Module 6: Solving Quadratic Equations

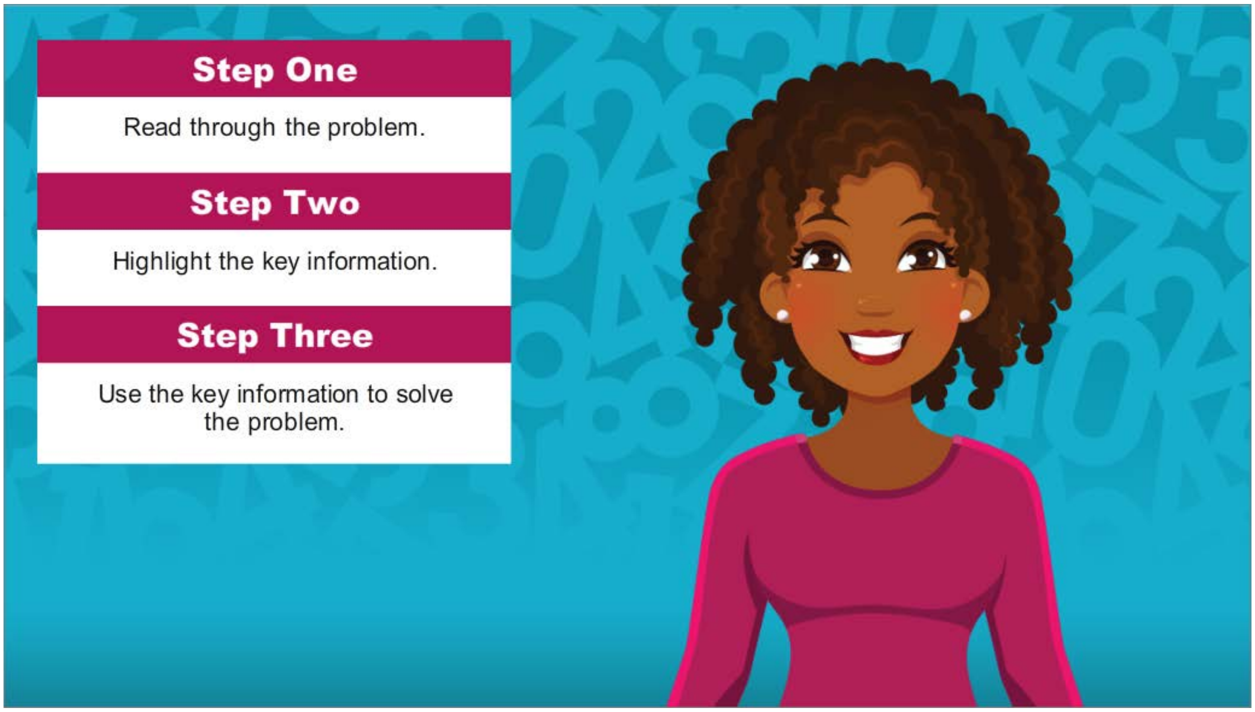
Topic 3 Content: Practical Problems Involving Quadratic Equations

Introduction

Hi there! I'm so glad to have you here for this lesson in Algebra I, where you will learn how to use quadratic equations to model and solve practical problems.

Module 6: Solving Quadratic Equations
Topic 3 Content: Practical Problems Involving Quadratic Equations

Anticipatory Set



Use the following steps to guide you in the process of using quadratic equations to solve real-world problems.

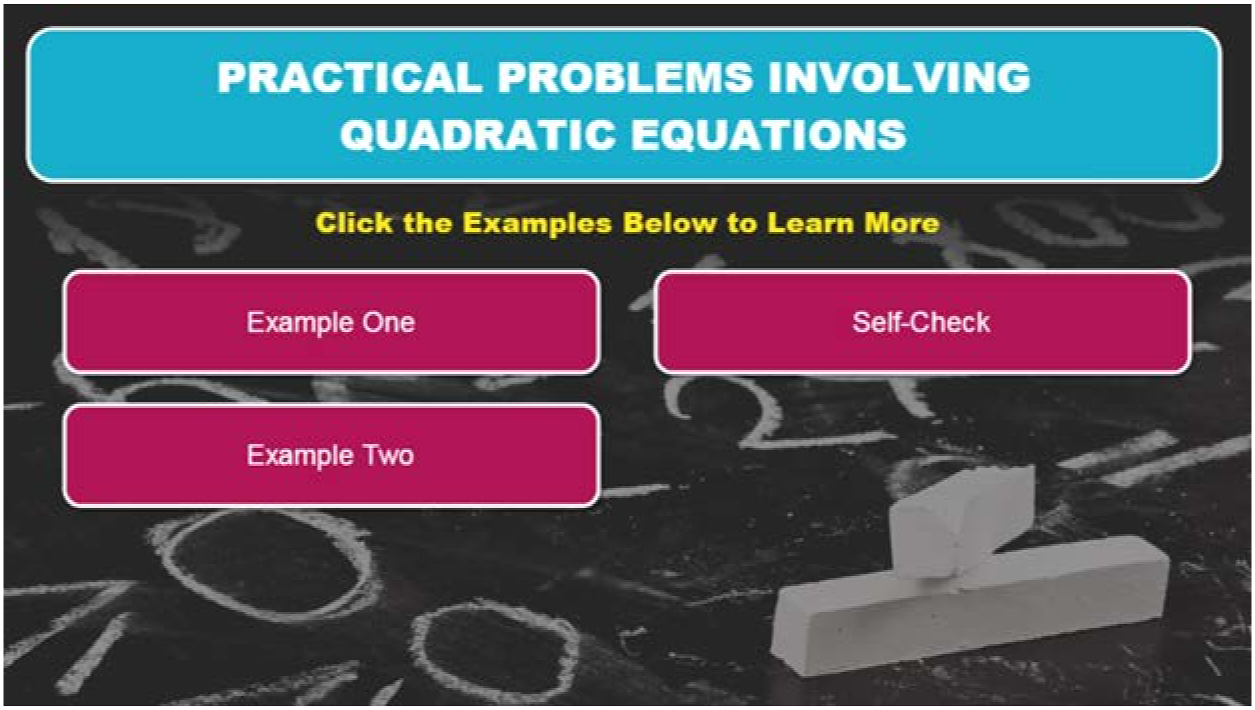
Step 1: Read through the problem.

Step 2: Highlight the key information.

Step 3: Use the key information to solve the problem.

Keep these steps in mind while you work through the following examples.

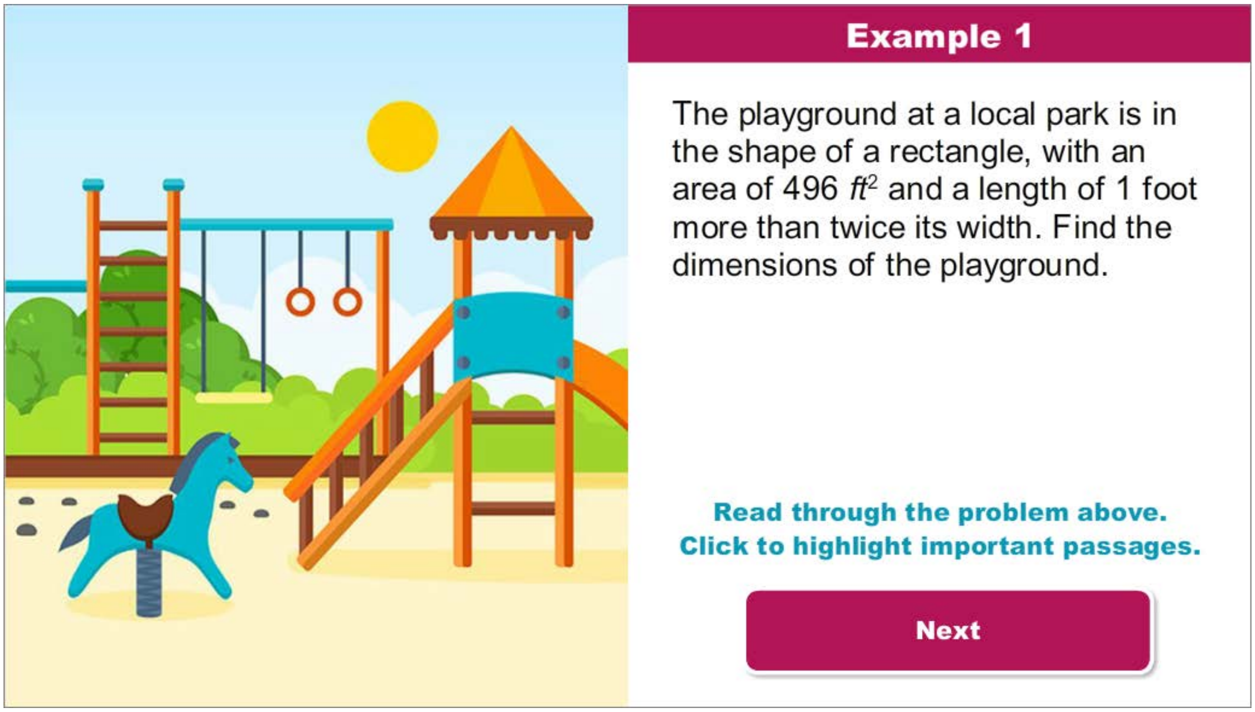
Practical Problems Involving Quadratic Equations



Click the examples below to learn more.

- Example One
- Example Two
- Self-Check

Example One

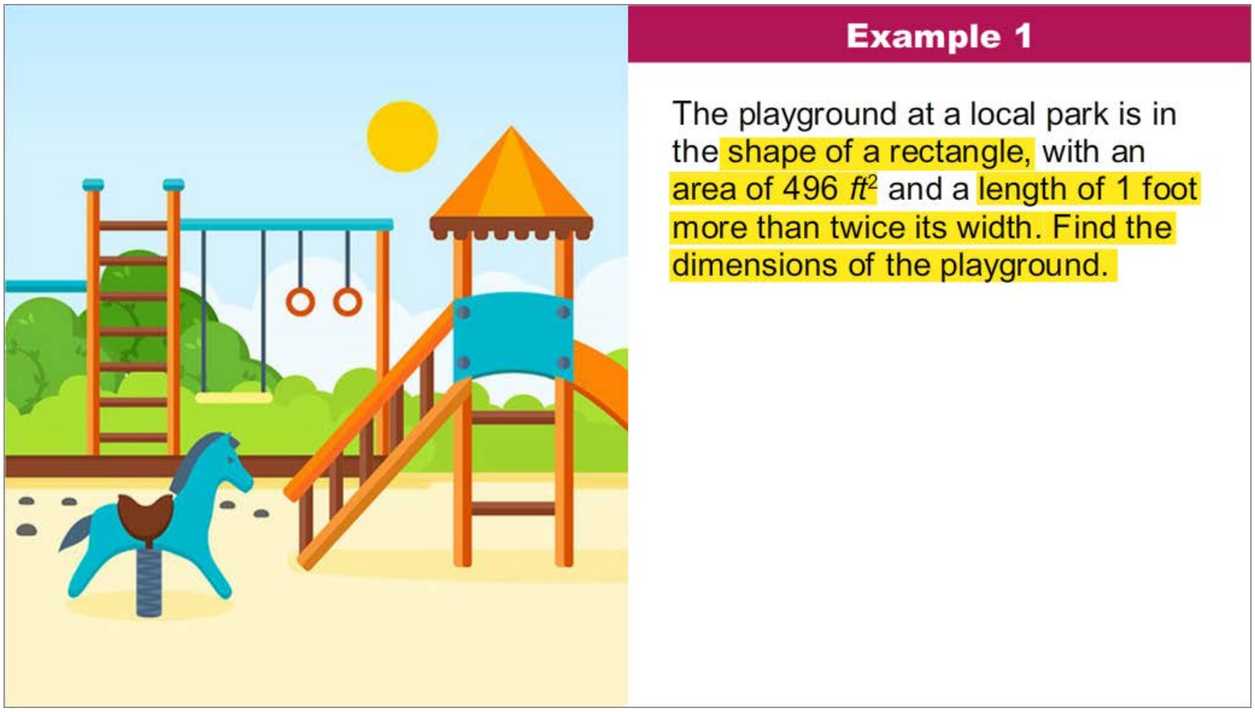


The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

Take a few moments to read Example 1. Highlight the information you think is necessary to solve the problem.

Read through the problem above. Click to highlight important passages.

Example One (continued)

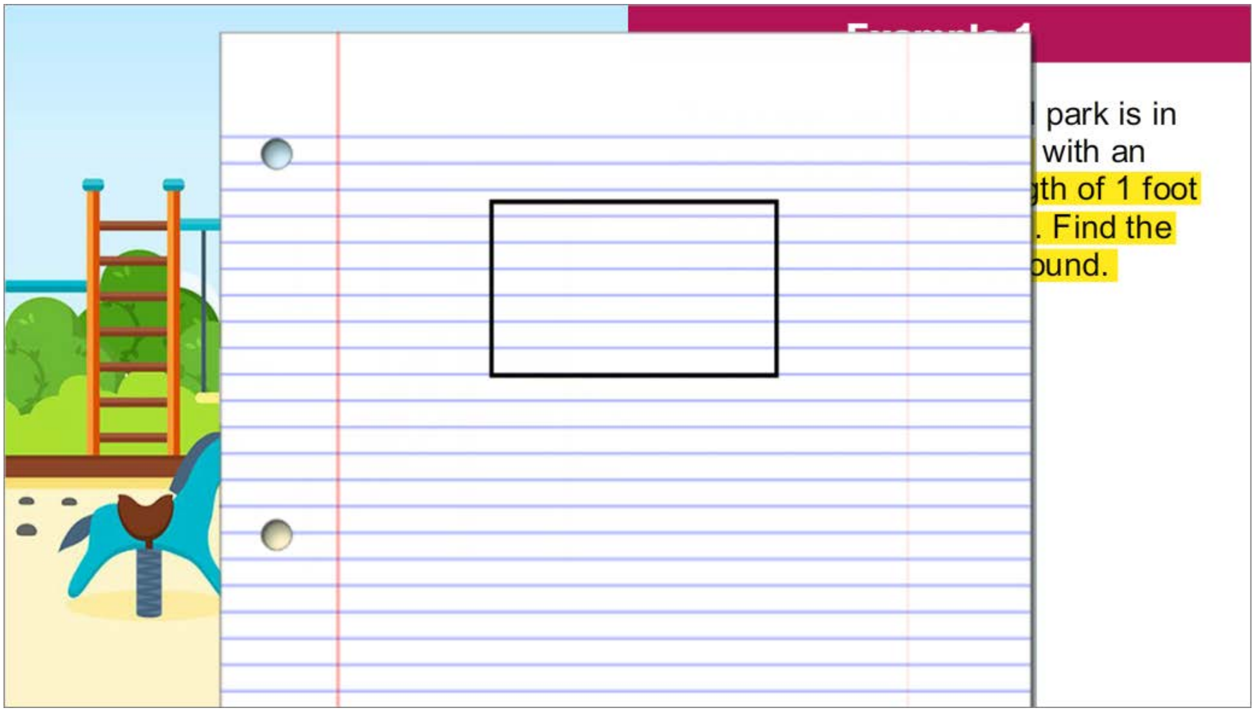


The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

Did you highlight the important facts?

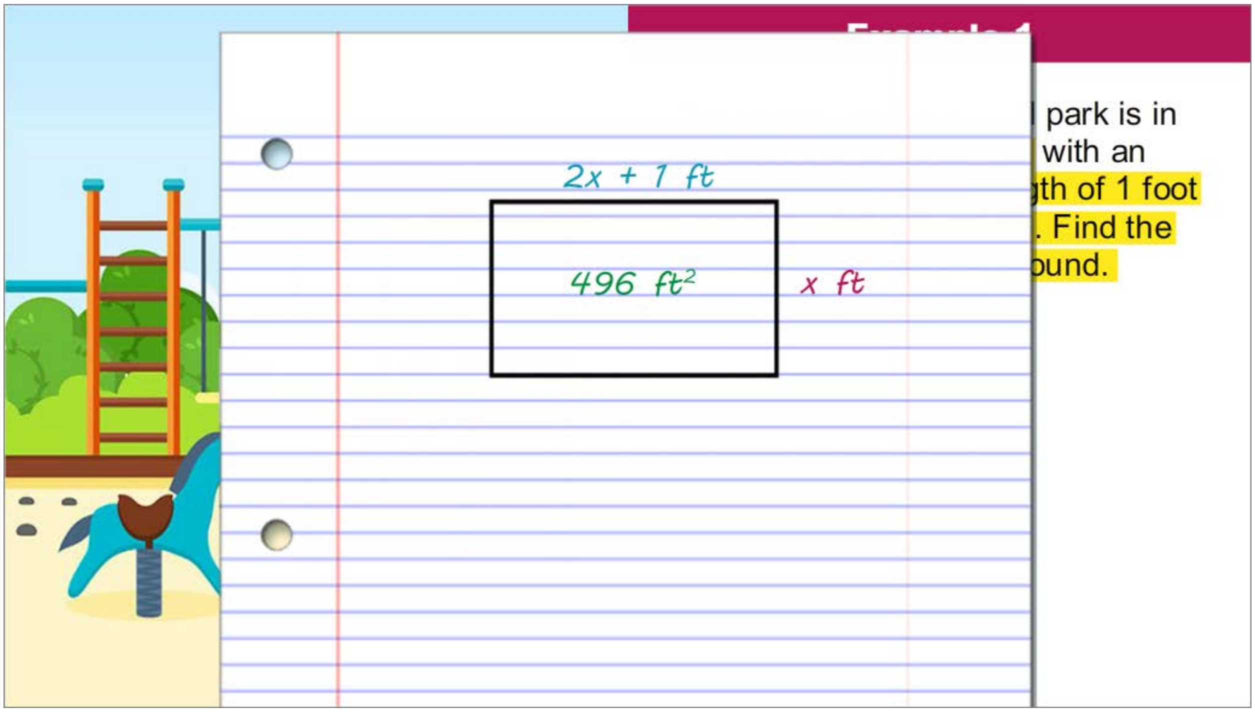
- The playground is in the shape of rectangle
- The area of the playground is $496\ ft^2$
- The length of the playground is 1 foot more than twice its width
- Find the dimensions of the playground

Example One (continued)



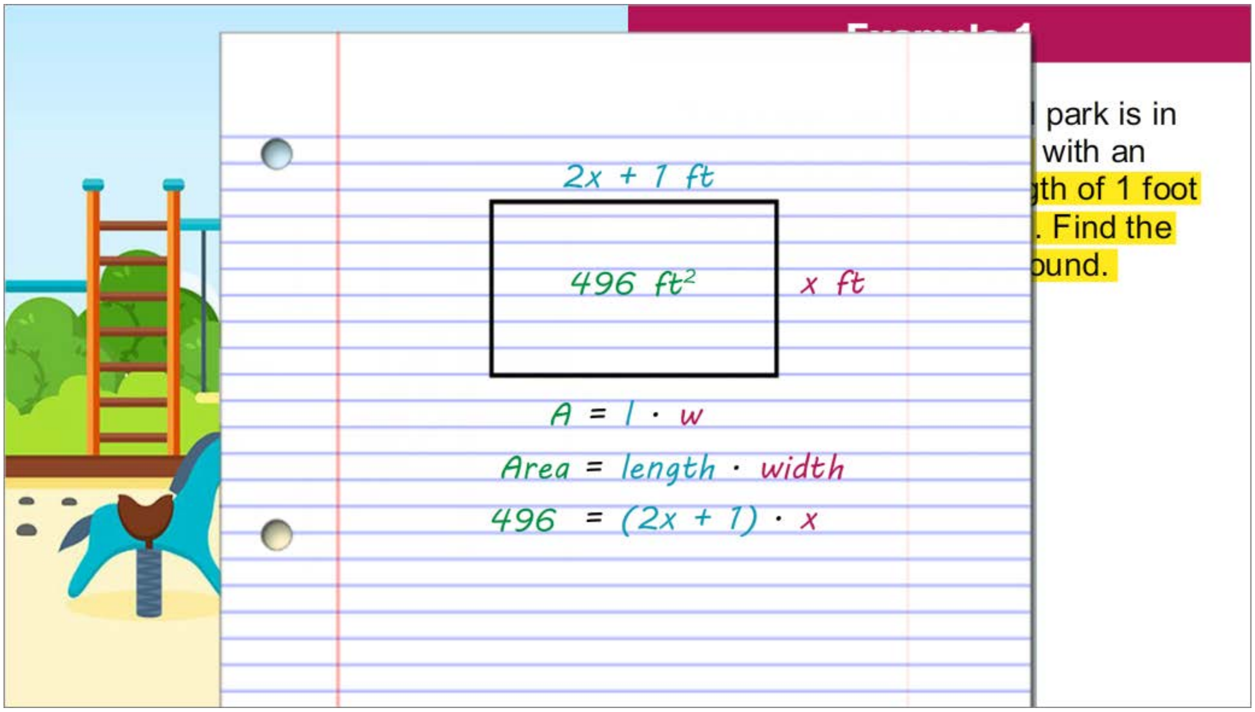
To solve this problem, you may find it helpful to start with a rough sketch of the situation. The playground is in the shape of a rectangle. On notebook paper, sketch a rectangle to represent the playground.

