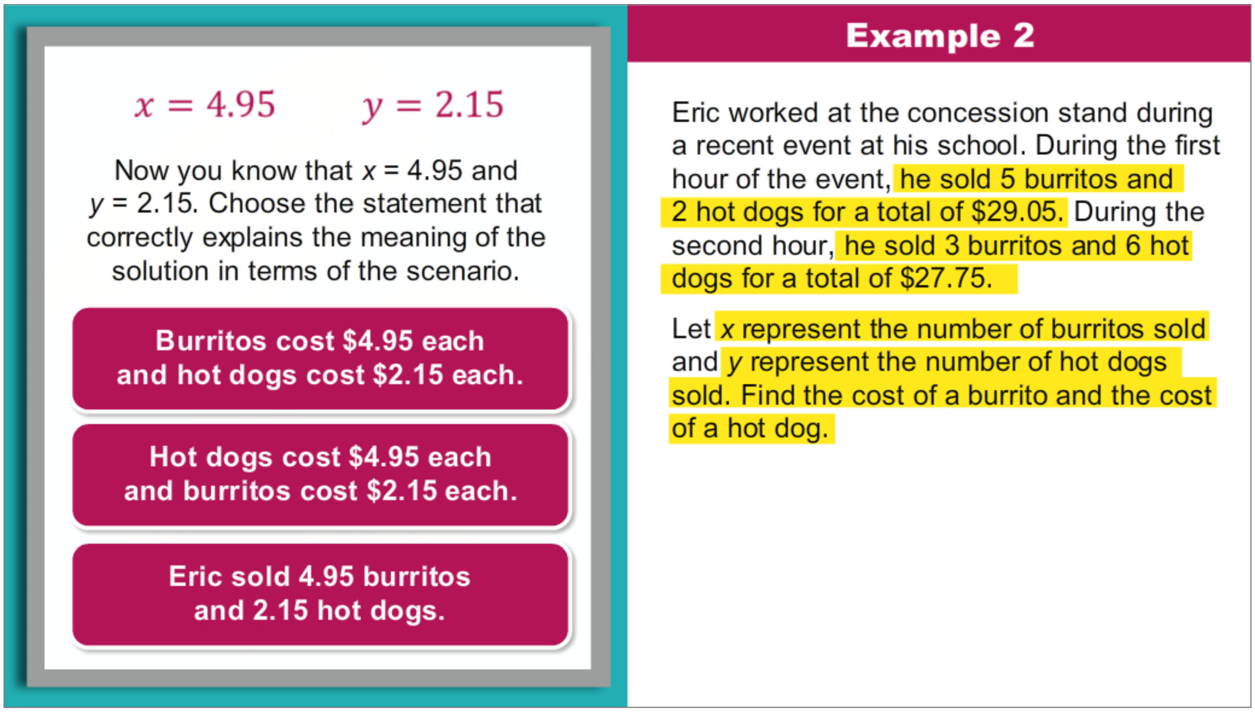


$$
\begin{aligned}
3x + 6y &= 27.75 \\
3(4.95) + 6y &= 27.75 \\
14.85 + 6y &= 27.75 \\
\underline{-14.85 \qquad} & \underline{\;\; - 14.85} \\
\frac{6y}{6} &= \frac{12.90}{6} \\
y &= 2.15
\end{aligned}
$$

Substituting 4.95 for x in either of the original equations will result in the same value. Below, you will find the work that would result from choosing the second equation.

$$y = 215$$

Example 2 (continued)

