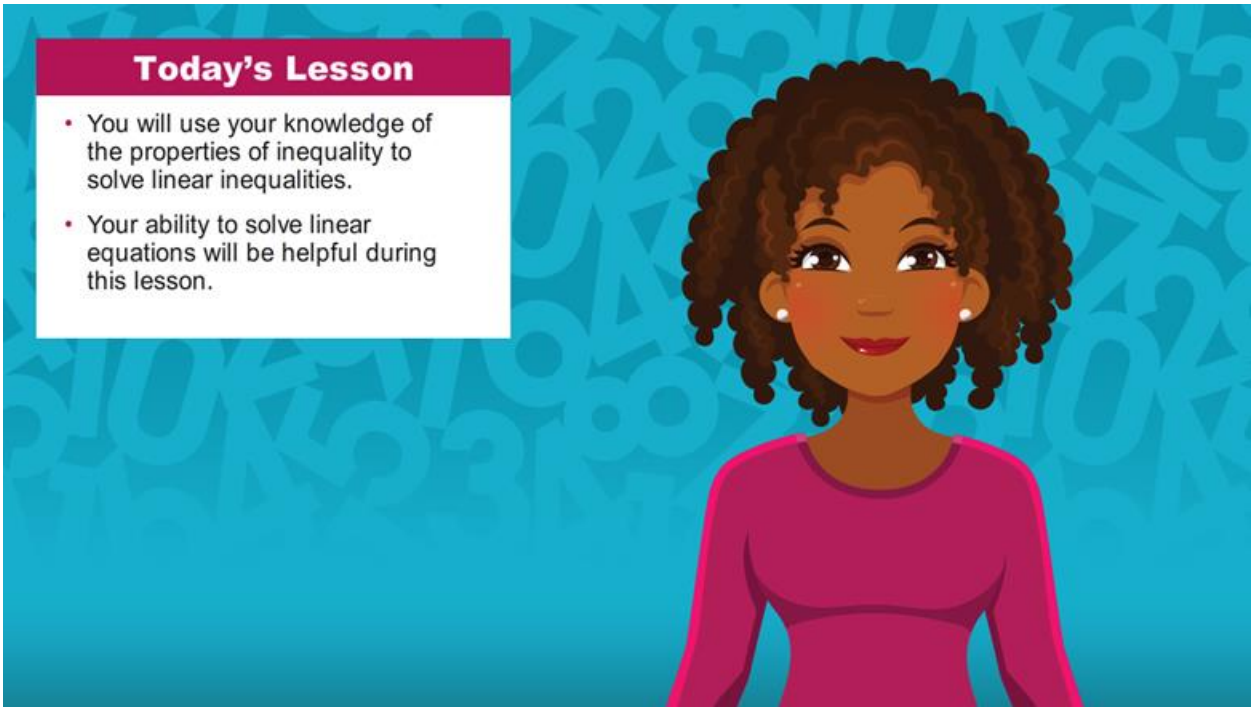
Module 7: Solving Linear Inequalities
Topic 1: Solving Linear Inequalities

Introduction



I'm glad you could join me for this lesson in Algebra I. In this lesson, we will focus on applying your knowledge of the properties of inequality, in order to solve linear inequalities in one variable. Your knowledge of how to solve linear equations will prove useful, as you solve these linear inequalities. So, before we begin solving linear inequalities, let's first take a look back at how to solve linear equations.

Module 7: Solving Linear Inequalities
Topic 1: Solving Linear Inequalities

Solving Linear Equations

What is the solution to the equation?

$3x - 6 = 9$

$+6 \quad +6$ — Add six to both sides of the equation in order to isolate x.

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$ — Divide each side of the equation by 3.

$x = 5$

You could use set notation to represent the solution: {5}.

You could also use a number line to represent the solution graphically.

By placing a point at 5 on the number line, you show that 5 is the solution to the equation.

We can verify this by substituting five back into the equation.