Example One (continued)



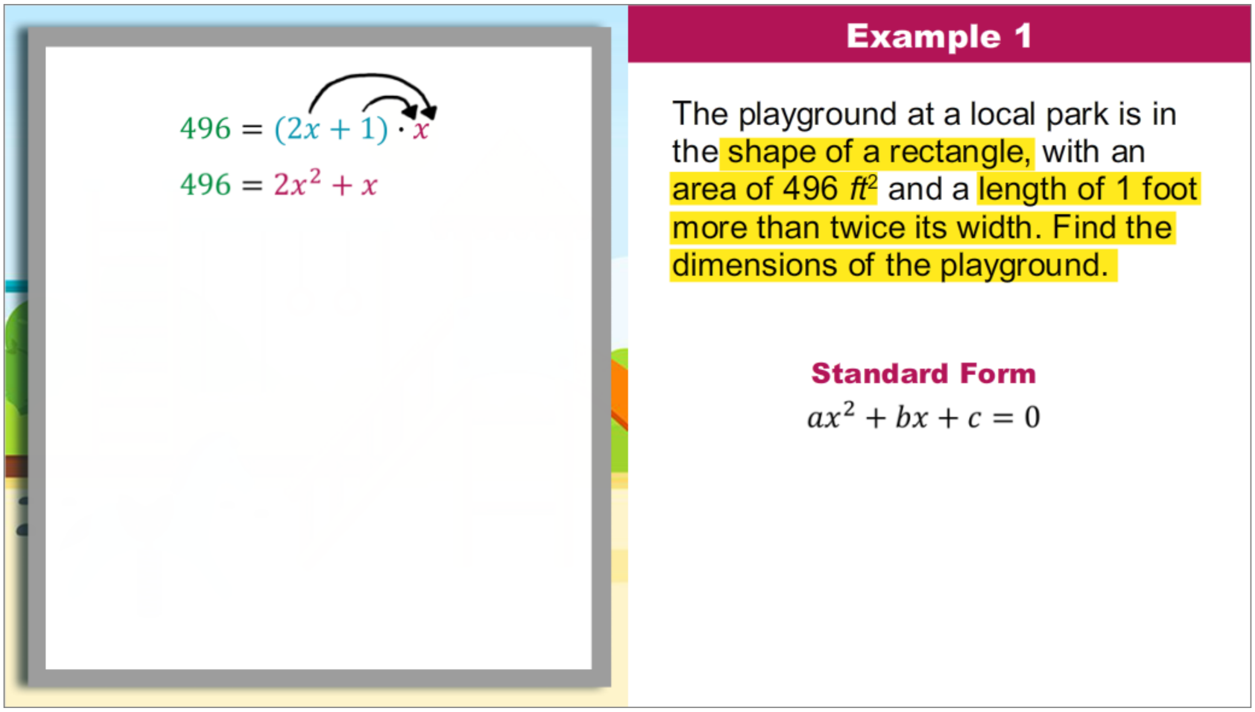
The area of the playground is 496 ft^2. Its length is 1 foot more than twice its width. Let x represent the width of the rectangle. Because the length is 1 foot more than twice its width, the length can be represented by the expression $2x + 1$.

Example One (continued)



Now it is time to set up the equation. The formula for the area of a rectangle is $A = l \cdot w$. A represents the area of the rectangle, l represents its length and w represents its width. In the equation, substitute 496 for A, the expression $2x + 1$ for l, and x for w.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

Standard form: $a^2 + bx + c = 0$

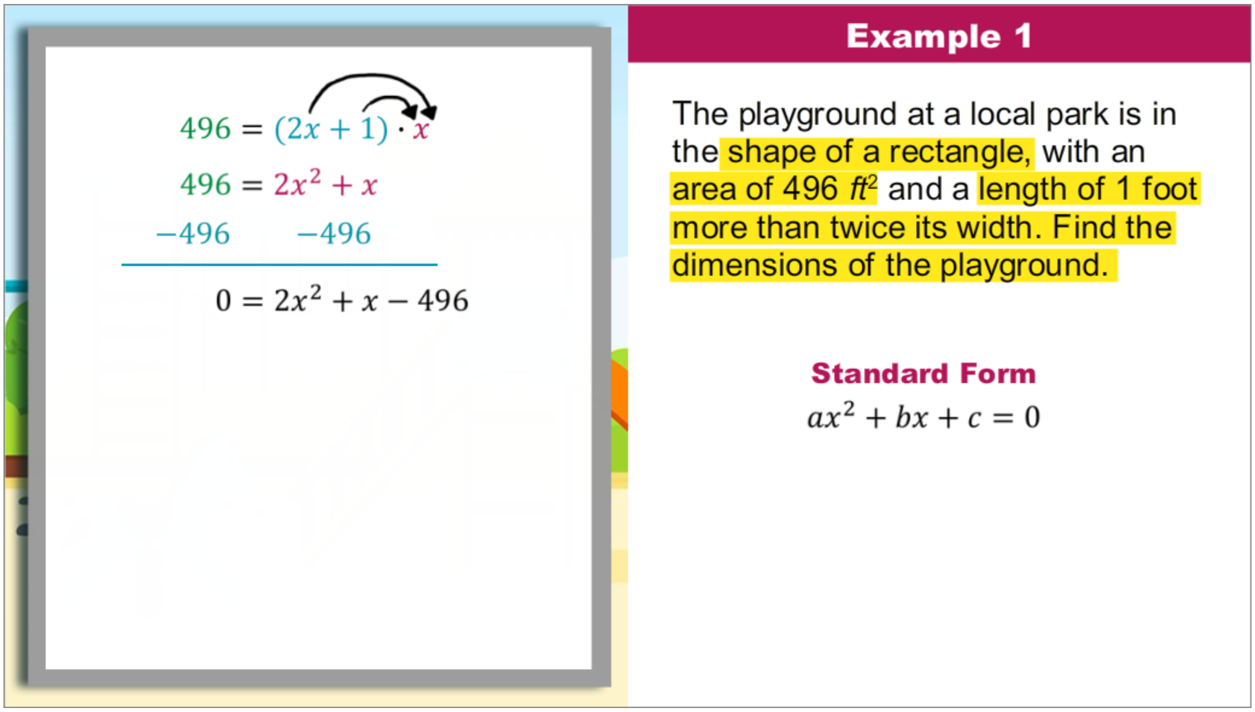
$496 = (2x^2 + 1) \cdot x$

$496 = 2x^2 + x$

Now that you have written the equation to model the situation, solve for x. Begin by applying the distributive property to the right side of the equation.

Notice that the equation now includes a square term. Therefore, it is a quadratic equation. To solve a quadratic equation begin by representing it in standard form.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

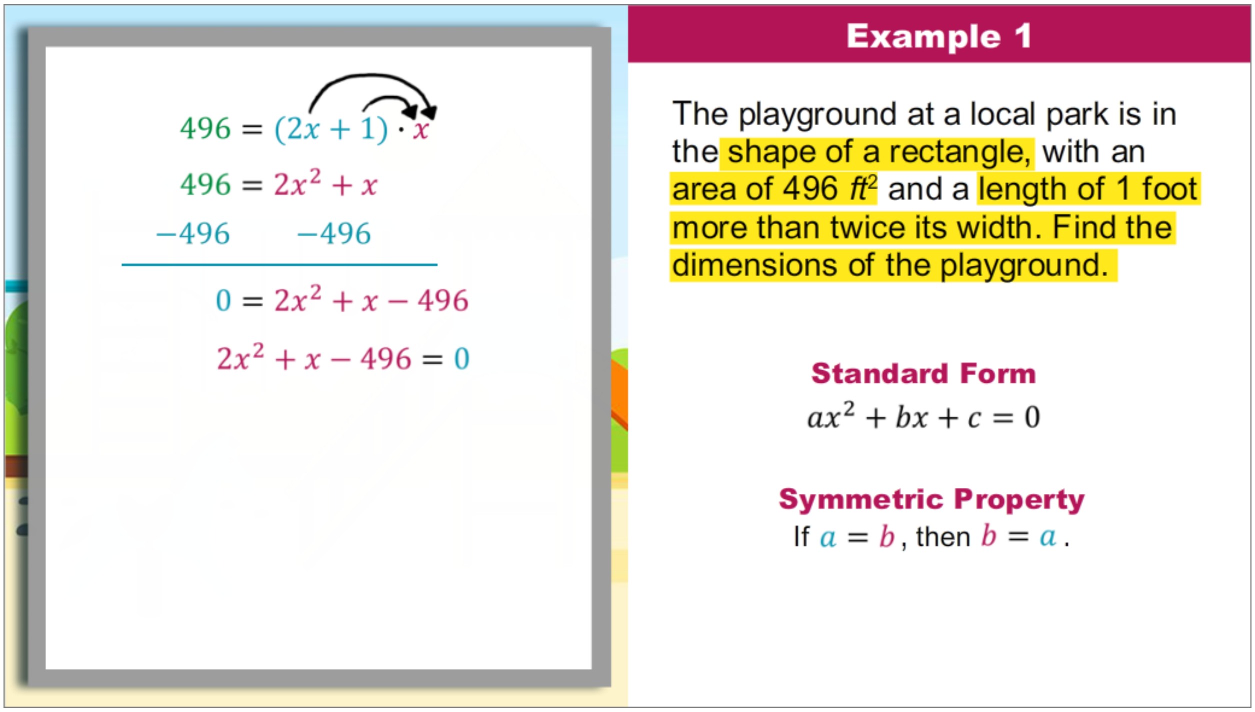
Standard form: $a^2 + bx + c = 0$

$$496 = (2x^2 + 1) \cdot x$$

$$496 = 2x^2 + x$$
$$-\quad 496 - 496$$
$$0 = 2x^2 + x - 496$$

One strategy is to subtract 496 from each side of the equation. The result is $0 = 2x^2 + x - 496$.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

Standard form: $a^2 + bx + c = 0$

Symmetric Property: If $a = b$, then $b = a$.

$$496 = (2x^2 + 1) \cdot x$$

$$496 = 2x^2 + x$$

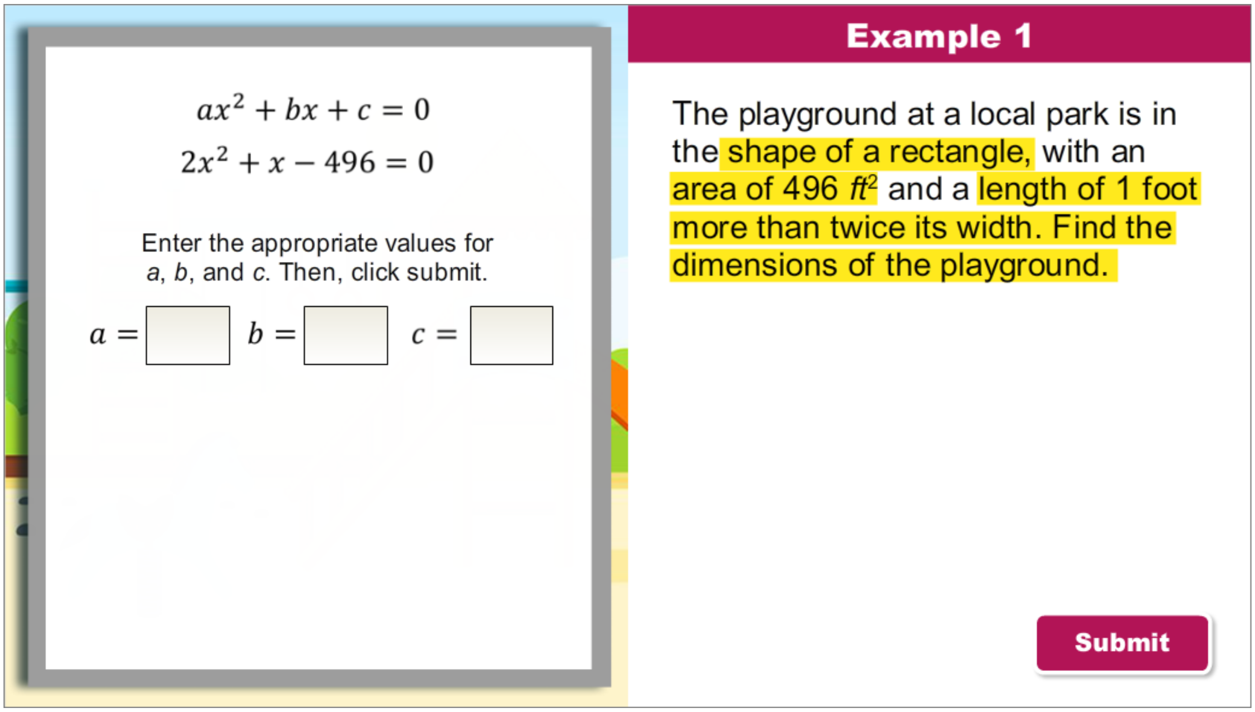
$$-\quad 496 \quad -496$$

$$0 = 2x^2 + x - 496$$

$$2x^2 + x - 496 = 0$$

The Symmetric Property states that the order of equality does not affect an equation. Therefore, you can represent the equation as $2x^2 + x - 496 = 0$.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

Standard form: $a^2 + bx + c = 0$

Symmetric Property: If $a = b$, then $b = a$.

$$ax^2 + bx + c = 0$$
$$2x^2 + x - 496 = 0$$

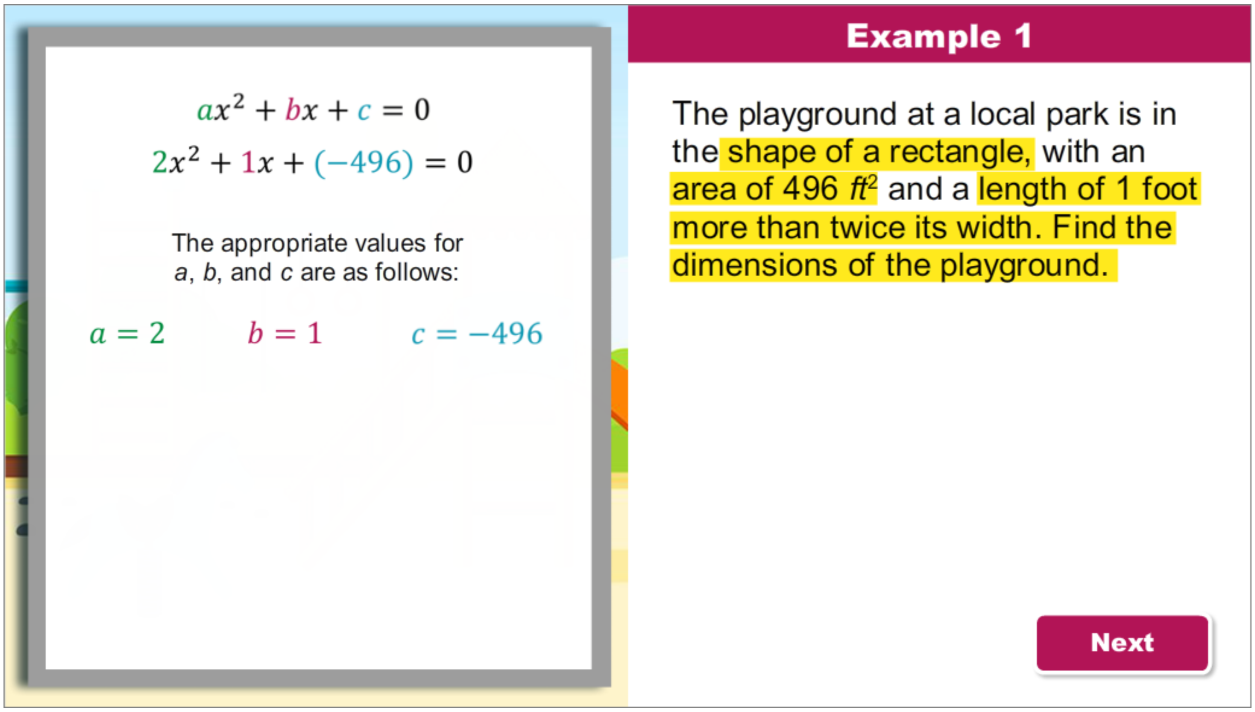
You have learned two methods of solving a quadratic equation, factoring and the quadratic formula. You may find it more efficient to use the quadratic formula for this example.

Start by identifying a, b, and c.

$$a = ? \qquad b = ? \qquad c = ?$$

Enter the appropriate values for *a*, *b*, and *c*. Then, click submit.