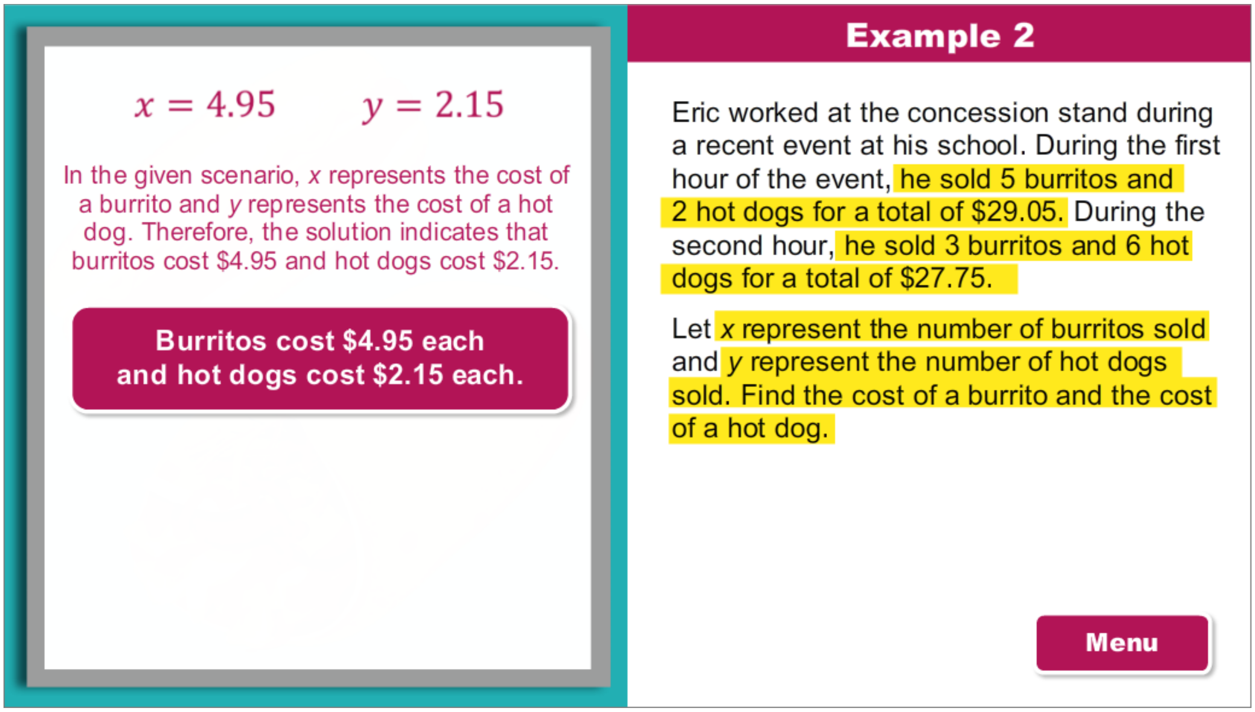


$$x = 4.95 \quad y = 2.15$$

Now you know that $x = 4.95$ and $y = 2.15$. Choose the statement that correctly explains the meaning of the solution in terms of the scenario.

A) Burritos cost $4.95 each and hot dogs cost $2.15 each.
B) Hot dogs cost $4.95 each and burritos cost $2.15 each.
C) Eric sold 4.95 burritos and 2.15 hot dogs.

Example 2 (continued)