$3x - 6 = 9$

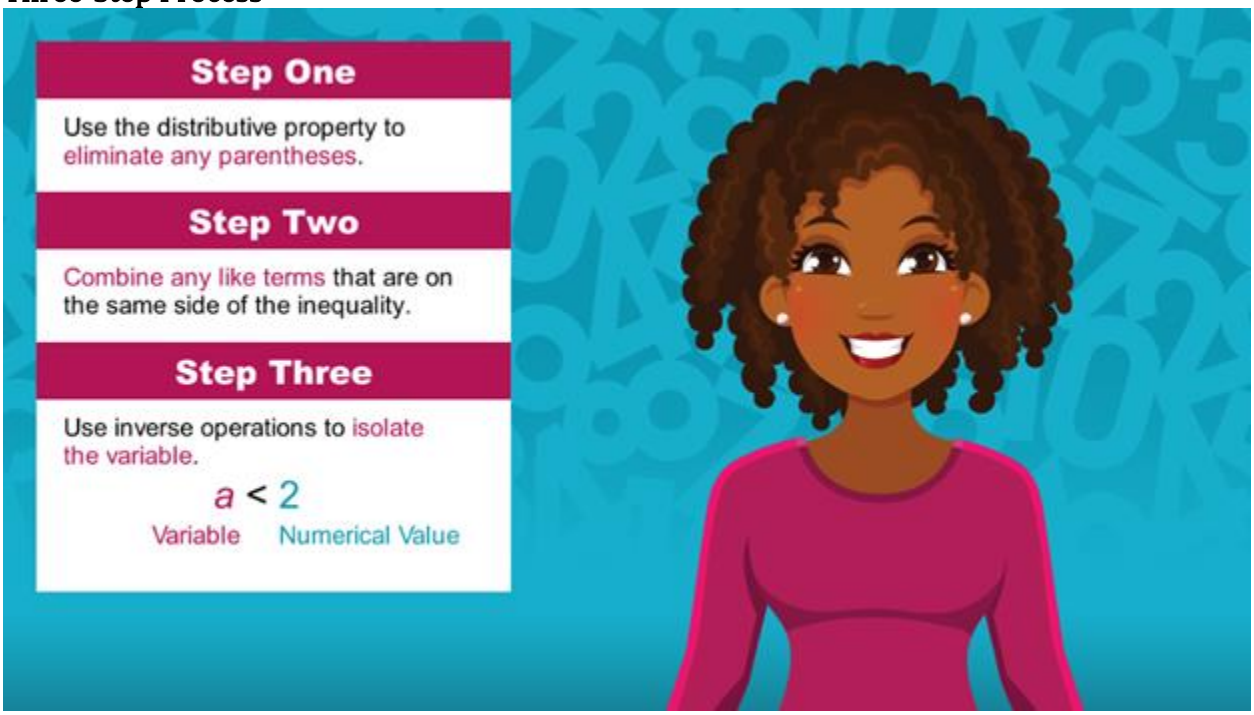
$3(5) - 6$

$15 - 6$

$9 = 9$

The left side of the equation has a value of 9, just like the right side. So, 5 is the solution to the equation.

Three-Step Process



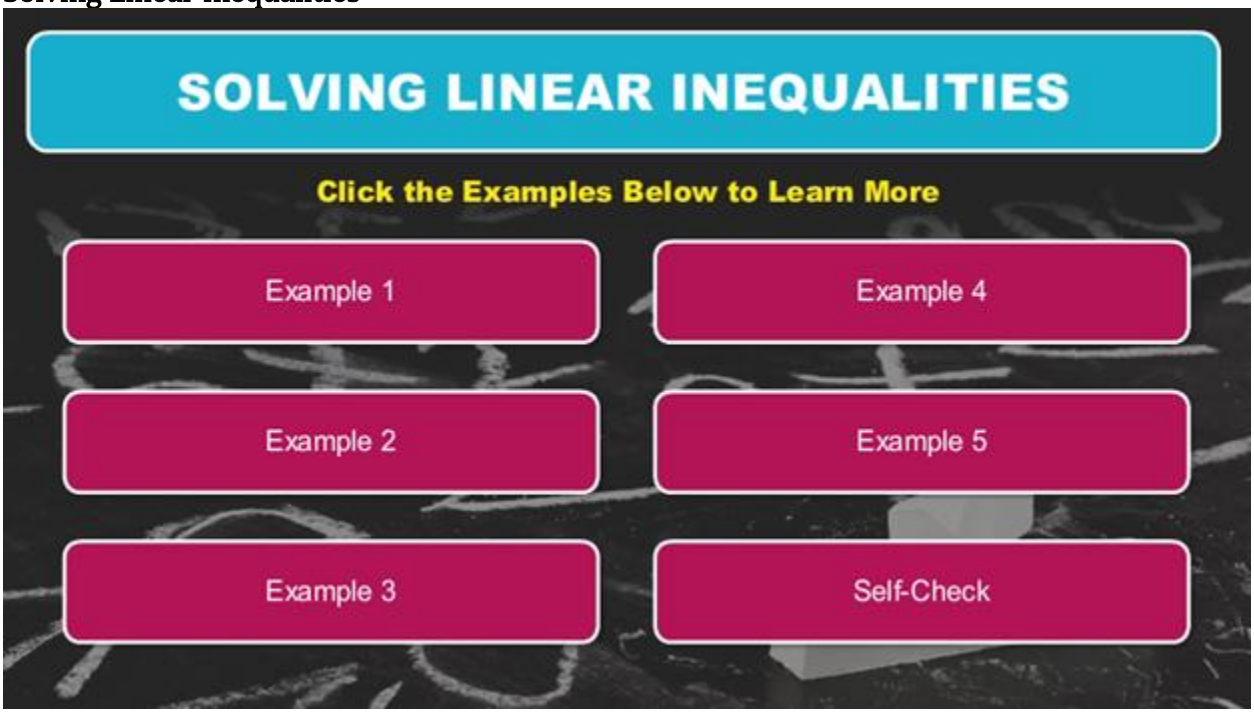
The process to solving an inequality is very similar to the process of solving an equation. The major difference between solving an equation and solving an inequality is how you interpret the solution.