Example One (continued)



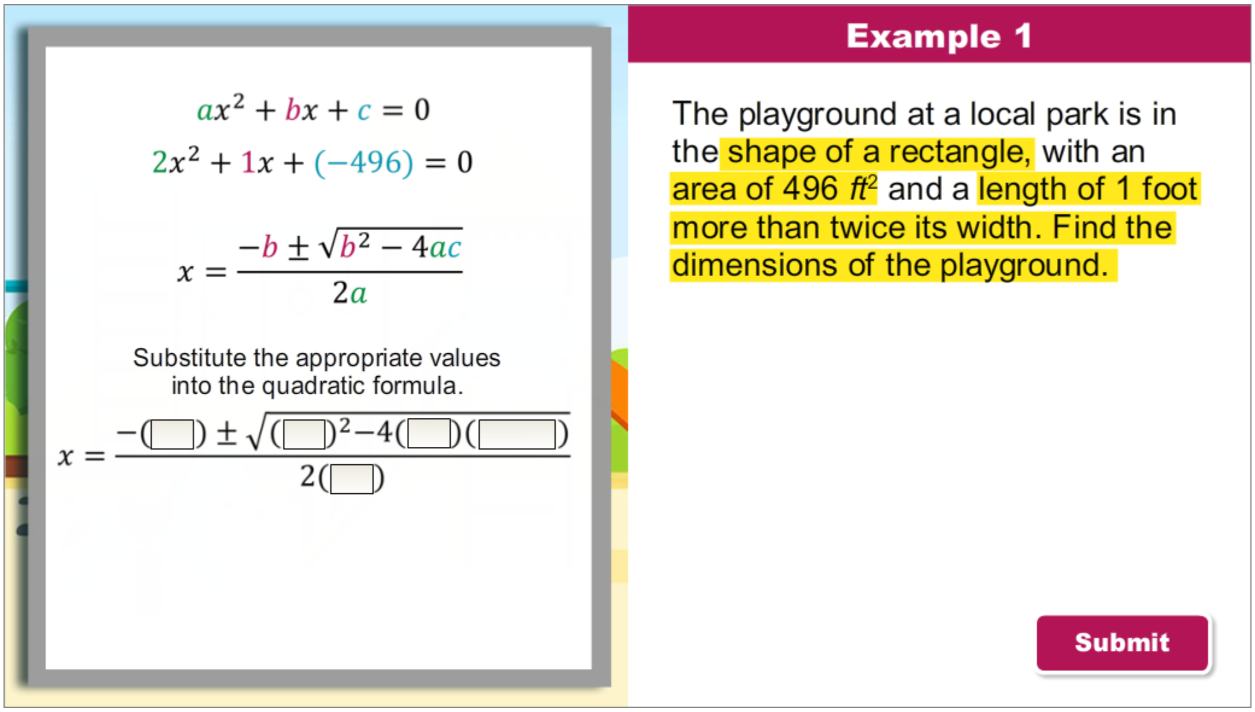
The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

$$ax^2 + bx + c = 0$$
$$2x^2 + 1x - 496 = 0$$

The appropriate values for a, b, and c are as follows:

$a = 2 \quad b = 1 \quad c = -496$

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

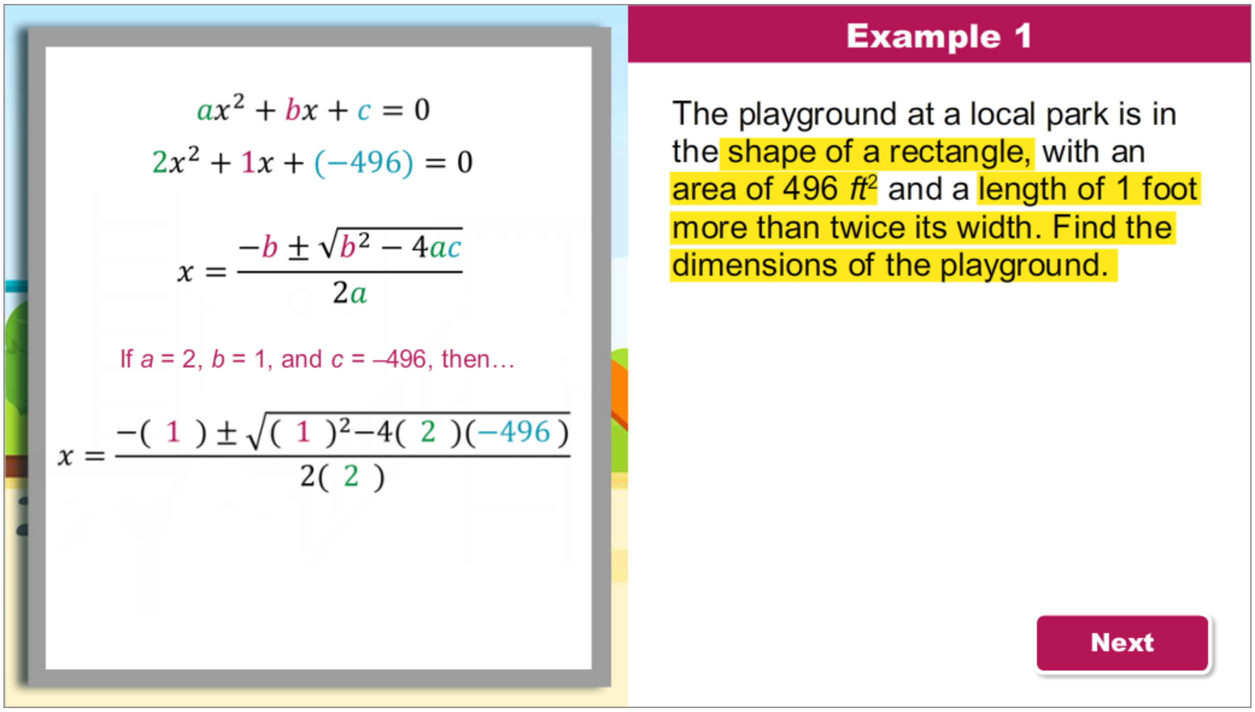
$$ax^2 + bx + c = 0$$
$$2x^2 + 1x - 496 = 0$$

Now that you have found that $a = 2, b = 1, c = -496$, substitute the values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(?) \pm \sqrt{(?)^2 - 4(?)(?)}}{2(?)}$$

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

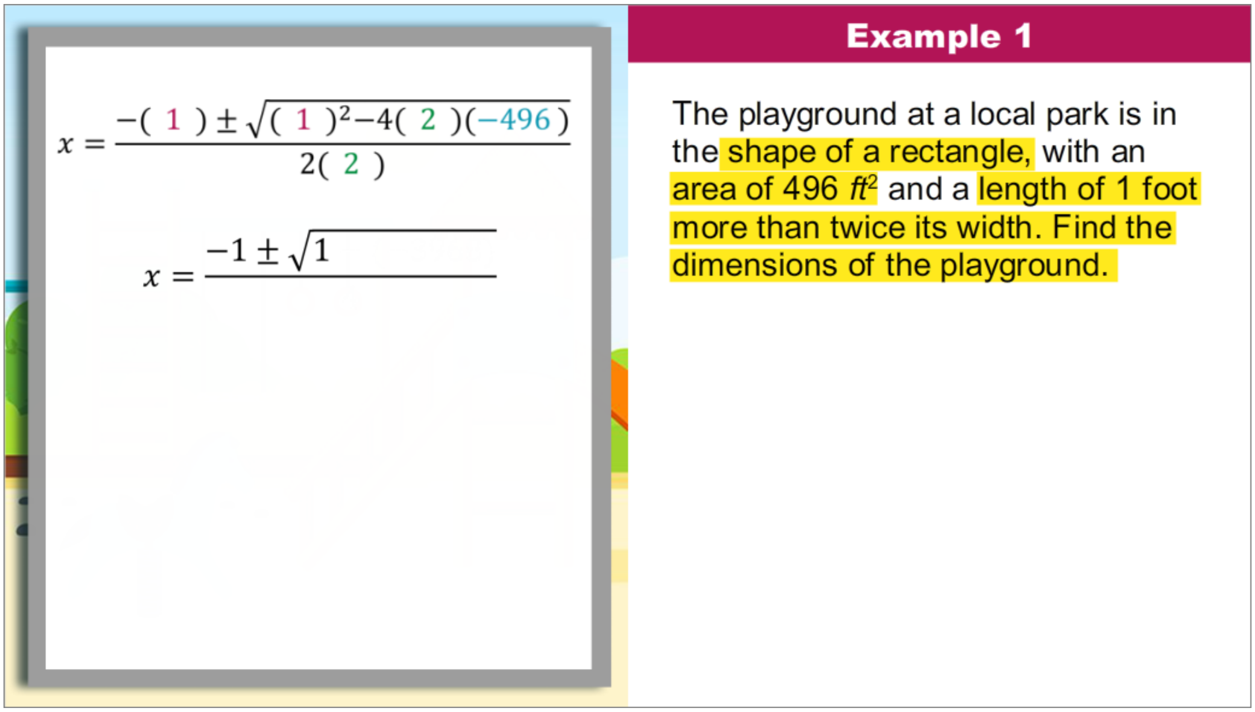
$$ax^2 + bx + c = 0$$
$$2x^2 + 1x - 496 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $a = 2, b = 1, c = -496$, then…

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-496)}}{2(2)}$$

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-496)}}{2(2)}$$

Now, begin to simplify the expression to the right of the equals sign. Start with the numerator.

$$x = \frac{-1 \pm \sqrt{1}}{}$$

The opposite of 1 is -1.

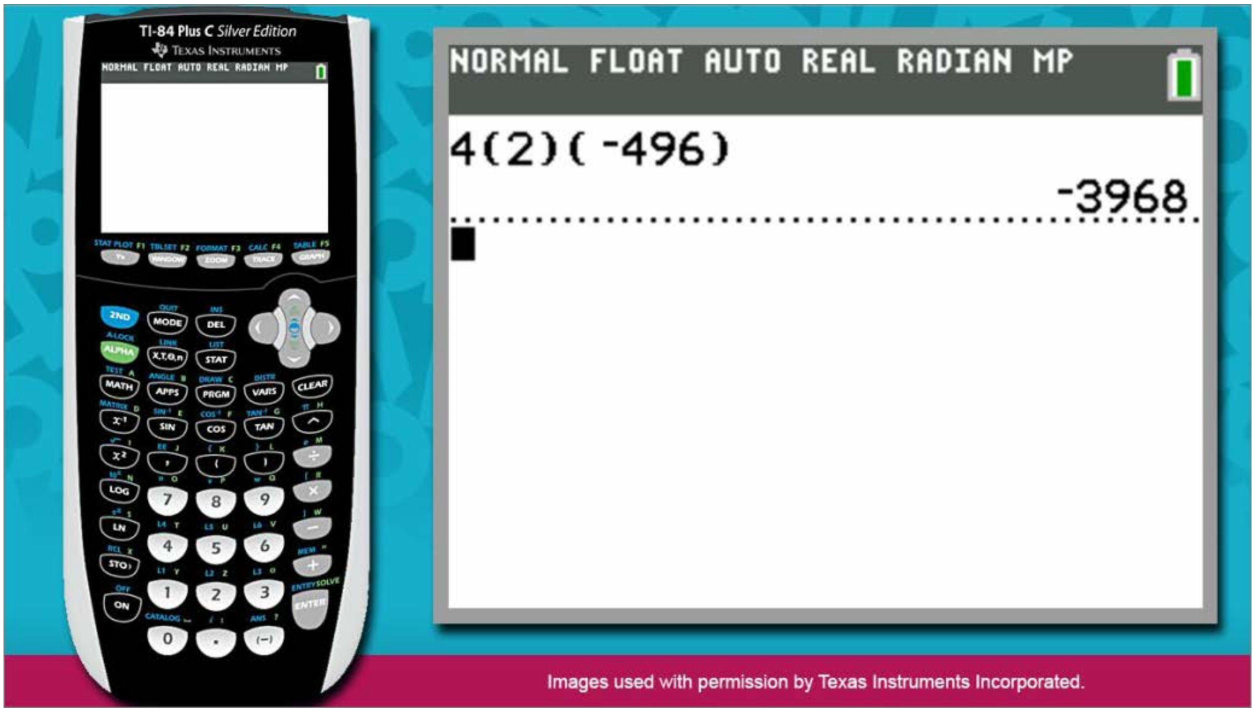
$1^2 = 1$

To find the product of $4, 2,$ and -496, you may want to use the calculator.

Example One (continued)

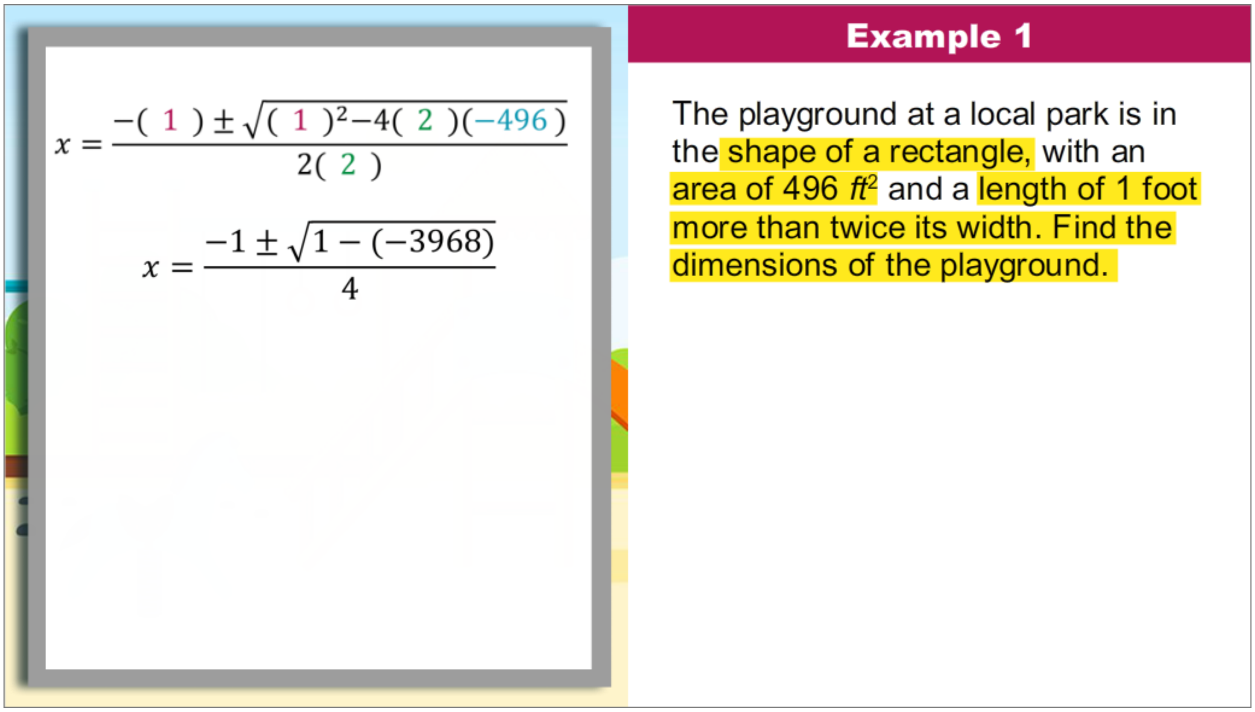
Input the product into the calculator.

Example One (continued)



After pressing enter you find that the product is -3968.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

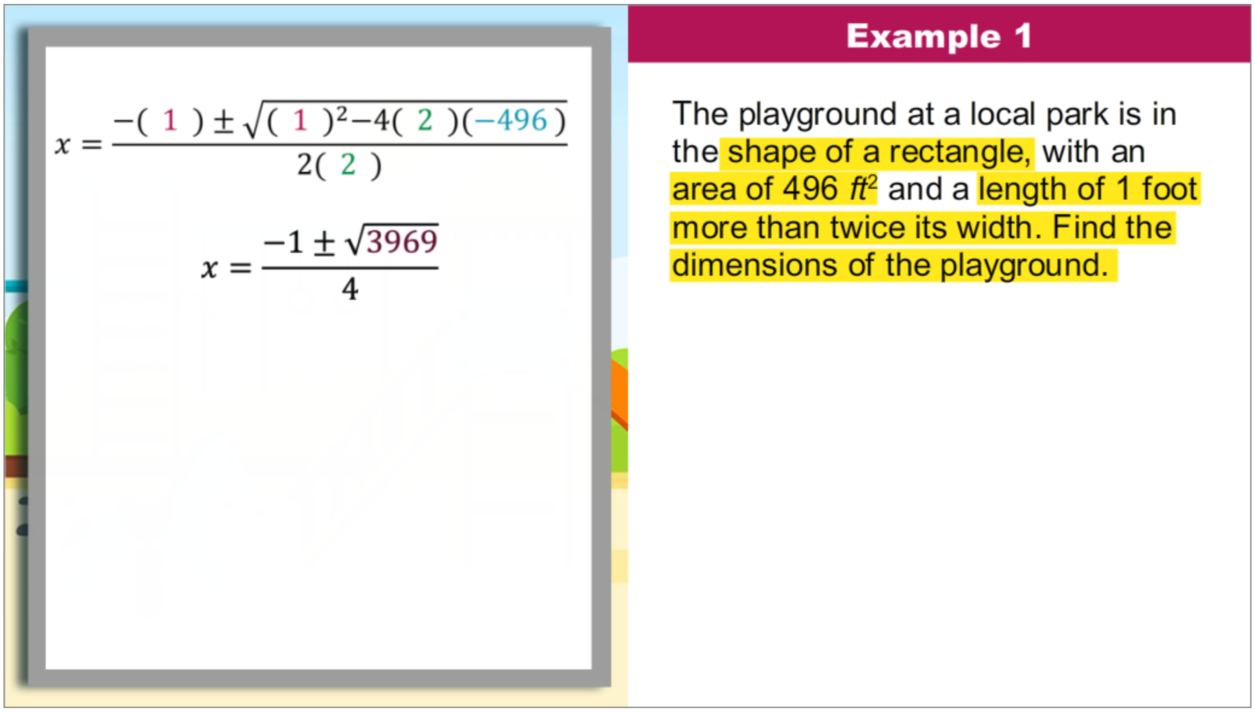
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(2)(-496)}}{2(2)}$$

$$x=\frac{-1\pm\sqrt{1-(-3968)}}{4}$$

Next, simplify the product in the denominator.

$$2(2)=4$$

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-496)}}{2(2)}$$

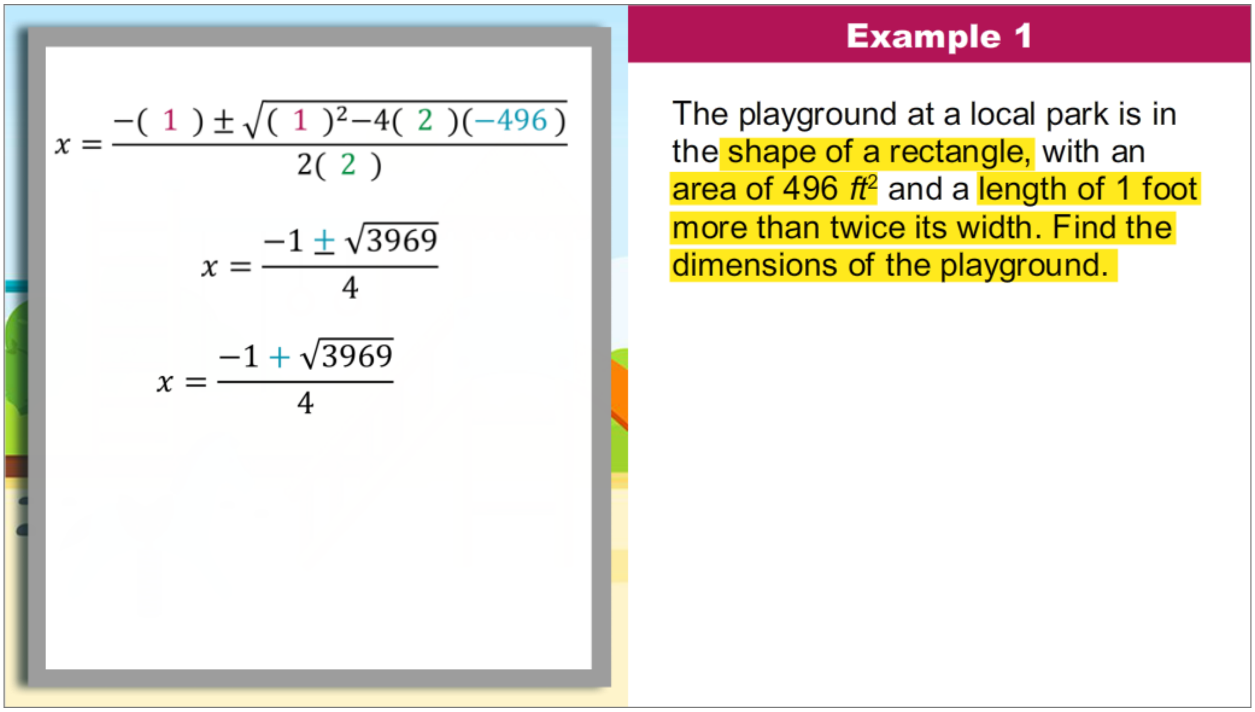
$$x = \frac{-1 \pm \sqrt{3968}}{4}$$

Now, simplify the expression underneath the radical symbol.