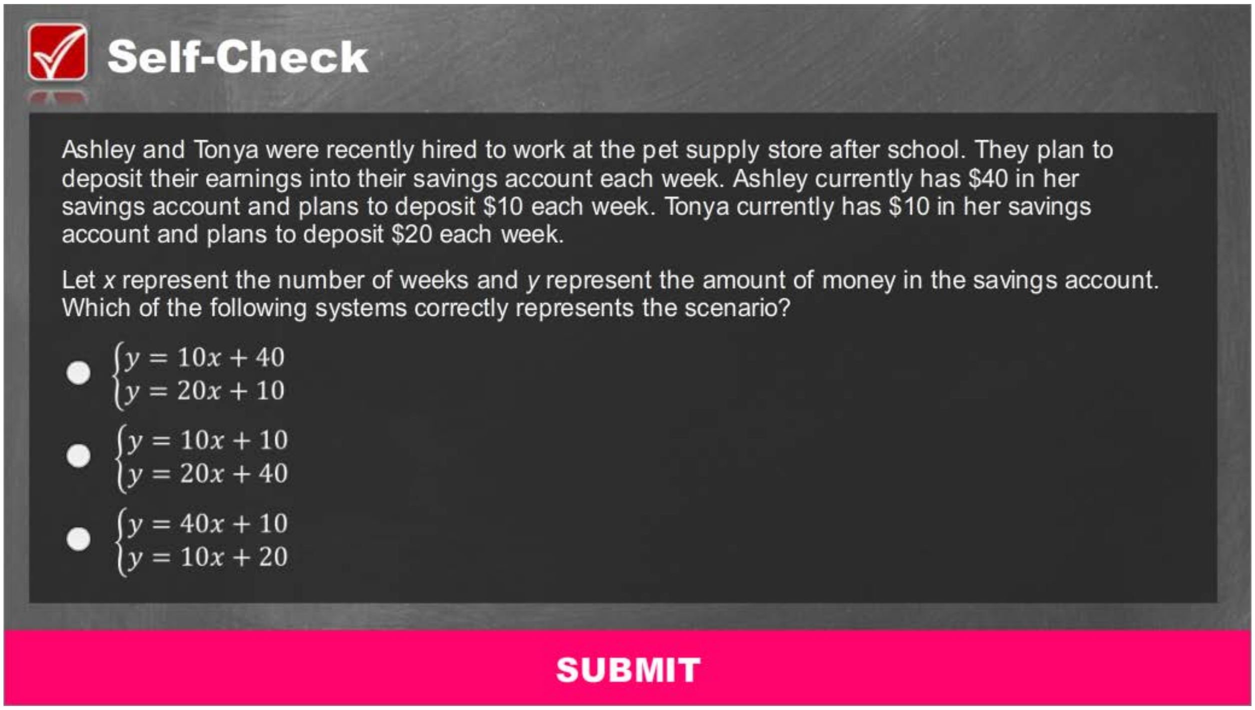


$$x = 4.95 \quad y = 2.15$$

In the given scenario, x represents the cost of a burrito and y represents the cost of a hot dog. Therefore, the solution $x = 4.95$ and $y = 2.15$ indicates that a burrito costs \$4.95 and a hot dog costs \$2.15.

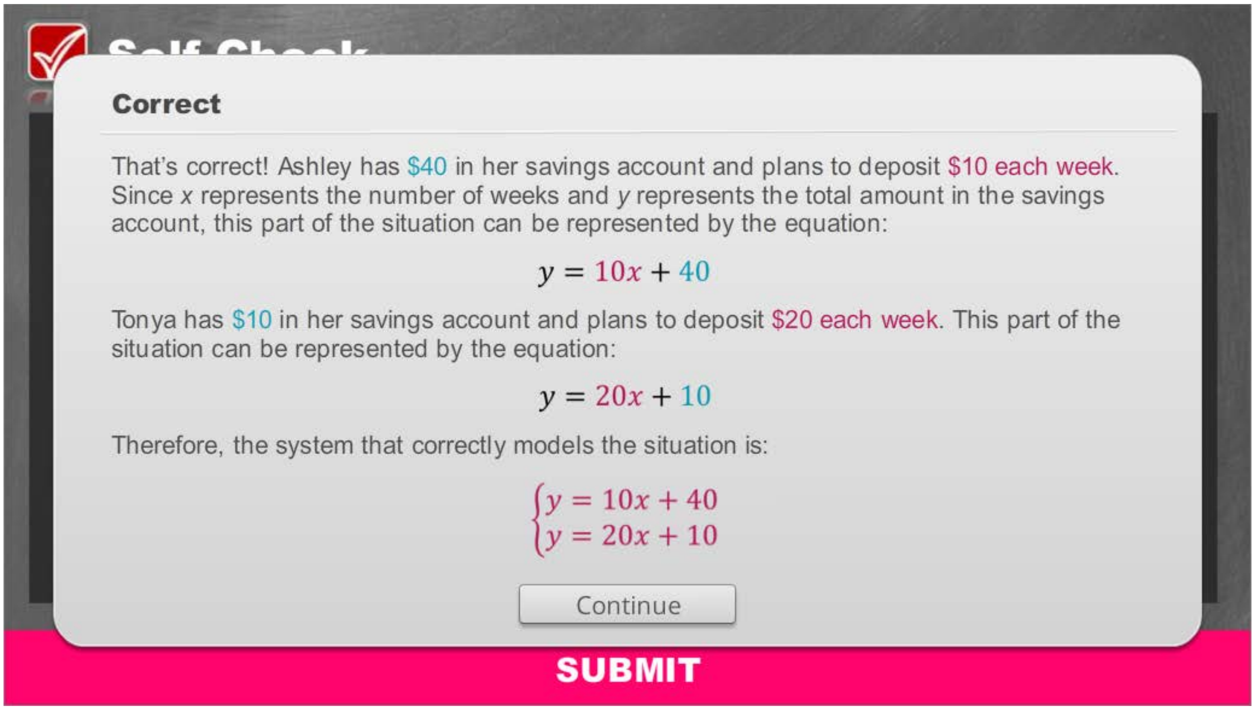
Burritos cost \$4.95 each and hot dogs cost \$2.15 each.