Step 1: Are there any ***parentheses***?
If so, use the distributive property to eliminate them.

Step 2: Are there any ***like terms to combine?***
Remember to only combine the like terms that are on the same side of the inequality.

Step 3: Now, it's time to ***isolate the variable.***
When solving an inequality, the goal is typically to end with only your variable on the left side of the inequality sign and your numerical value on the right side.

Solving Linear Inequalities



Click the examples below to learn more.

Example 1
What is the solution to the inequality?
$3x - 6 > 9$

Step 1: Are there any ***parentheses***?
This inequality does not have any parentheses, so you will not need to use the distributive property. So, move on to Step 2.

Step 2: Are there any ***like terms to combine?***
In this case, you have no like terms to combine. So, move on to Step 3.

Step 3: Now, it's time to ***isolate the variable.***
You can begin using inverse operations to isolate the variable.

$$3x - 6 > 9$$
$$+6 \quad +6$$

Add 6 to each side of the inequality.

$$3x > 15$$
$$\frac{3x}{3} > \frac{15}{3}$$

Divide each side by 3.

$$x > 5$$

You can use the number line to represent the solutions to this inequality, graphically.

The solutions are all values of x that are greater than 5. The value of 5, itself, is not actually included in the solutions. To show this relationship on the number line, you would sketch an open circle at 5, and then highlight all of the values that are larger than 5, which would be all of the values to the right of 5.

Example 1 (continued)
If you substitute any value larger than 5 into the inequality, it will hold true. Take a look.

For example, 6 is a number that is greater than 5. Substitute 6 into your original inequality, and verify that it is a solution.

$3x - 6 > 9$

$3(6) - 6$

$18 - 6$

$12 > 9$

12 is greater than 9, so the inequality holds true; 6 is a solution to the inequality. Now substitute another value larger than five, and verify that it is also a solution to the inequality. Try 8.

$3x - 6 > 9$

$3(8) - 6$

$24 - 6$

$22 > 9$

22 is greater than 9, so the inequality holds true; 8 is a solution to the inequality.

You could continue this process forever, choosing numbers larger than 5, and substituting them into your inequality. For all values of x larger than 5, the inequality will hold true. The group of numbers larger than 5 is known as the solution set of the inequality. Set notation is a way to represent the solution set.

To represent the solution set, $x > 5$, in set notation, you would write: $\{x: x > 5\}$ or $\{x | x > 5\}$. Both examples would be read as: the set of all x such that x is greater than 5. Meaning that x is the group of all values that are greater than 5.

Example 2
Solve the inequality.

$2(4-x)+4 \leq 14$

Step 1: Are there any ***parentheses***?
This inequality does have parentheses, so you will need to use the distributive property to simplify the left side.

$2(4-x)+4 \leq 14$ Use the distributive property to eliminate the parentheses.

$8-2x+4 \leq 14$

Step 2: Do you have any ***like terms to combine?***
In this case, you do have some like terms to combine. Remember, you only combine like terms that are on the same side of the inequality.

$8-2x+4 \leq 14$ Combine 8 and 4.

$12-2x \leq 14$

Step 3: Now, it's time to ***isolate the variable.***
You can begin using inverse operations to isolate the variable.

$12-2x \leq 14$

$-12 \quad -12$ Subtract 12 from each side of the inequality.