$1 - (-3968)$ can be represented as $1 + 3968$.

This sum equals 3969.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-496)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{3968}}{4}$$

$$x = \frac{-1 + \sqrt{3969}}{4}$$

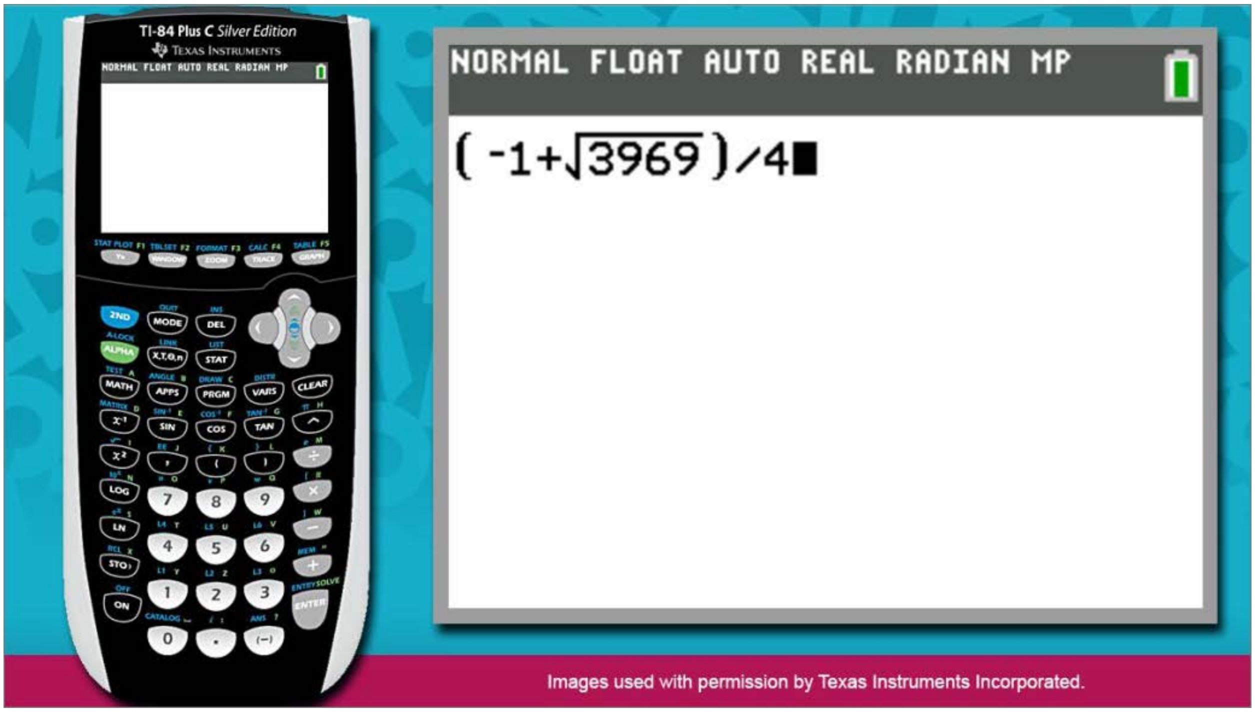
There are two solutions to the quadratic equation. One of the solutions is $x = \frac{-1+\sqrt{3969}}{4}$.

You may want to use the calculator to simplify the expression to the right of equals sign.

Example One (continued)

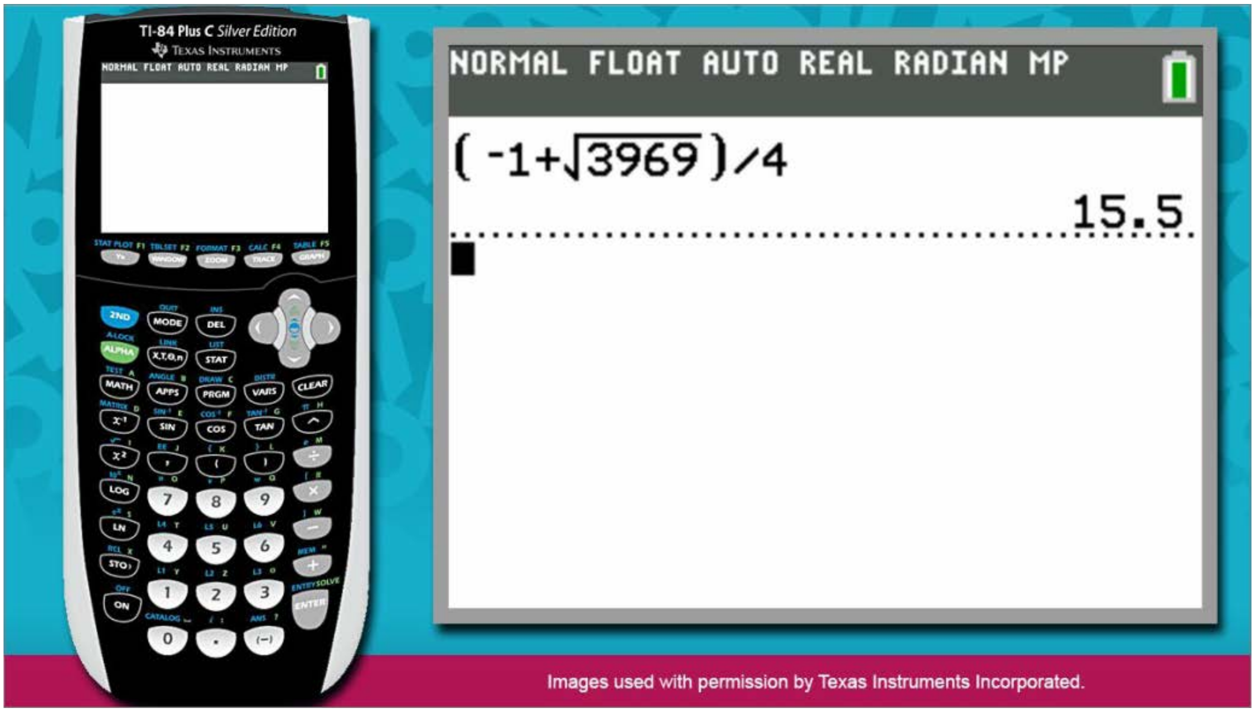
Remember to place parentheses around the expression in the numerator.

Example One (continued)



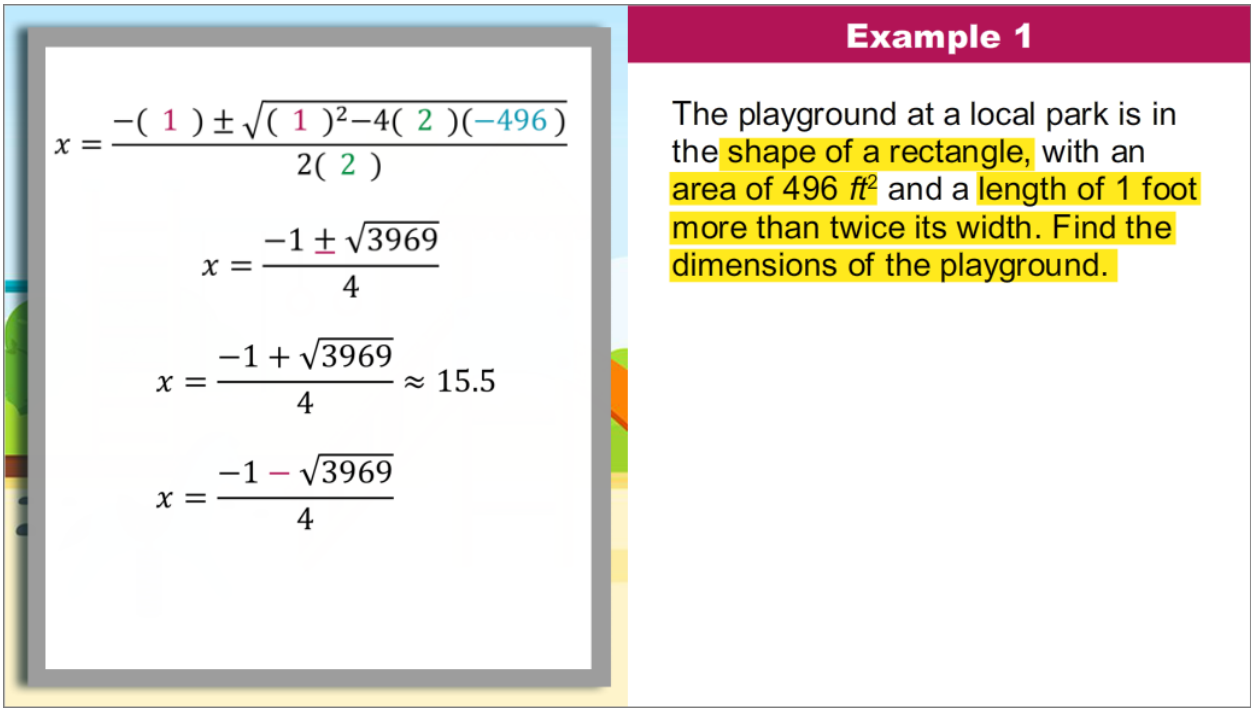
Then, divide by 4.

Example One (continued)



After pressing ENTER, you find that the result is 15.5.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-496)}}{2(2)}$$

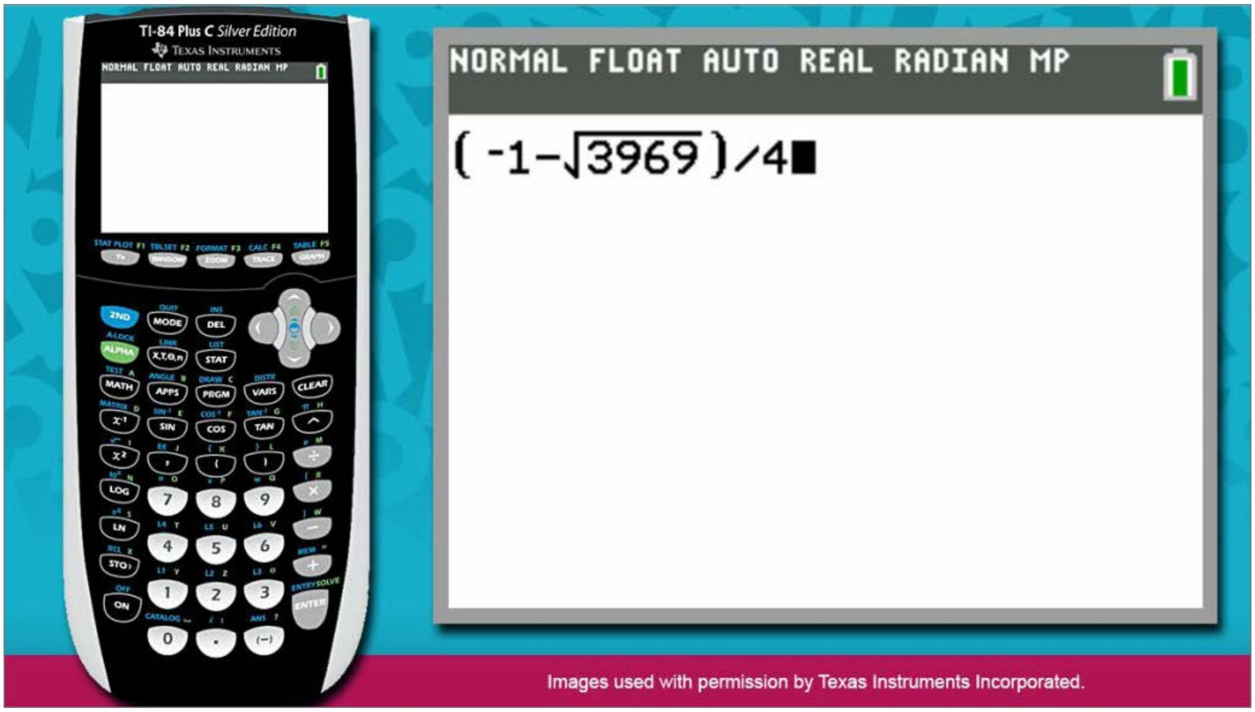
$$x = \frac{-1 \pm \sqrt{3968}}{4}$$

$$x = \frac{-1 - \sqrt{3969}}{4}$$

Now that you know that one of the solutions is 15.5, you can move on to determine the second solution, where $x = \frac{-1-\sqrt{3969}}{4}$.

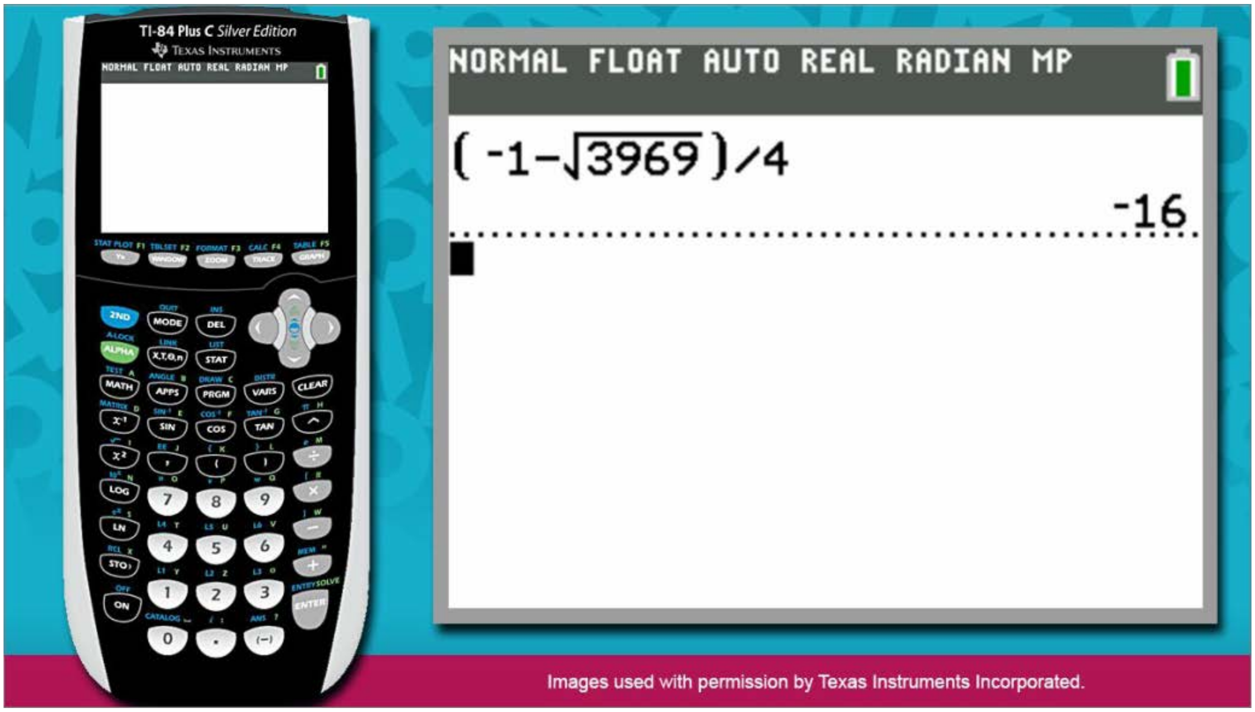
Again, you may want to use the calculator to determine this value.

Example One (continued)



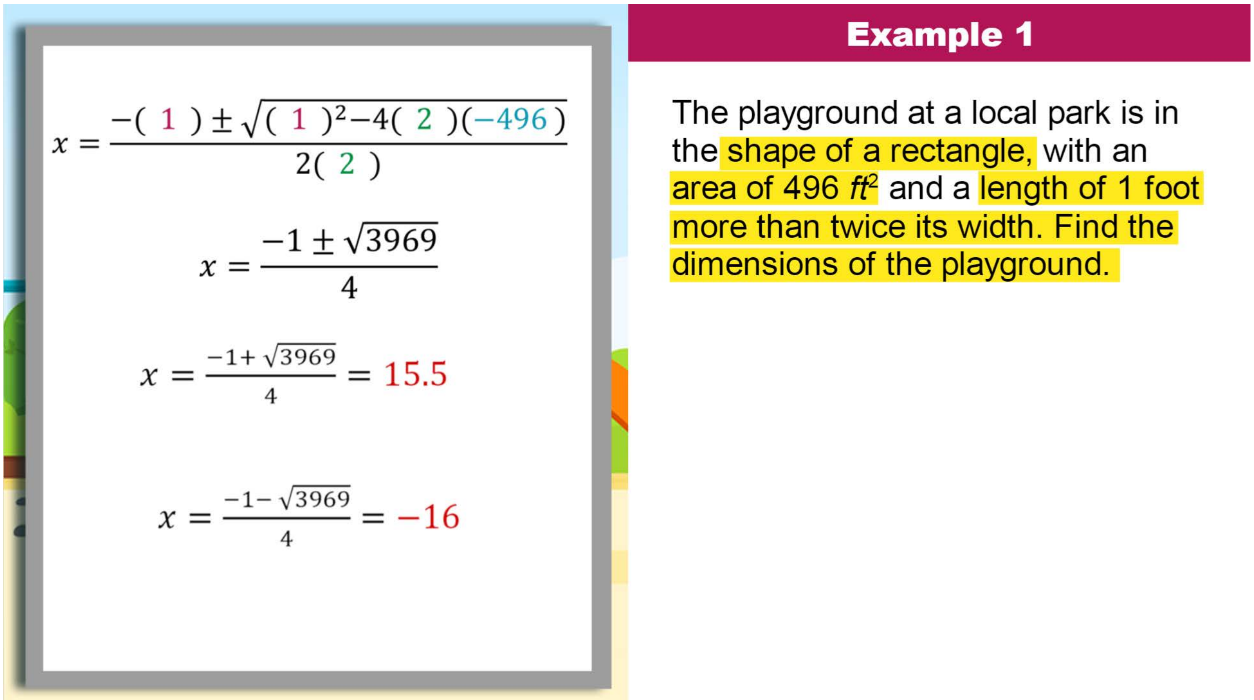
Remember to place parentheses around the expression in the numerator before dividing by 4.