Self-Check 1



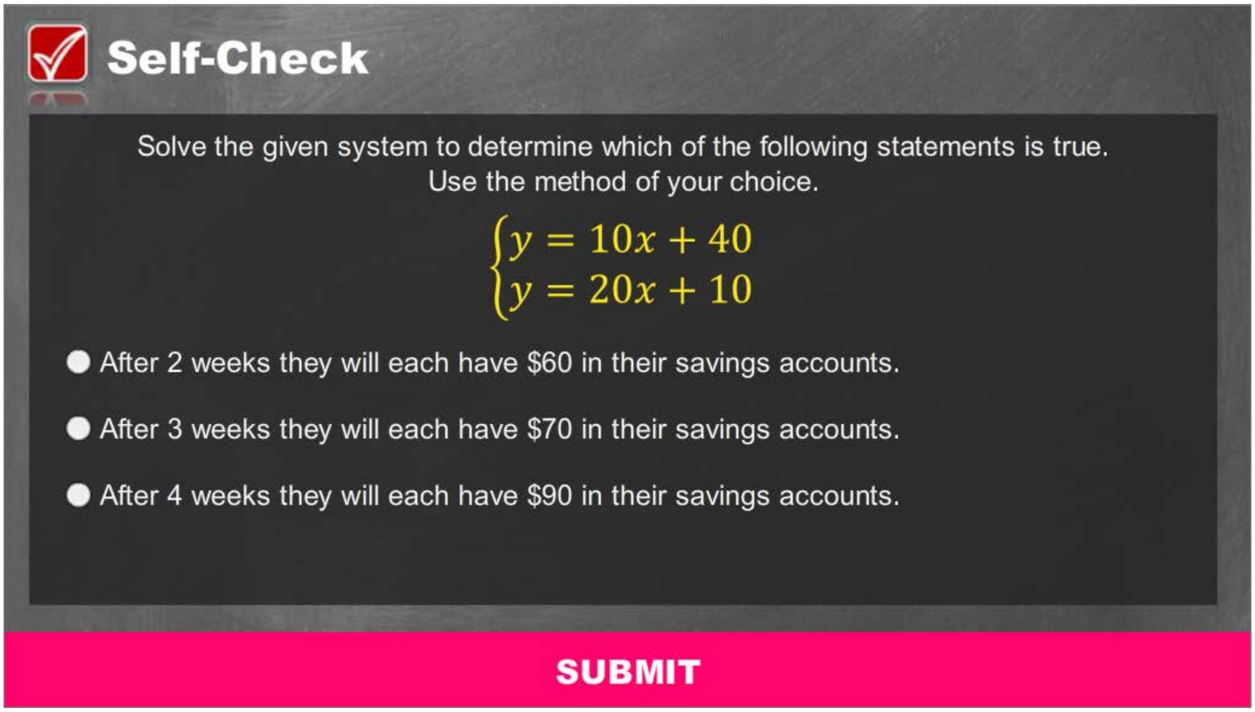
Solve the problem in the image above to check your understanding of the content.