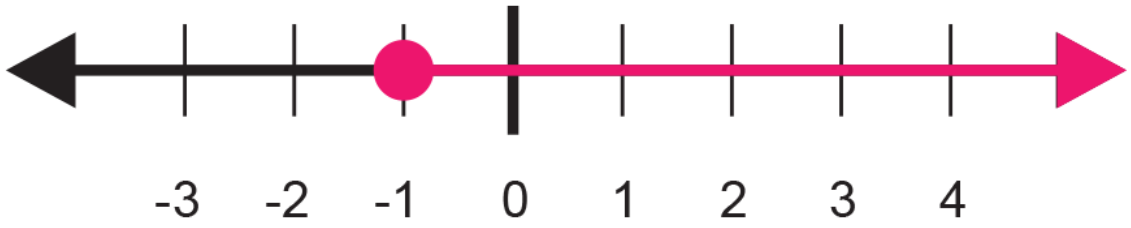
$-2x \leq 2$

$\frac{-2x}{-2} \leq \frac{2}{-2}$ Divide each side by -2. Remember when working with inequalities, when you multiply or divide both sides by a negative number, you must flip your inequality sign.

$x \geq -1$

In set notation, you can represent the solution set as: $\{x: x \geq -1\}$ or $\{x | x \geq -1\}$.

Example 2 (continued)
You can use the number line to represent the solution set, graphically.



The solution set includes all values of x that are greater than or equal to -1. The value of -1 is included in the solution set. To show this relationship on the number line, you would sketch a closed circle at -1, to show that it is included in the solution set, and then highlight all of the values that are greater than -1, the numbers to the right, to show that they are also included.

Example 3
Solve for x.

$2 + \frac{3}{4}x < 11$

Step 1: Are there any ***parentheses***?
This inequality does not have any parentheses, so we will not need to use the distributive property. So, move on to Step 2.

Step 2: Do you have any ***like terms to combine?***
In this case, we have no like terms to combine. So, move on to Step 3.

Step 3: Now, it's time to ***isolate the variable.***
You can begin using inverse operations to isolate the variable.

$2 + \frac{3}{4}x < 11$

$-2 \quad -2$ Subtract 2 from each side of the inequality.

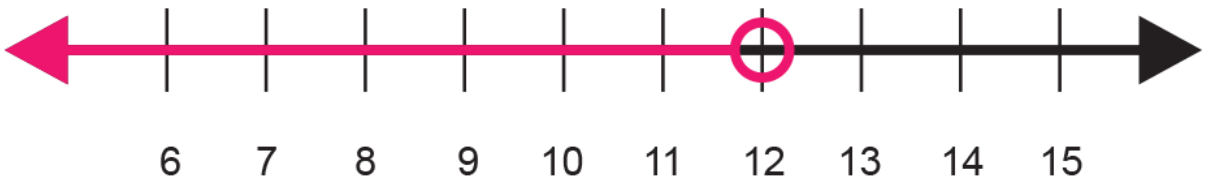
$4 \cdot \frac{3}{4}x < 9 \cdot 4$ Multiply each side by 4.

$\frac{3x}{3} < \frac{36}{3}$ Divide each side of the inequality by 3.

$x < 12$

In set notation, you can represent the solution set as: $\{x: x < 12\}$ or $\{x | x < 12\}$.

You can use the number line to represent the solution set, graphically.



The solution set includes all values of x that are less than 12. 12 is ***not*** included in the solution set. To show this relationship on the number line, sketch an open circle at 12, to show that it is ***not*** included in the solution set, and then highlight all of the values that are less than 12, the numbers to the left, to show that they ***are*** included in the solution set.

Example 4
Solve for x.

$-4x + 9 > x + 21$

Step 1: Are there any ***parentheses***?
This inequality does not have any parentheses, so we will not need to use the distributive property. So, move on to Step 2.

Step 2: Do you have any ***like terms to combine?***
In this case, we have no like terms to combine. So, move on to Step 3.

Step 3: Now, it's time to ***isolate the variable.***
You can begin using inverse operations to isolate the variable.

Generally when working with inequalities with variables on both sides, the goal is to end up with your variable term on the left side and your constant term on the left side. Keep that goal in mind while using inverse operations to isolate the variable.

$-4x + 9 > x - 21$

$-9 \quad -9$ Subtract 9 from each side of the inequality.

$-4x > x - 30$

$-x \quad -x$ Subtract x from each side of the inequality.