Example One (continued)



After pressing ENTER, you find that the result is -16.

Example One (continued)



The playground at a local park is in the shape of a rectangle, with an area of $496\ ft^2$ *and a length* 1 *foot more than twice its width. Find the dimensions of the playground.*

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-496)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{3968}}{4}$$

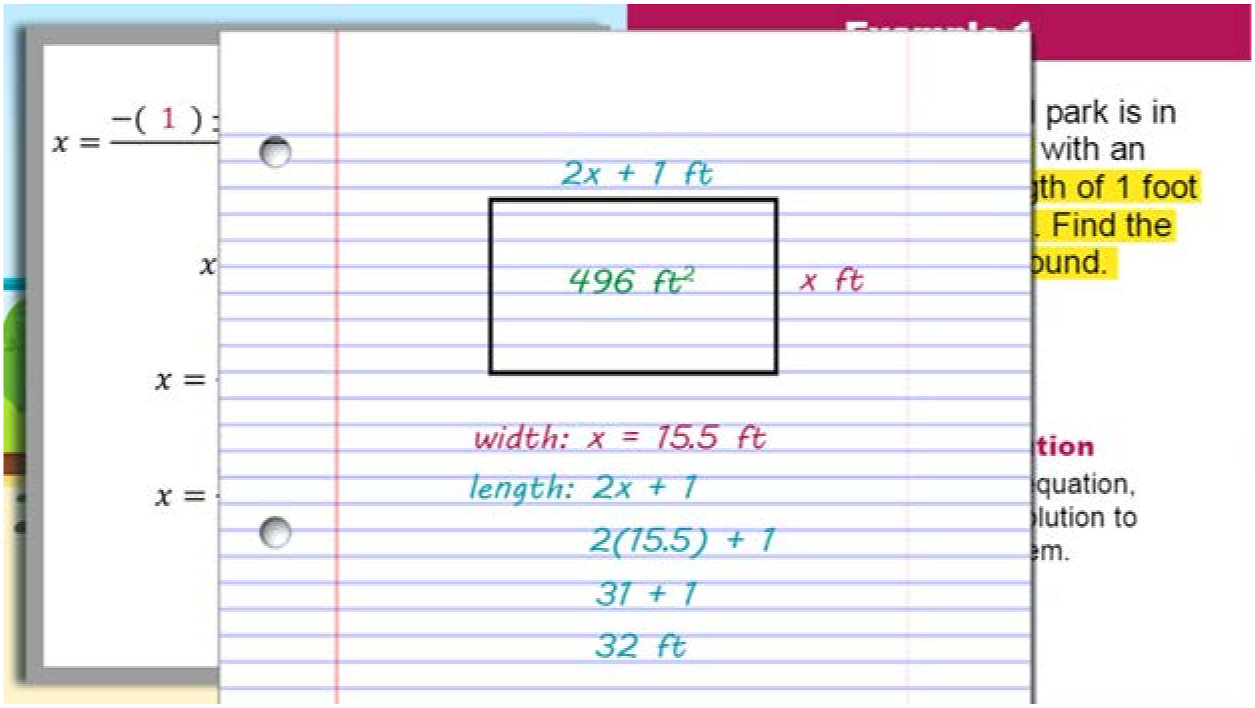
$$x = \frac{-1 + \sqrt{3969}}{4} = 15.5$$

$$x = \frac{-1 - \sqrt{3969}}{4} = -16$$

You have determined both solutions to the quadratic equation: $x = 15.5$ or $x = -16$.

Because x represents the width of the rectangle, -16 is not a reasonable solution. -16 is referred to as an extraneous solution, as it is a valid solution to the equation, but not a reasonable solution to the problem.

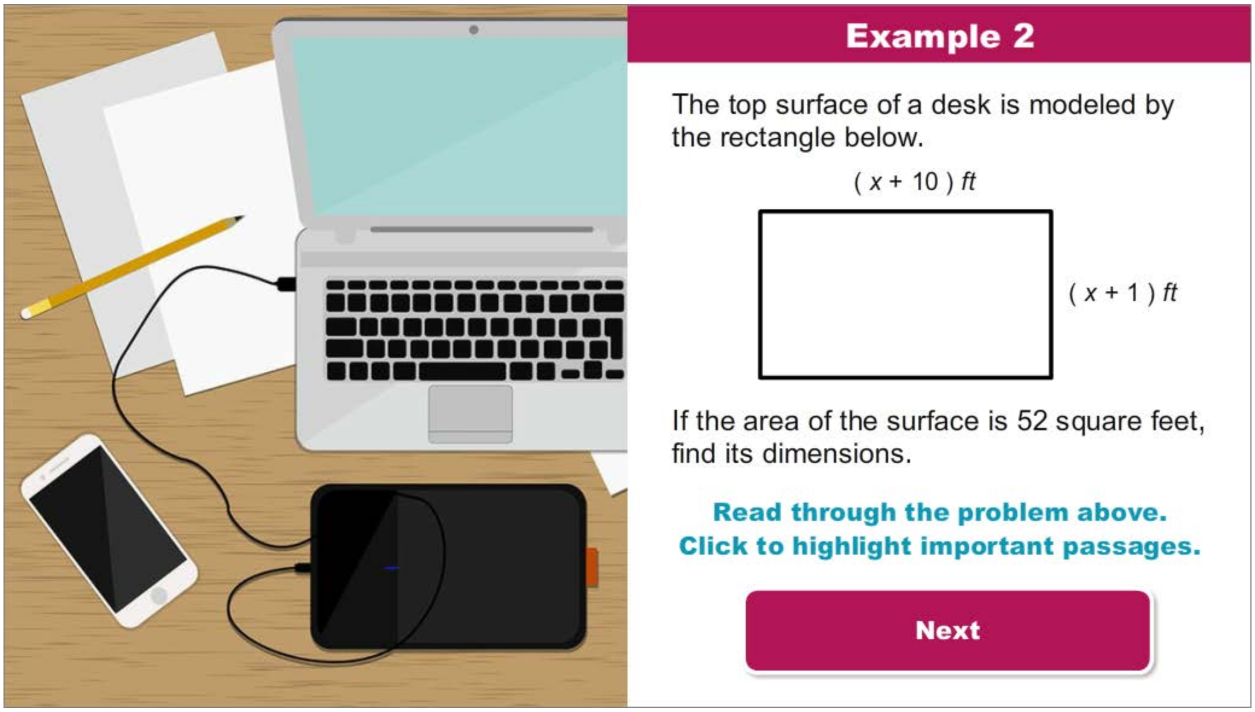
Example One (continued)



width: $x = 15.5$
length: $2x + 1$
$2(15.5) + 1$
$31 + 1$
$32\ ft$

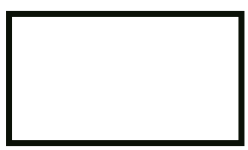
Therefore, the rectangular-shaped playground has a width 15.5 ft. After substituting 15.5 for x in the expression $2x + 1$, you find that the length of the playground is 32 ft.

Example Two



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$



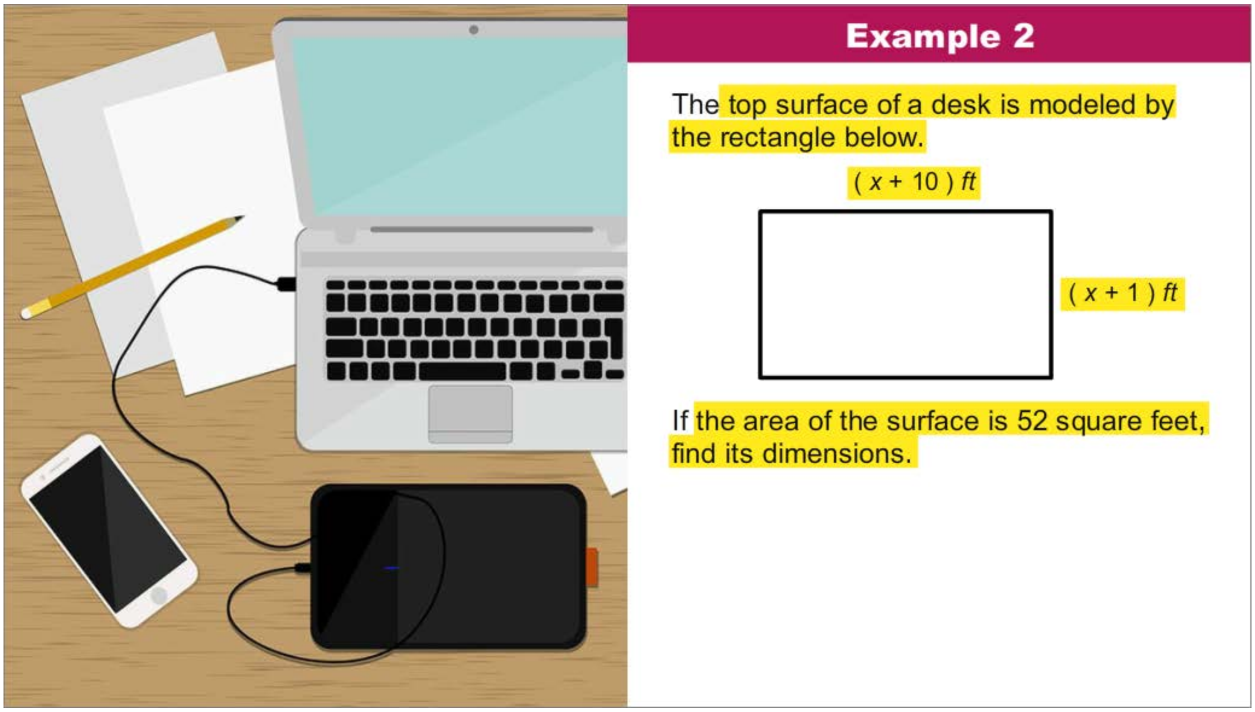
$(x + 1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

Take a few moments to read Example 2. Highlight the information you think is necessary to solve the problem.

Read through the problem above. Click to highlight the important passages.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$

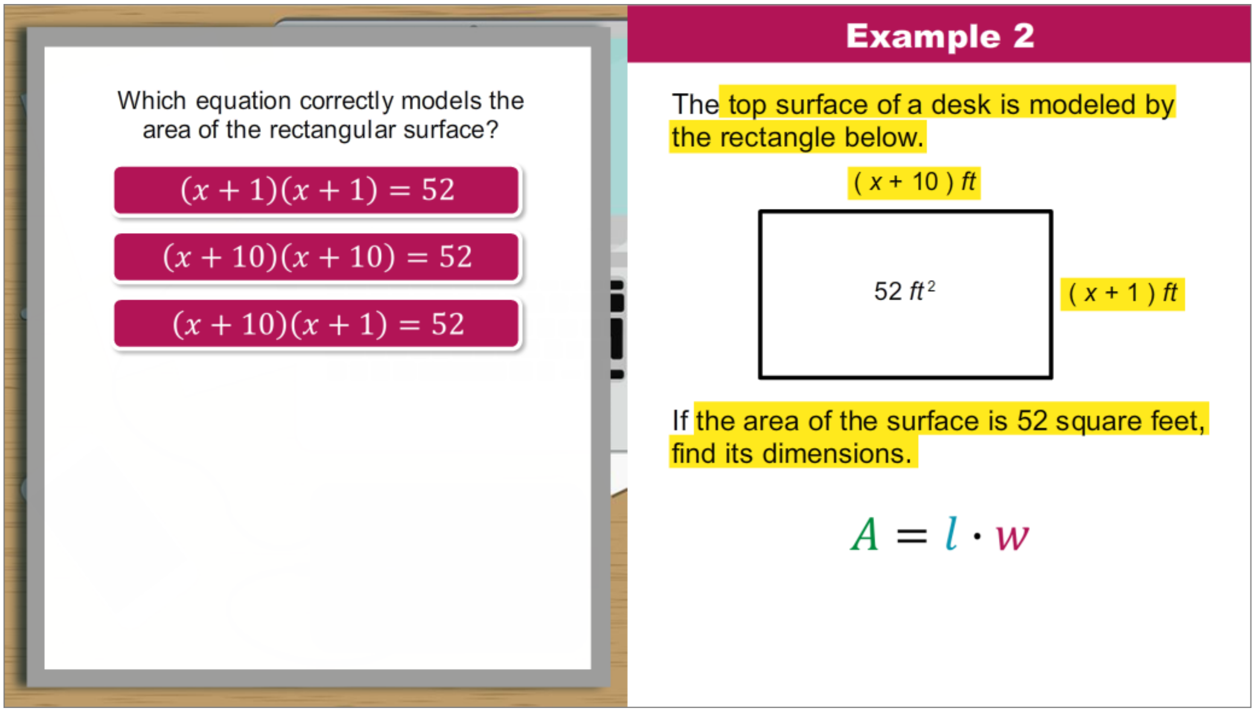
$(x + 1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

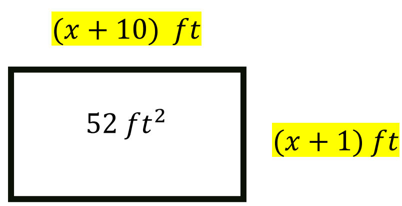
Did you highlight the important facts?

- The top surface of the desk is a rectangle
- The length of the rectangle is represented by $x + 10$
- The width of the rectangle is represented by $x + 1$
- The area of the surface is 52 square feet
- Find the dimensions of the rectangle.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.



If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

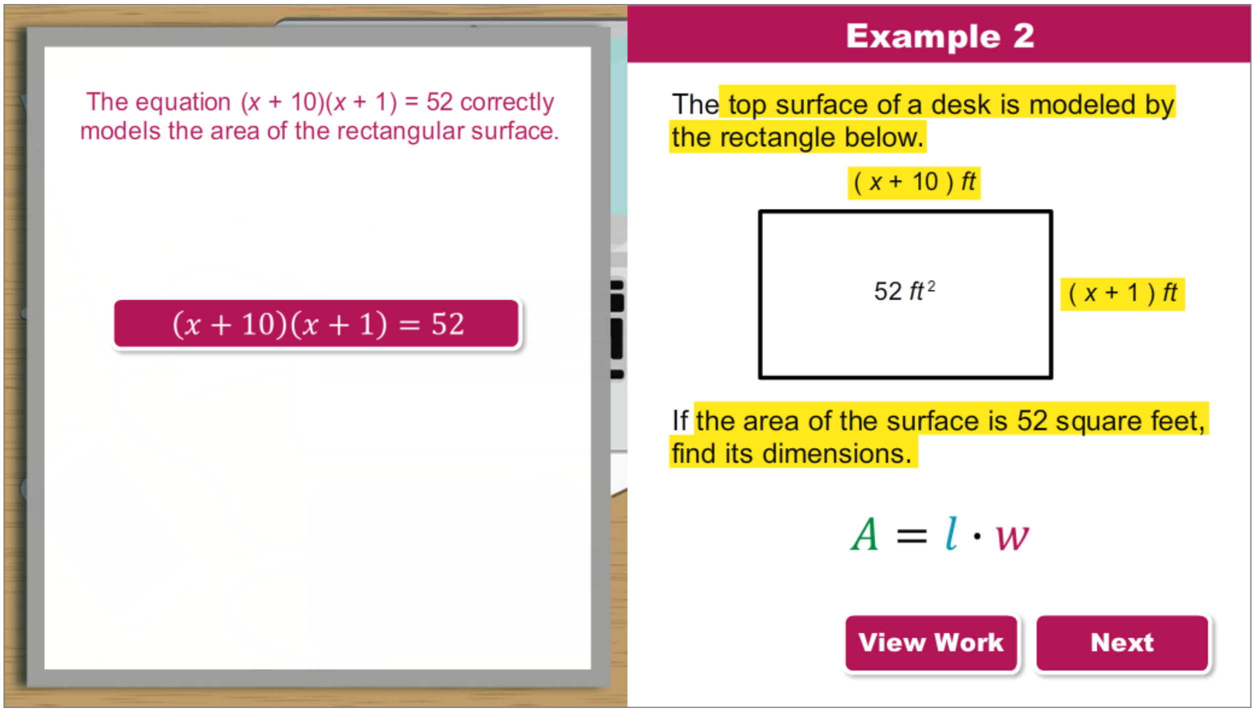
The formula for the area of a rectangle is $A = l \cdot w$.

The area of the rectangular surface is 52 square feet.

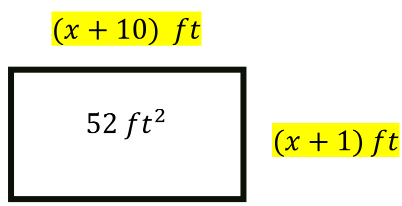
Which of the following equations correctly models the area of the rectangular surface?

A) $(x+1)(x+1) = 52$
B) $(x+10)(x+10) = 52$
C) $(x+10)(x+1) = 52$

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

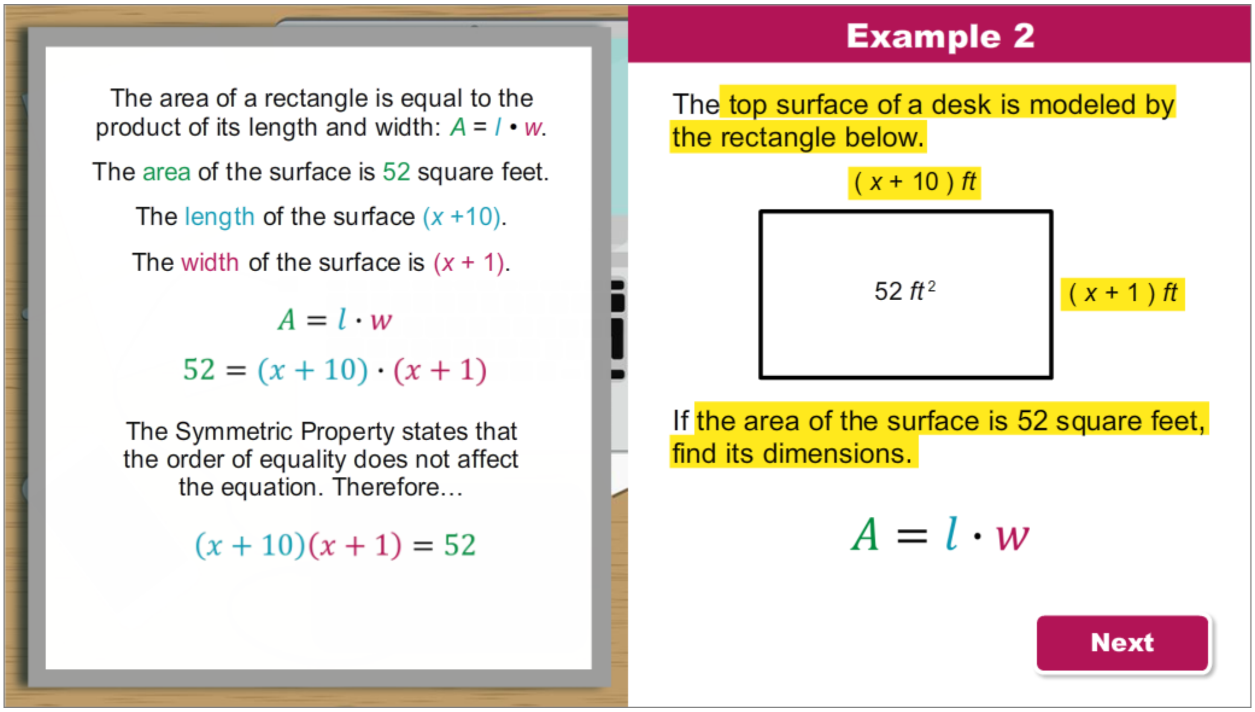


If the area of the surface is 52 *square feet, find its dimensions.*

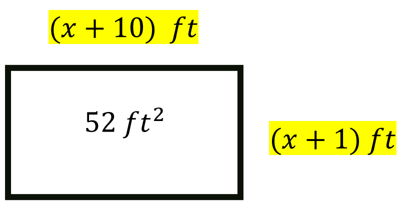
$$A = l \cdot w$$

The equation $(x + 10)(x + 1) = 52$ correctly models the area of the rectangular surface.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.