Self-Check 1: Answer



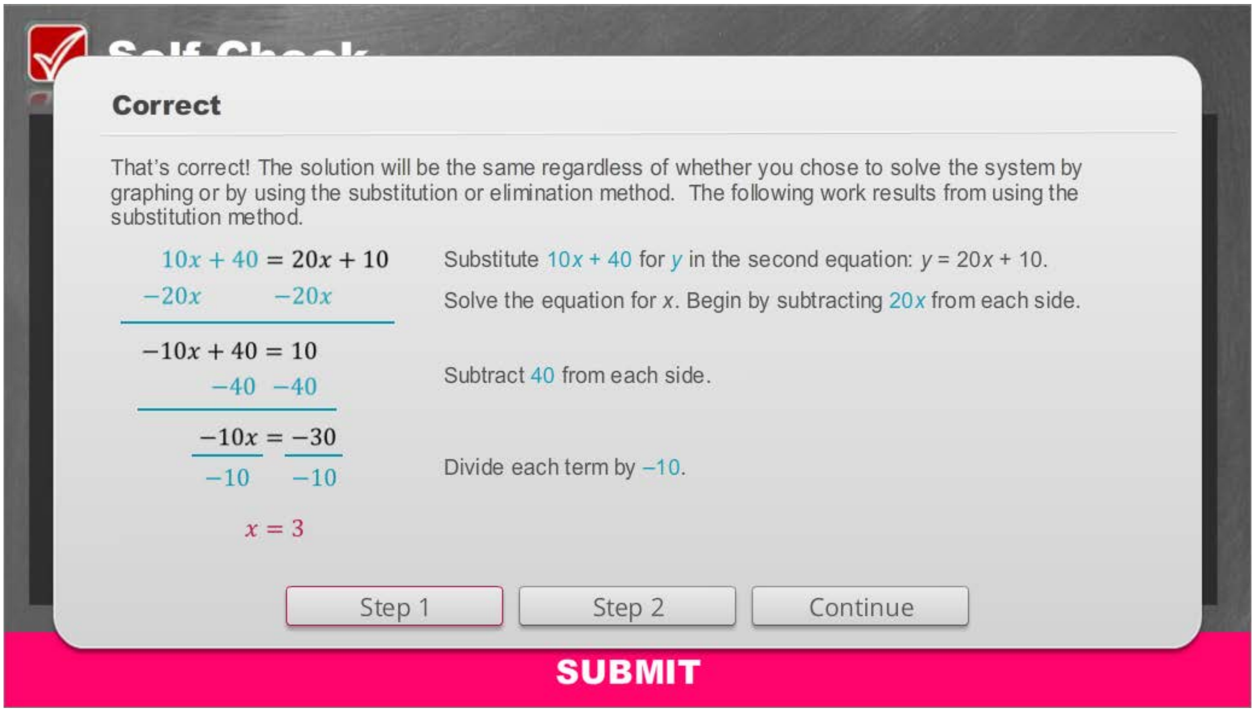
For your reference, the image above shows the correct solution to the self-check problem.