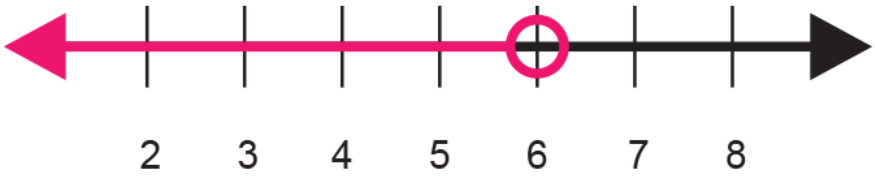
$-5x > -30$

$\frac{-5x}{-5} > \frac{-30}{-5}$ Divide each side of the inequality by -5. Recall the division property of inequality. Because you divided each side of the inequality by a negative number you must flip the inequality sign.

$x < 6$

In set notation, the solution set can be represented as $\{x: x < 6\}$ or $\{x | x < 6\}$

You can use the number line to represent the solution set, graphically.



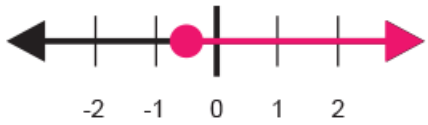
Because the solution set only includes those values less than 6, sketch an open circle at 6, and highlight the values that are less than 6.

Example 5
Which of the following graphs correctly represents the solution set of the inequality?

$5(x + 2) - 3 \leq 3x + 6$

A)

C)



B)

D)

To answer this question, you will first need to solve the inequality for x. Then you will be able to determine which of the graphs correctly represents the solution set. Let the steps to solving inequalities guide you through the process.

Step 1: Are there any ***parentheses***?
This inequality does have parentheses, so you will need to use the distributive property to eliminate them.

$5(x + 2) - 3 \leq 3x + 6$

$5x + 10 - 3 \leq 3x + 6$

Step 2: Do you have any ***like terms to combine?***
In this case, you do have some like terms to combine. Remember, you only combine like terms that are on the same side of the inequality.

$5x + 10 - 3 \leq 3x + 6$

$5x + 7 \leq 3x + 6$

On the left side of the inequality, you combine 10 and -3. There are no like terms to combine on the right side of the inequality, so you can move on to Step 3.

Example 5 (continued)

Step 3: Now, it's time to ***isolate the variable***.

You can begin using inverse operations to isolate the variable.

$$5x + 7 \leq 3x + 6$$

$-7 \qquad -7$ Subtract 7 from each side of the inequality.

$$5x \leq 3x - 1$$

$-3x \quad -3x$ Subtract $3x$ from each side of the inequality.

$$2x \leq -1$$

$\frac{2x}{2} \leq \frac{-1}{2}$ Divide each side of the inequality by 2.

$$x \leq -\frac{1}{2}$$

Now that you have represented the solution set algebraically, you can identify its graphic representation.

Because the solution set includes all values of x that are less than or equal to $-\frac{1}{2}$ ...

Will you need an open circle or a closed circle at $-\frac{1}{2}$?

Will you need to highlight the values to the left of $-\frac{1}{2}$ or to the right of $-\frac{1}{2}$?

Example 5 (continued)
Which of the graphs meets these requirements?

A)

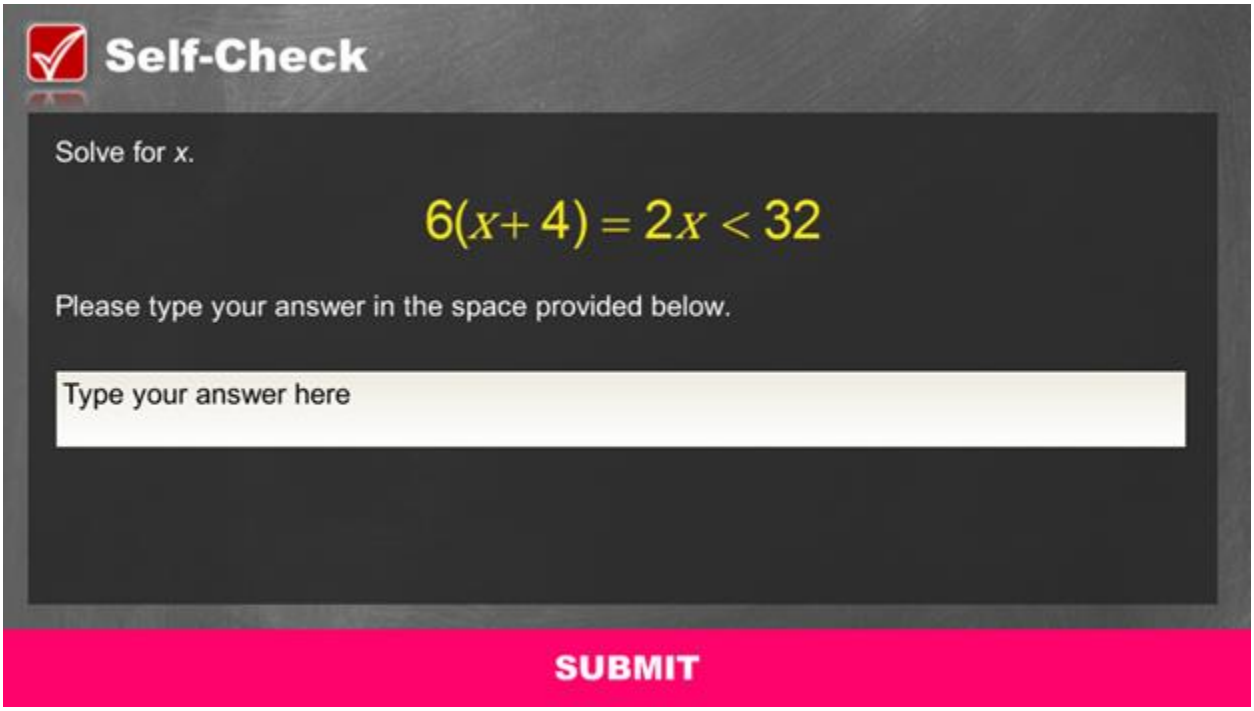
C)