If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

The area of a rectangle is equal to the product of its length and width: $A = l \cdot w$.

The area of the surface is 52 square feet.

The length of the surface is $(x + 10)$.

The width of the surface is $(x + 1)$.

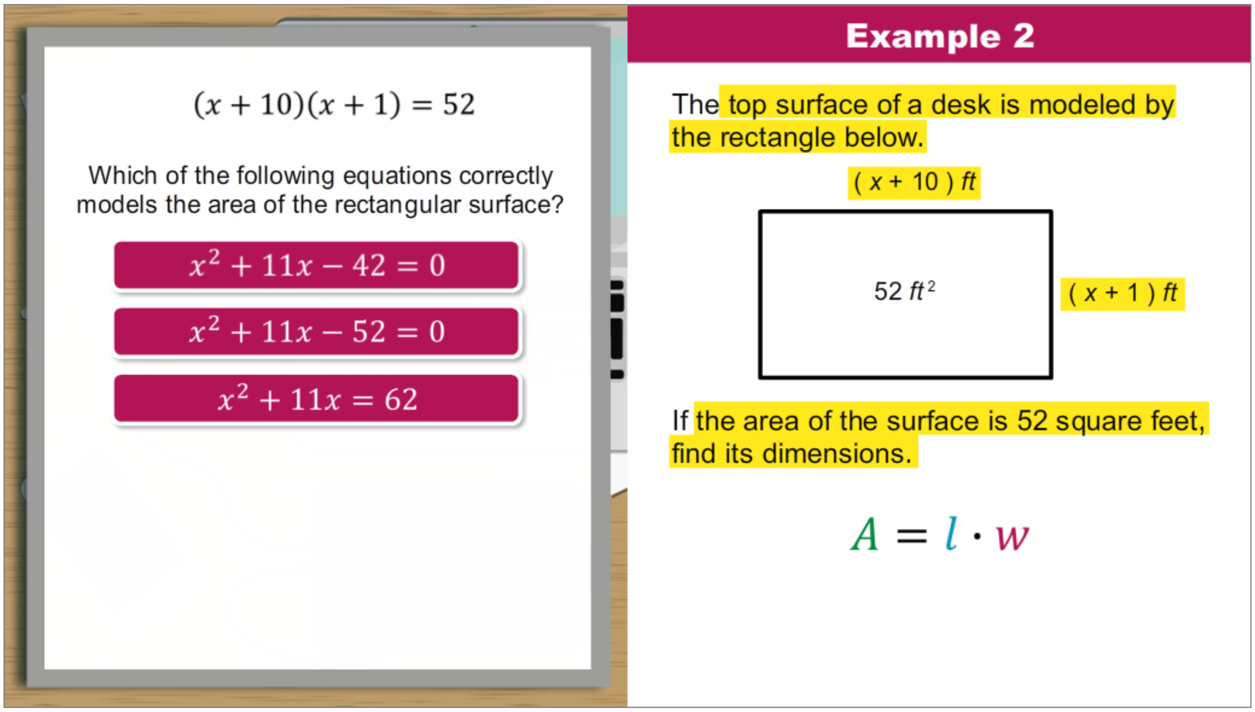
$$A = l \cdot w$$

$$52 = (x + 10) \cdot (x + 1)$$

The Symmetric Property states that the order of equality does not affect the equation.

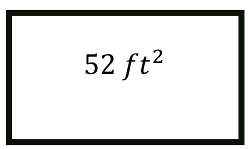
Therefore, $(x + 10)(x + 1) = 52$.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$



$(x + 1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

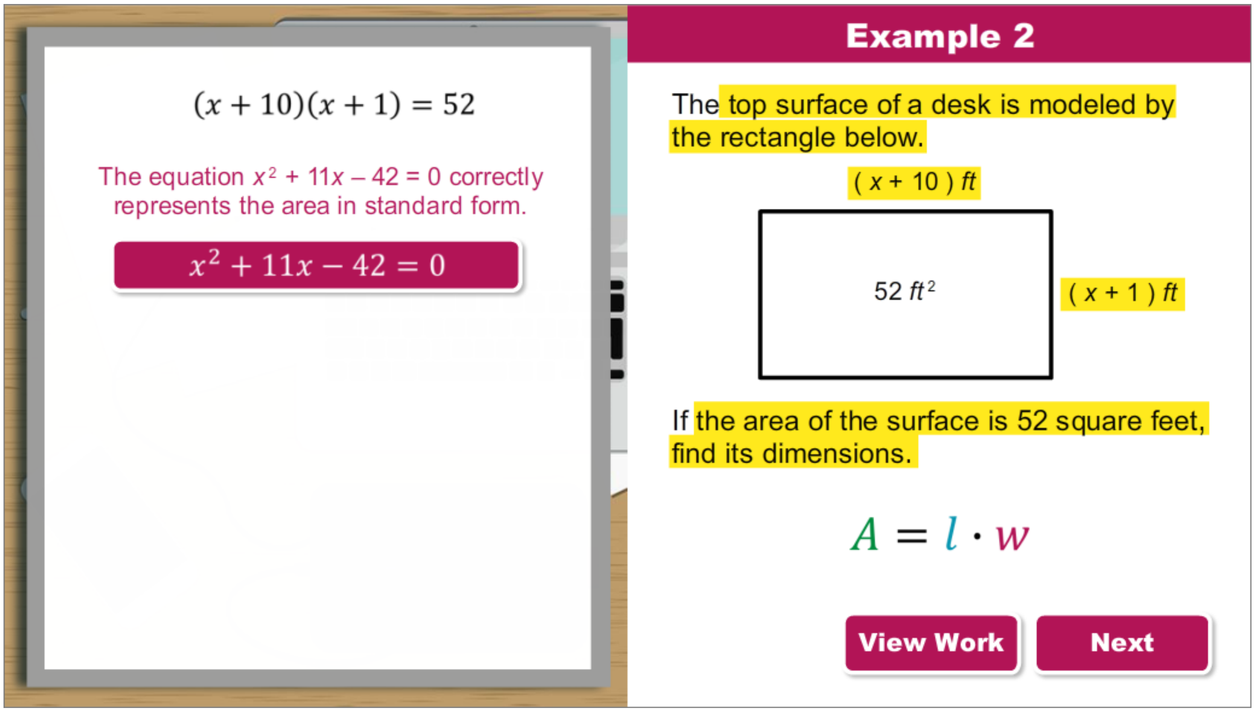
$$(x + 10)(x + 1) = 52$$

Now that you have written an equation to model the situation, solve for x. Begin by representing the quadratic equation in standard form.

Which of the following equations correctly models the area of the rectangular surface?

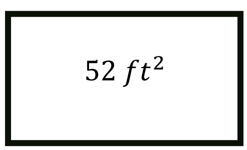
A) $x^2 + 11x - 42 = 0$
B) $x^2 + 11x - 52 = 0$
C) $x^2 + 11x = 62$

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$



$(x + 1)\ ft$

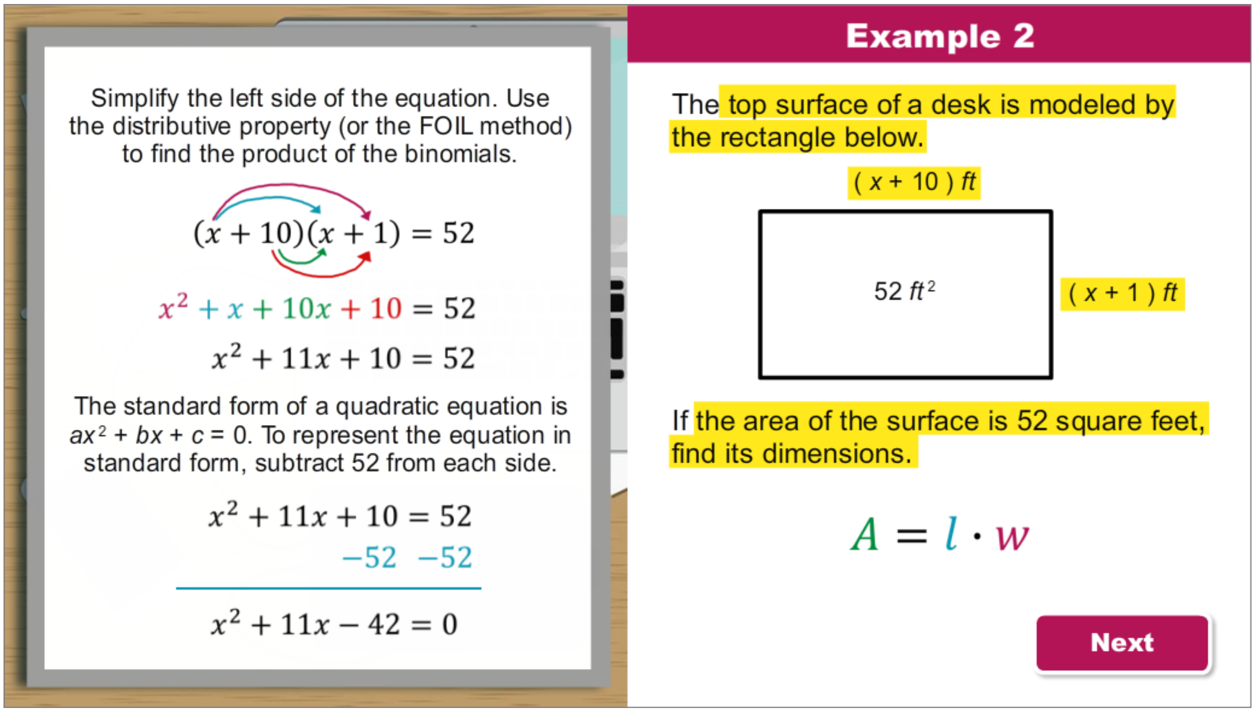
If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

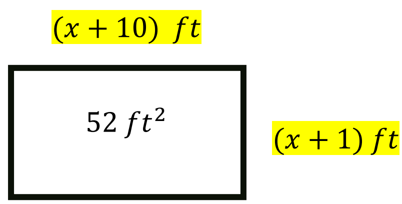
$$(x + 10)(x + 1) = 52$$

The equation $x^2 + 11x - 42 = 0$ correctly represents the area in standard form.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.



If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

$$(x + 10)(x + 1) = 52$$

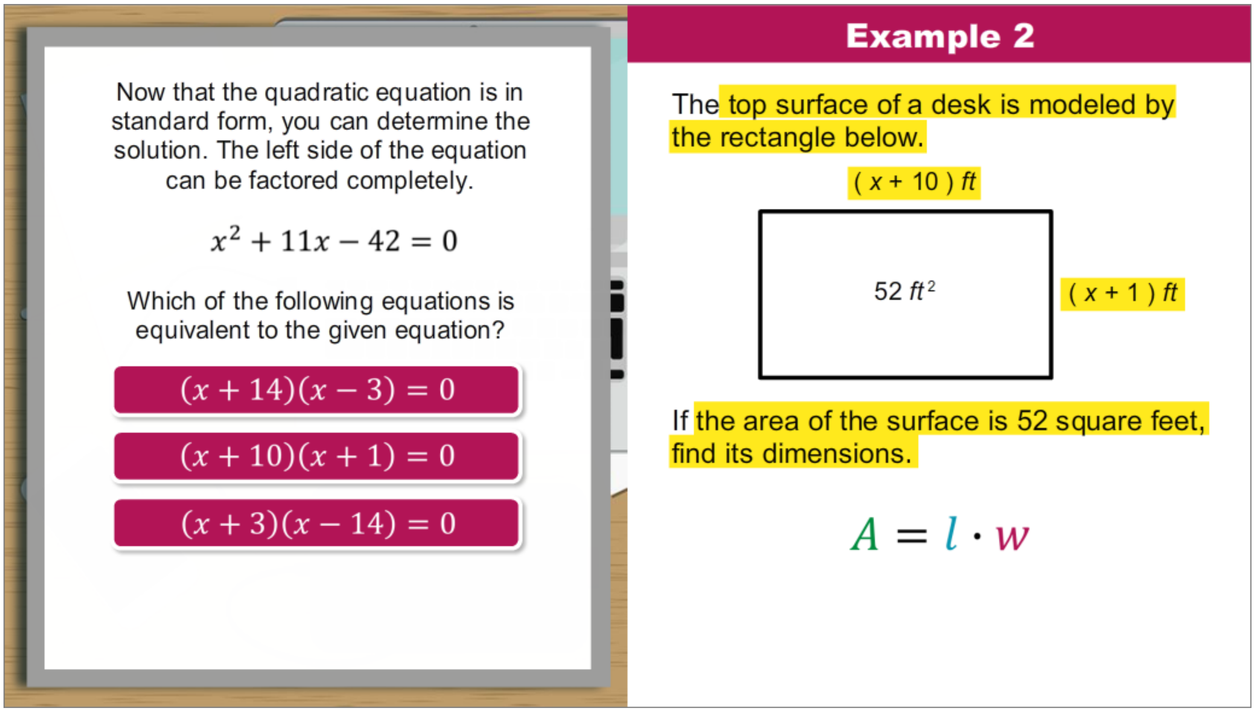
$$x^2 + x + 10x = 52$$
$$x^2 + 11x + 10 = 52$$

Simplify the left side of the equation. Use the distributive property (or the FOIL method) to find the product of the binomials.

$$x^2 + 11x + 10 = 52$$
$$-52 \quad -52$$
$$x^2 + 11x - 42 = 0$$

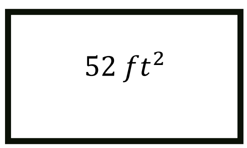
The standard form of a quadratic equation is $ax^2 + bx + c = 0$. To represent the equation in standard form, subtract 52 from each side.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$



$(x + 1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

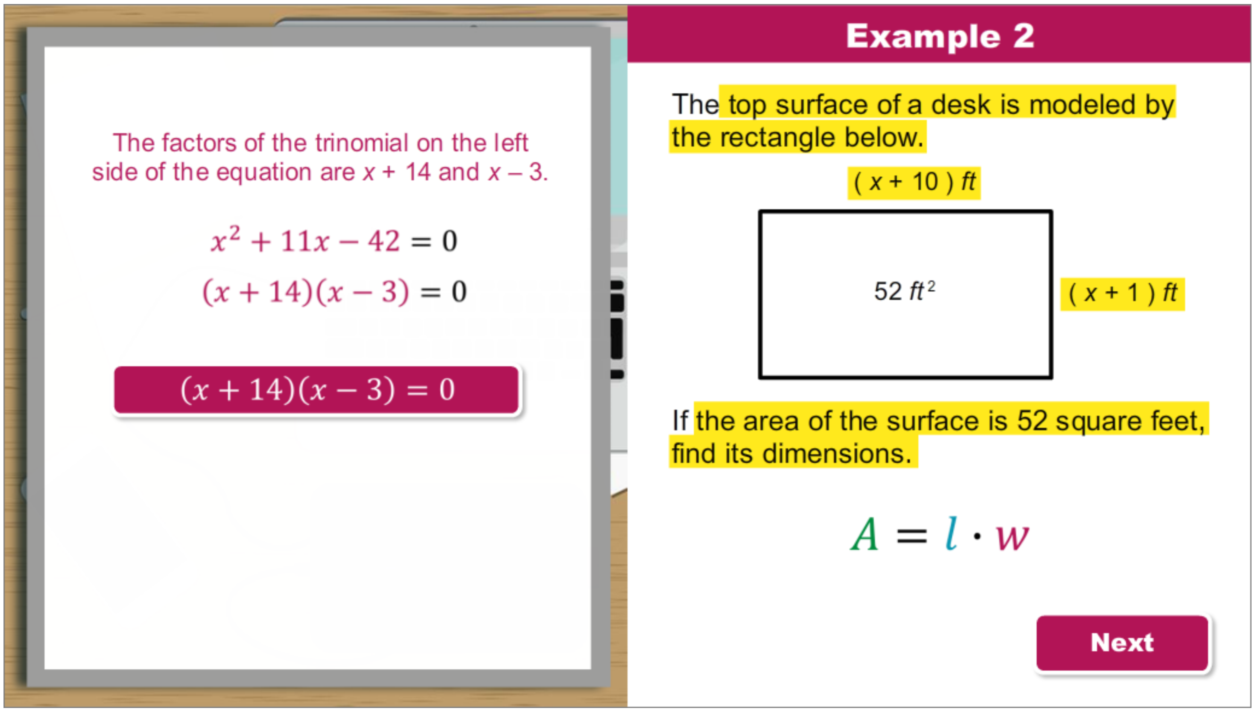
Now that the quadratic equation is in standard form you can determine the solution. The left side of the equation can be factored completely.

$$x^2 + 11x - 42 = 0$$

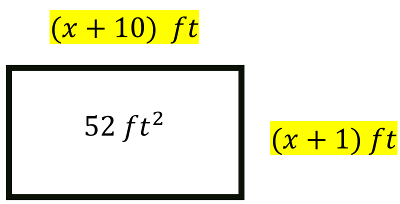
Which of the following equations is equivalent to the given equation?

A) $(x + 14)(x - 3) = 0$
B) $(x + 10)(x + 1) = 0$
C) $(x + 3)(x - 14) = 0$

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.