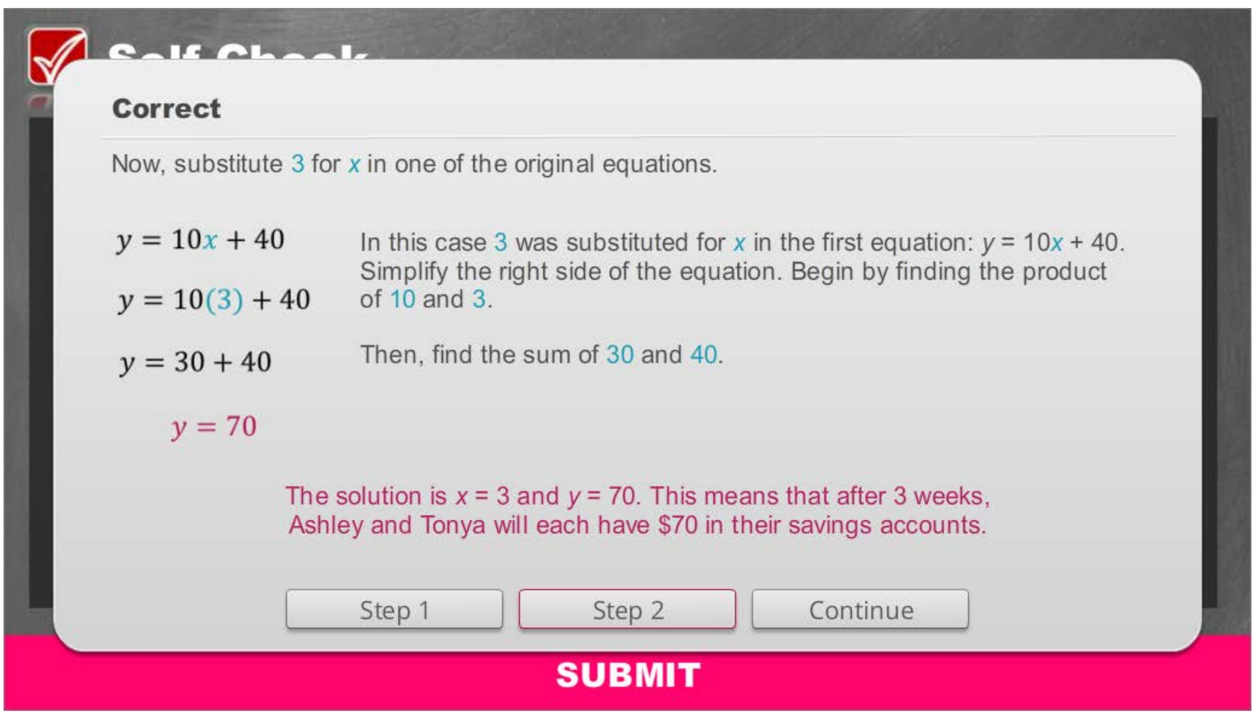
Self-Check 2



Solve the problem in the image above to check your understanding of the content.

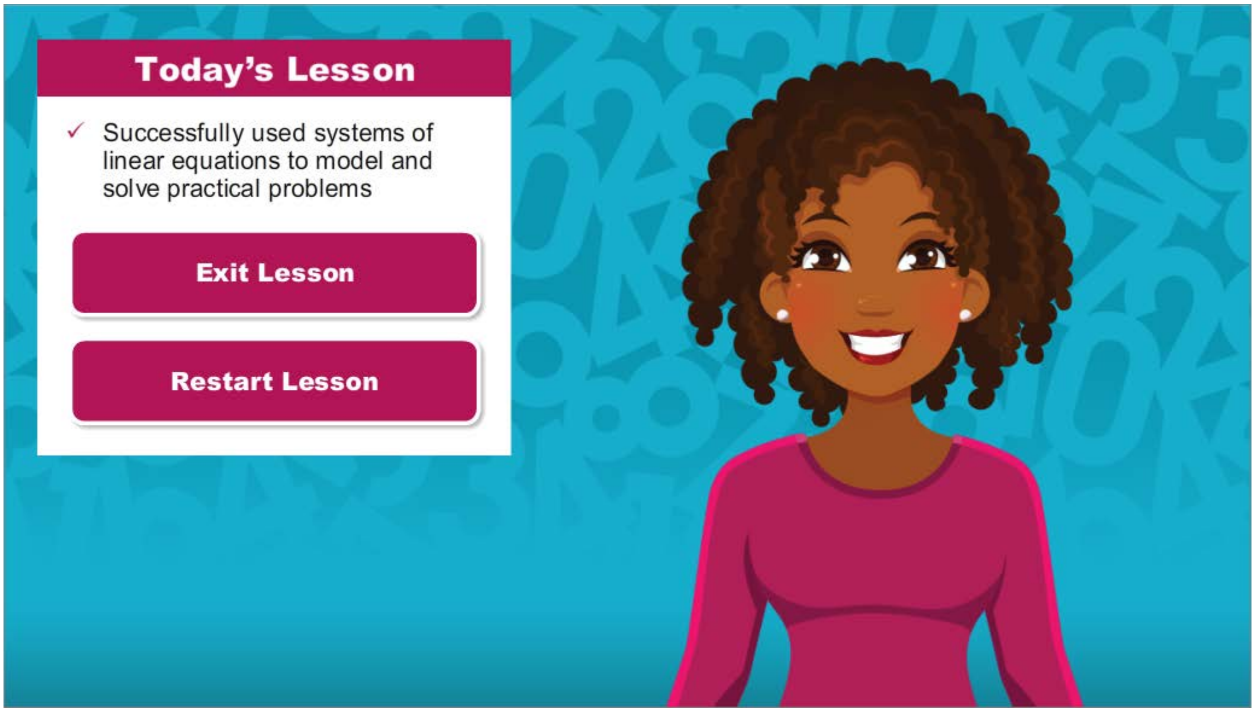
Self-Check 2: Answer





For your reference, the images above shows the correct solution to the self-check problem.

Conclusion



You have reached the conclusion of this lesson where you learned how to use systems of linear equations to model and solve practical problems.