If the area of the surface is 52 *square feet, find its dimensions.*

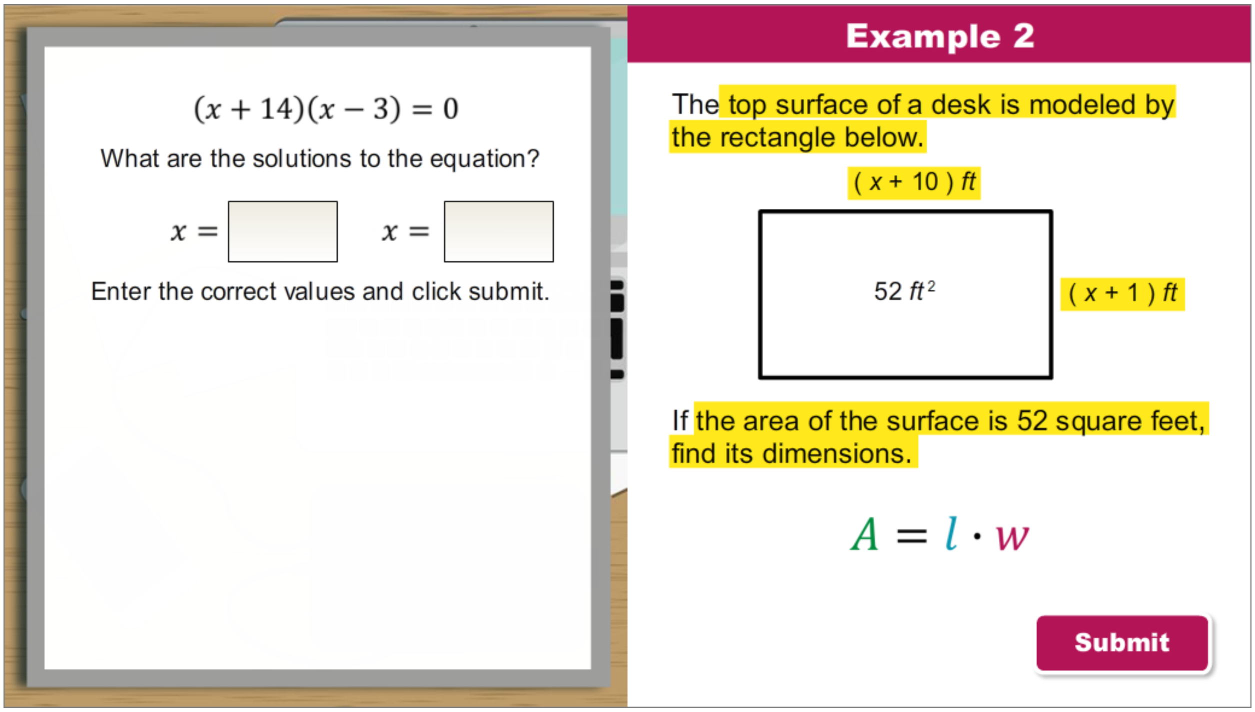
$$A = l \cdot w$$

The factors of the trinomial on the left side of the equation are $x + 14$ and $x - 3$.

$$x^2 + 11x - 42 = 0$$

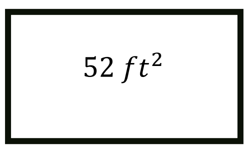
$$(x + 14)(x - 3) = 0$$

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$



$(x + 1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

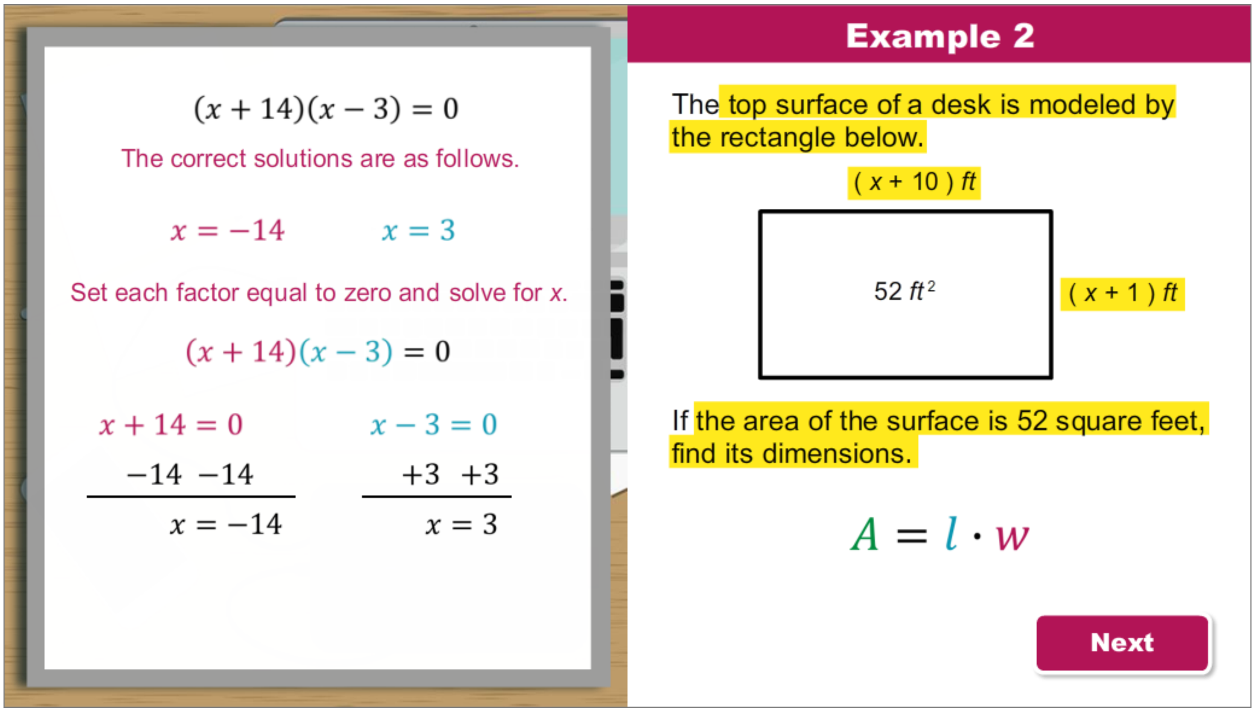
Continue to solve the quadratic equation by factoring, by now applying the Zero Product Property. Set each factor equal to 0. Then, solve for x.

$$(x + 14)(x - 3) \quad = \quad 0$$

What are the solutions to the equation? $x = ?$ and $x = ?$

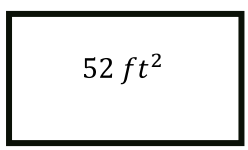
Enter the correct values and then click submit.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x + 10)\ ft$



$(x + 1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

$$(x + 14)(x - 3) \;=\; 0$$

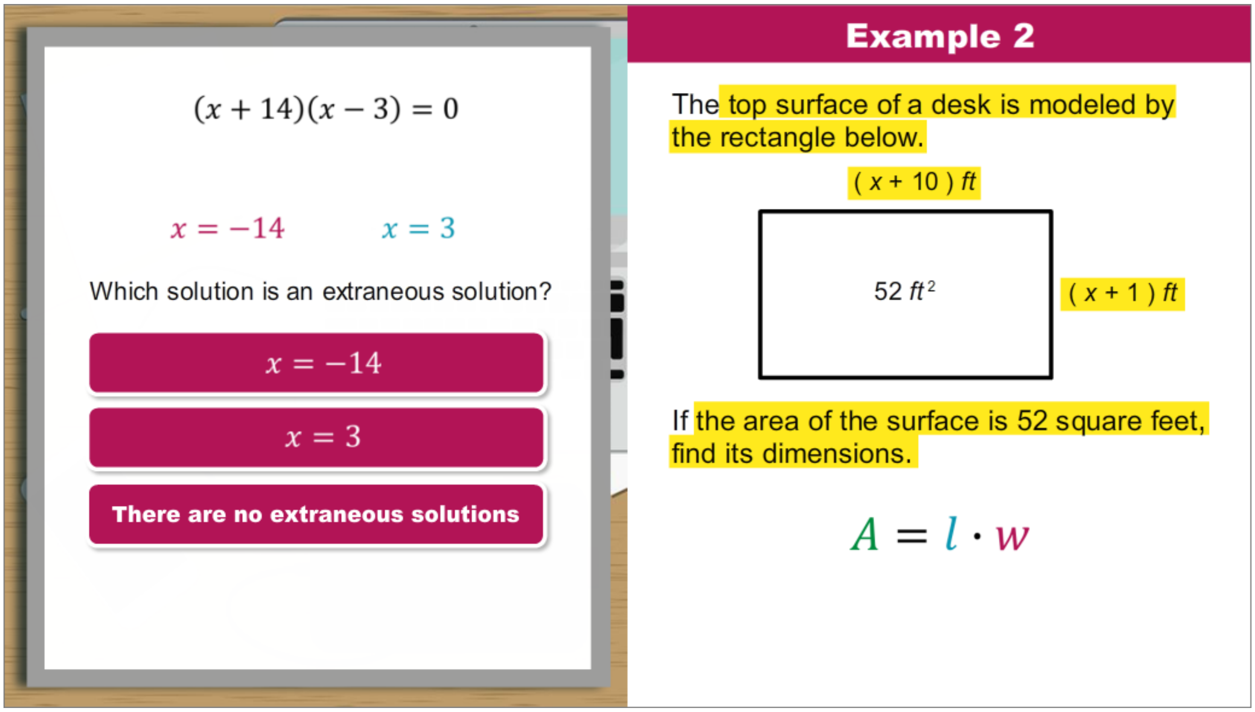
The correct solutions are as follows: $x = -14$, $x = 3$

Set each factor equal to zero and solve for x.

$$(x + 14)(x - 3) \;=\; 0$$

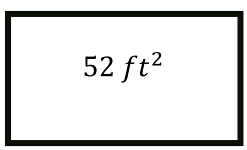
$$\begin{array}{r} x + 14 = 0 \\ -14 \quad -14 \\ \hline x = -14 \end{array} \qquad \begin{array}{r} x - 3 = 0 \\ +3 \quad +3 \\ \hline x = 3 \end{array}$$

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x+10)\ ft$



$(x+1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

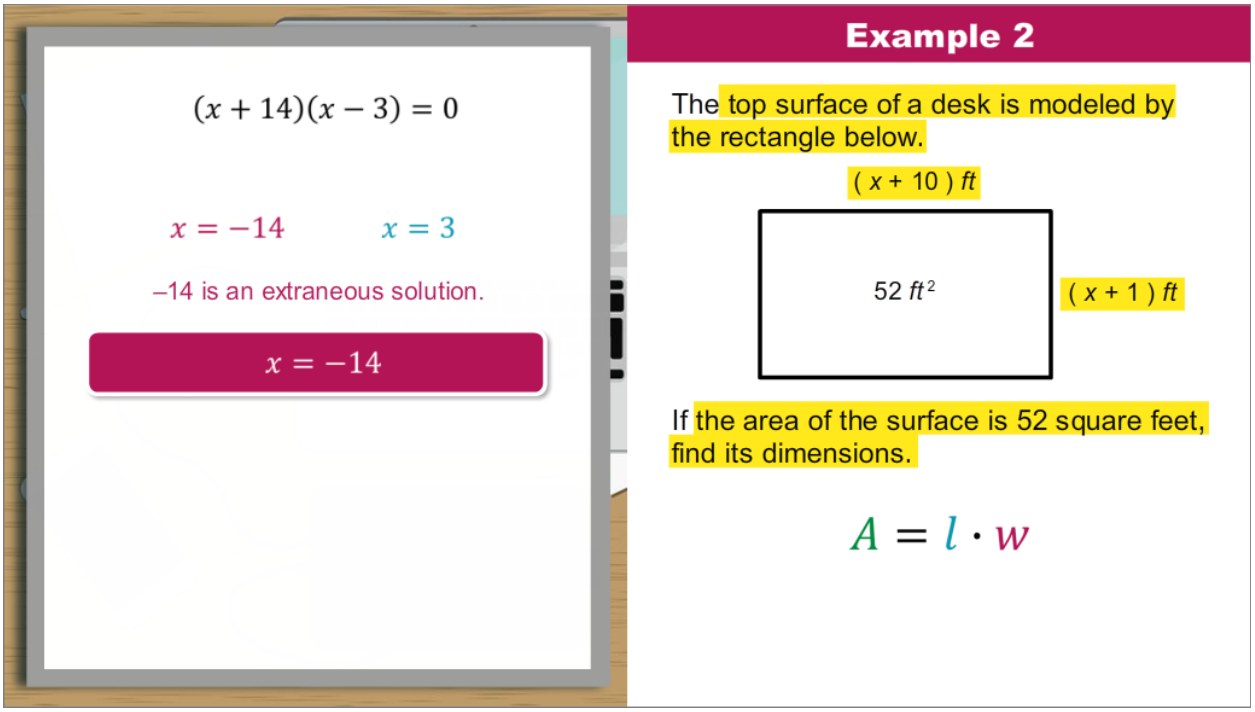
$$A = l \cdot w$$

$$(x+14)(x-3) \quad = \quad 0$$

Which of the solutions is an extraneous solution?

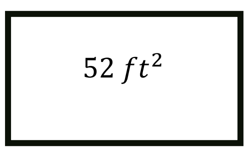
A) $x = 3$
B) $x = -14$
C) There are no extraneous solutions to the equation.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.

$(x+10)\ ft$



$(x+1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

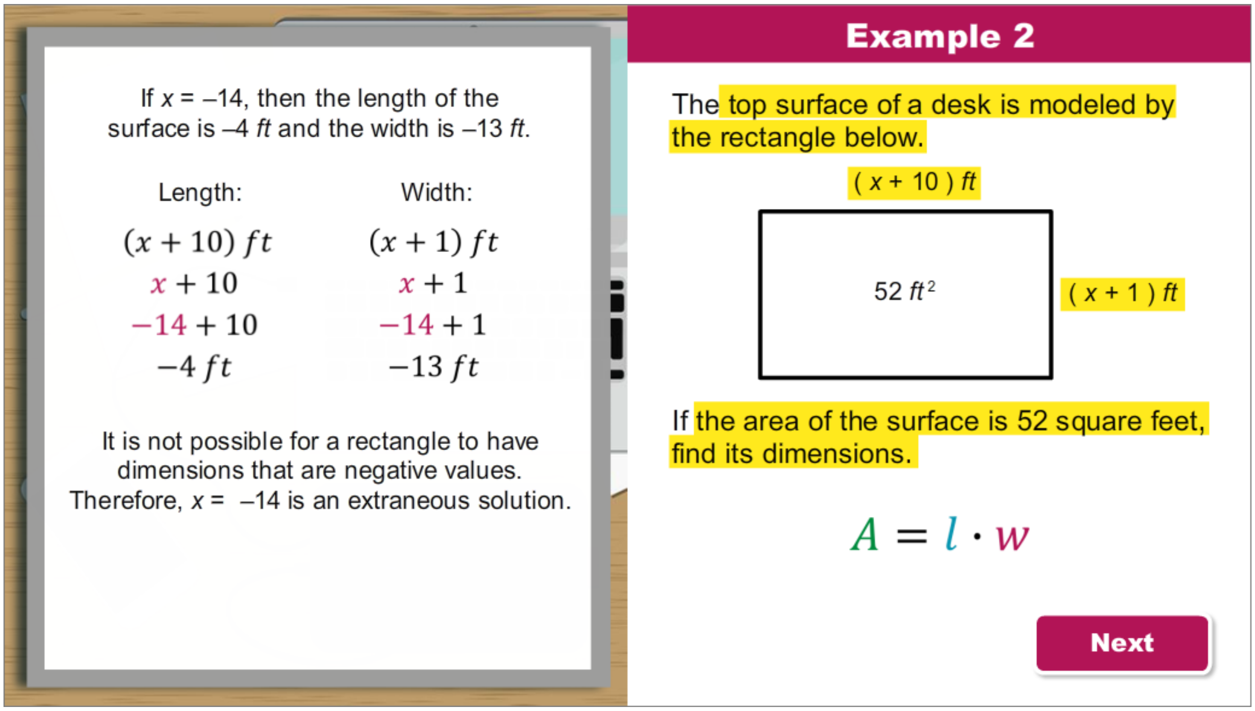
$$A = l \cdot w$$

$$(x+14)(x-3) = 0$$

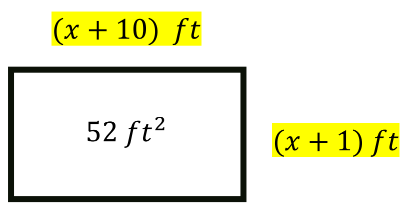
$$x=-14 \qquad x=3$$

-14 is an extraneous solution.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.



If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

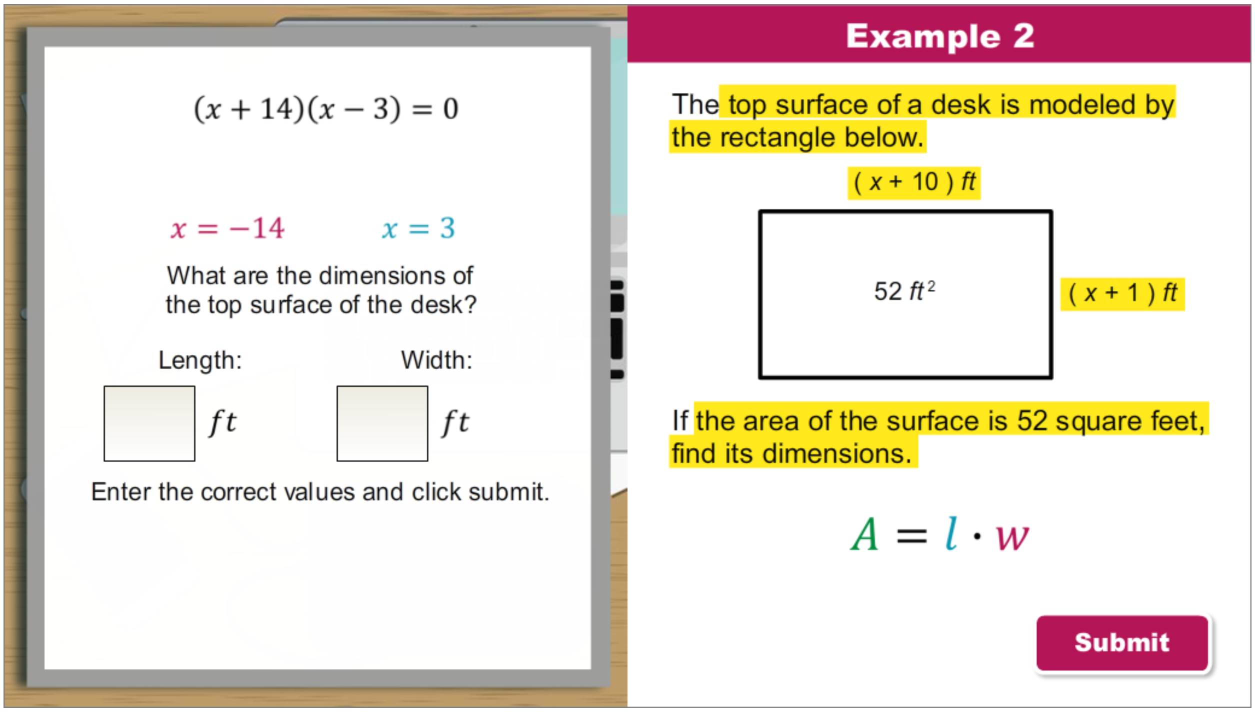
$$(x+14)(x-3) \quad = \quad 0$$

If $x = -14$, then the length of the surface is $-4\ ft$ and the width is $-13\ ft$.

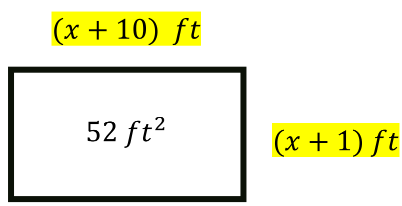
Length: $(x+10)\ ft$	Width: $(x+1)\ ft$
$x+10$	$x+1$
$-14+10$	$-14+1$
$-4\ ft$	$-13\ ft$

It is not possible for a rectangle to have dimensions that are negative values. Therefore, $x = -14$ is an extraneous solution.

Example Two (continued)



The top surface of a desk is modeled by the rectangle below.



If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

$$(x + 14)(x - 3) \quad = \quad 0$$

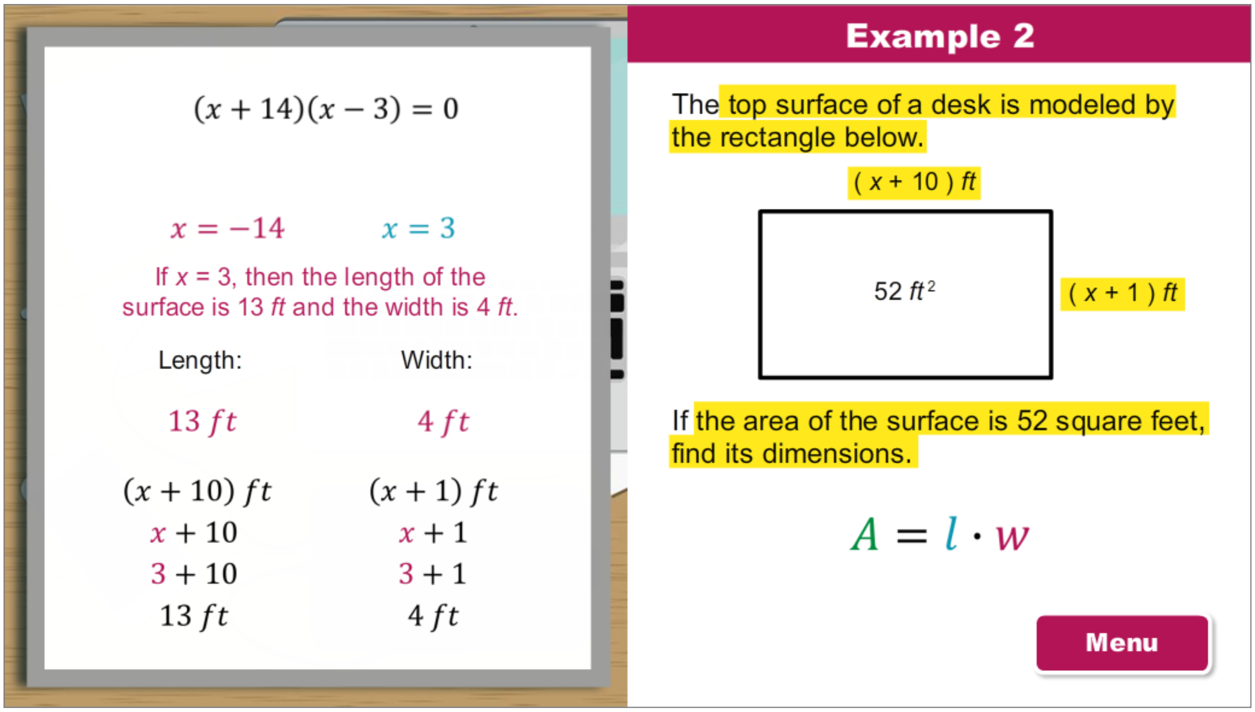
$$x = -14 \qquad x = 3$$

What are the dimensions of the top surface of the desk?

Length $= ?\ ft$ Width $= ?\ ft$

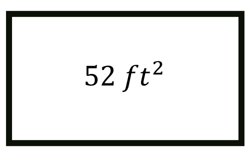
Enter the correct dimensions and click submit.

Example Two (continued)



he top surface of a desk is modeled by the rectangle below.

$(x+10)\ ft$



$(x+1)\ ft$

If the area of the surface is 52 *square feet, find its dimensions.*

$$A = l \cdot w$$

$$(x+14)(x-3) = 0$$

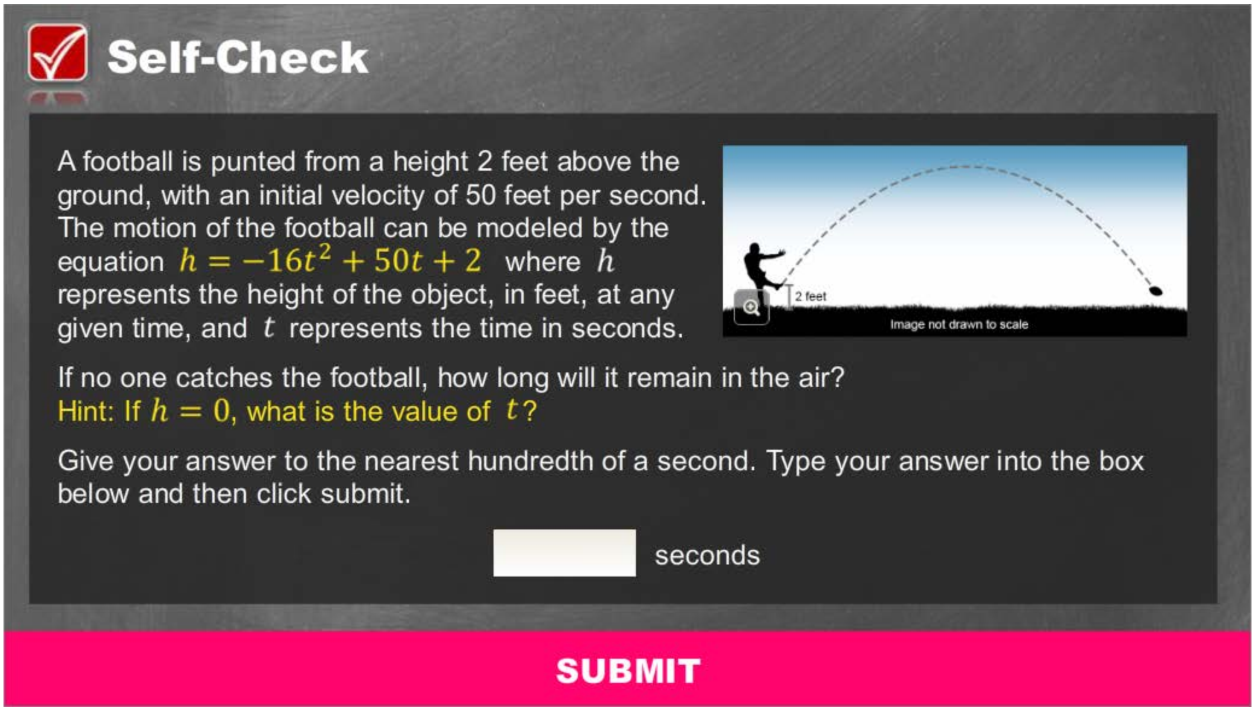
$$x = -14 \qquad x = 3$$

If $x = 3$, then the length of the surface is $13\ ft$ and the width is $4\ ft$.

Length: $(x+10)\ ft$
$x+10$
$3+10$
$13\ ft$

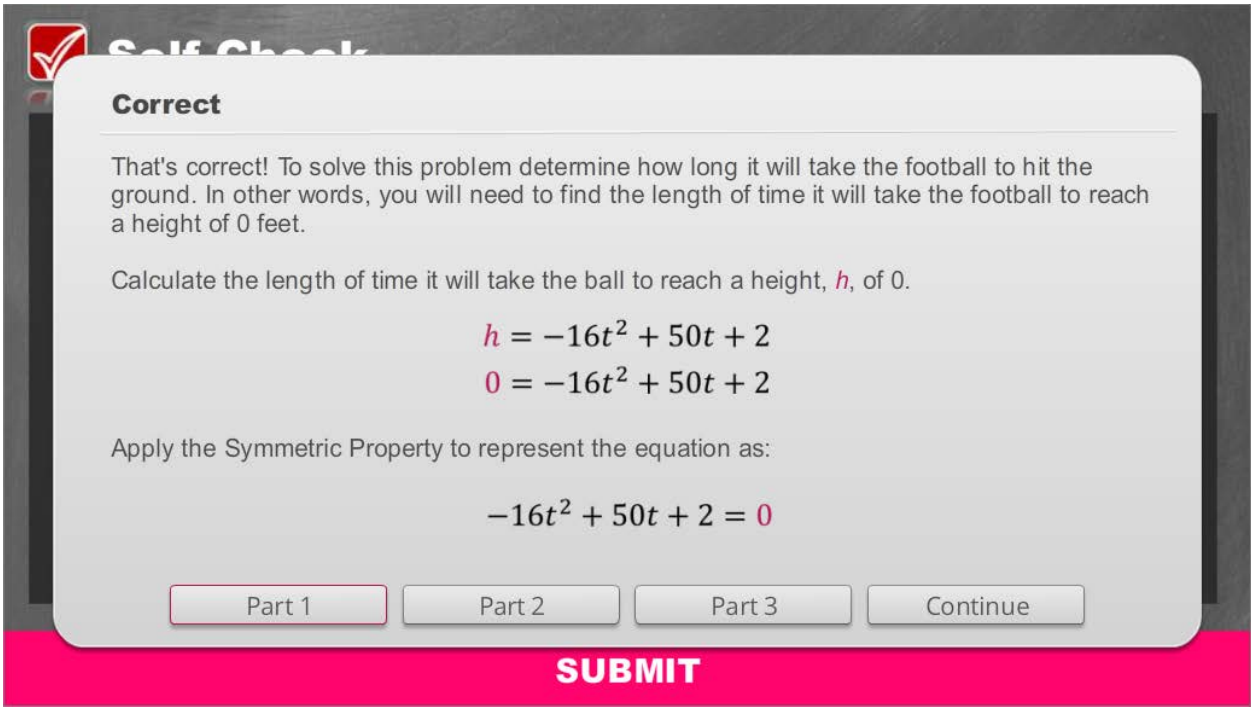
Width: $(x+1)\ ft$
$x+1$
$3+1$
$4\ ft$

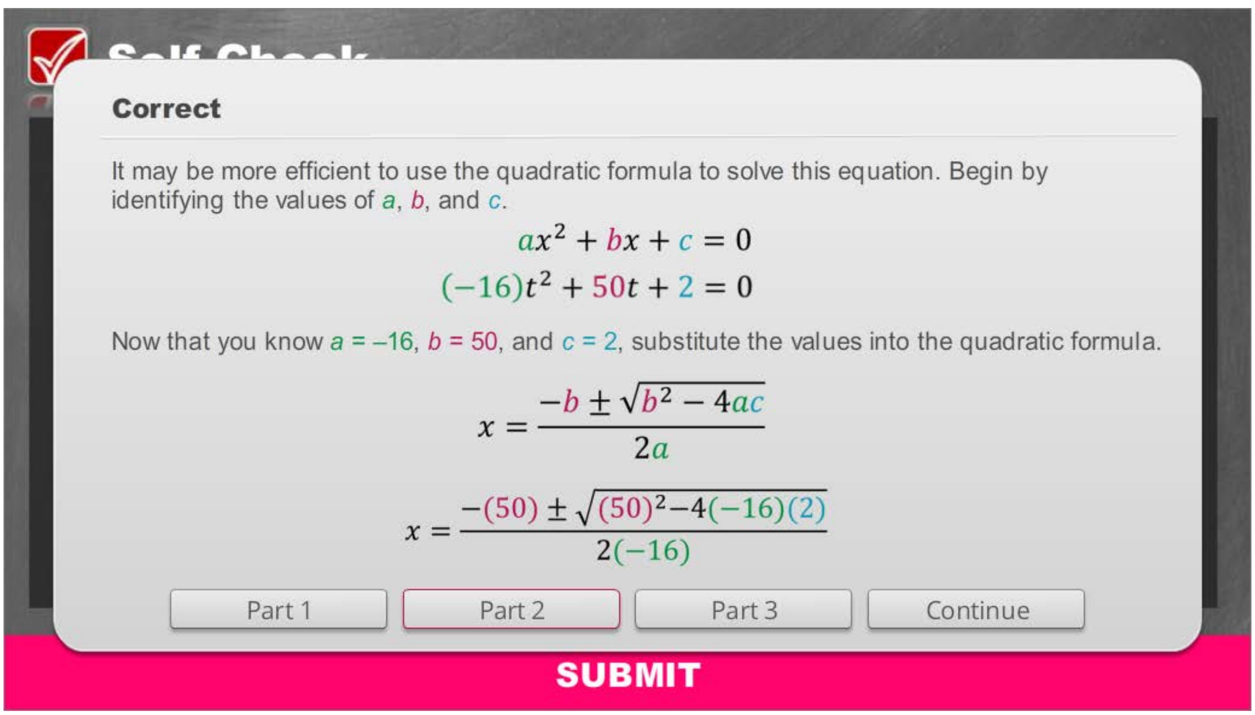
Self-Check



Solve the problem in the image above to check your understanding of the content.

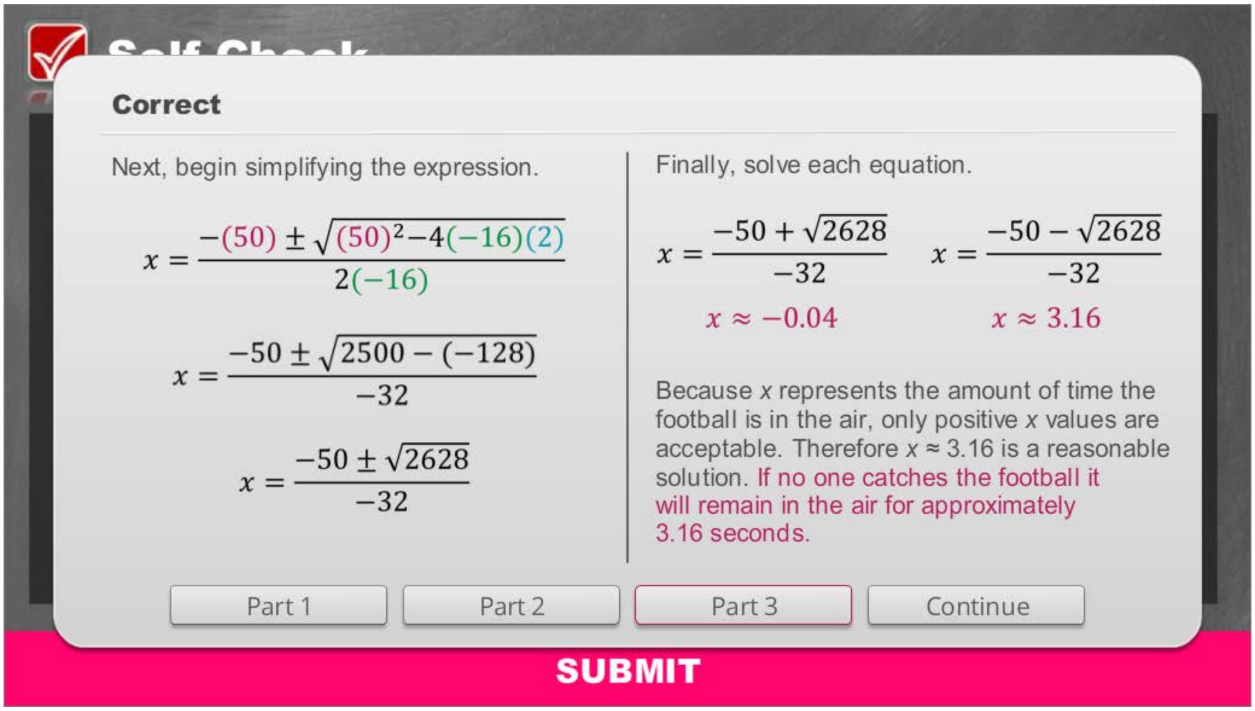
Self-Check: Answer





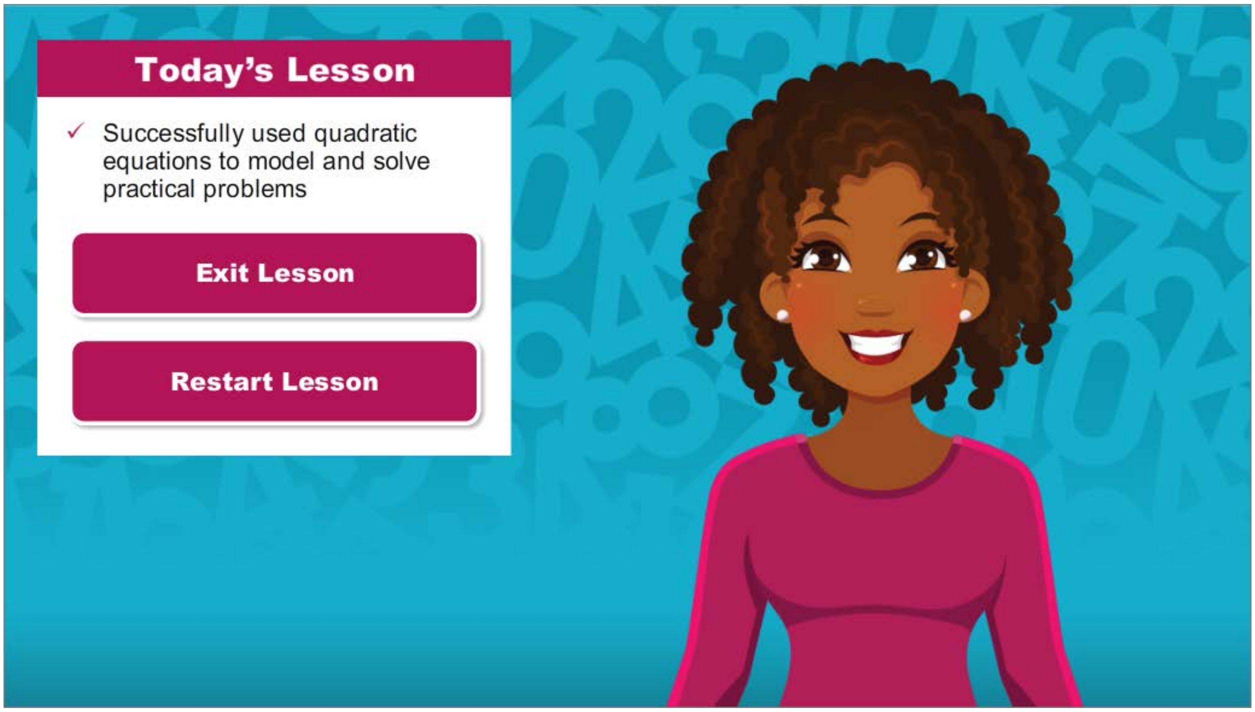
For your reference, the images above show the correct solution to the self-check problem.

Self-Check: Answer (continued)



For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



You have reached the conclusion of this lesson where you learned how to use quadratic equations to model and solve practical problems.