B)

D)

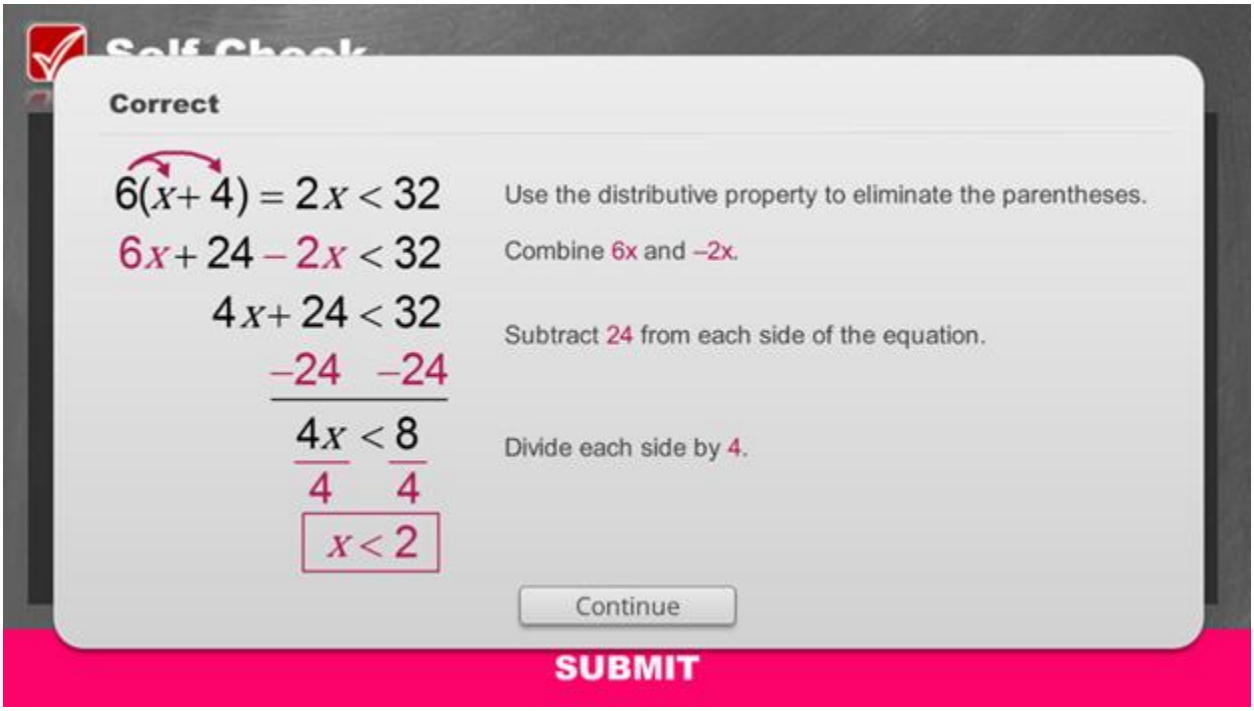
Answer choice A correctly represents the solution set.

Self-Check 1



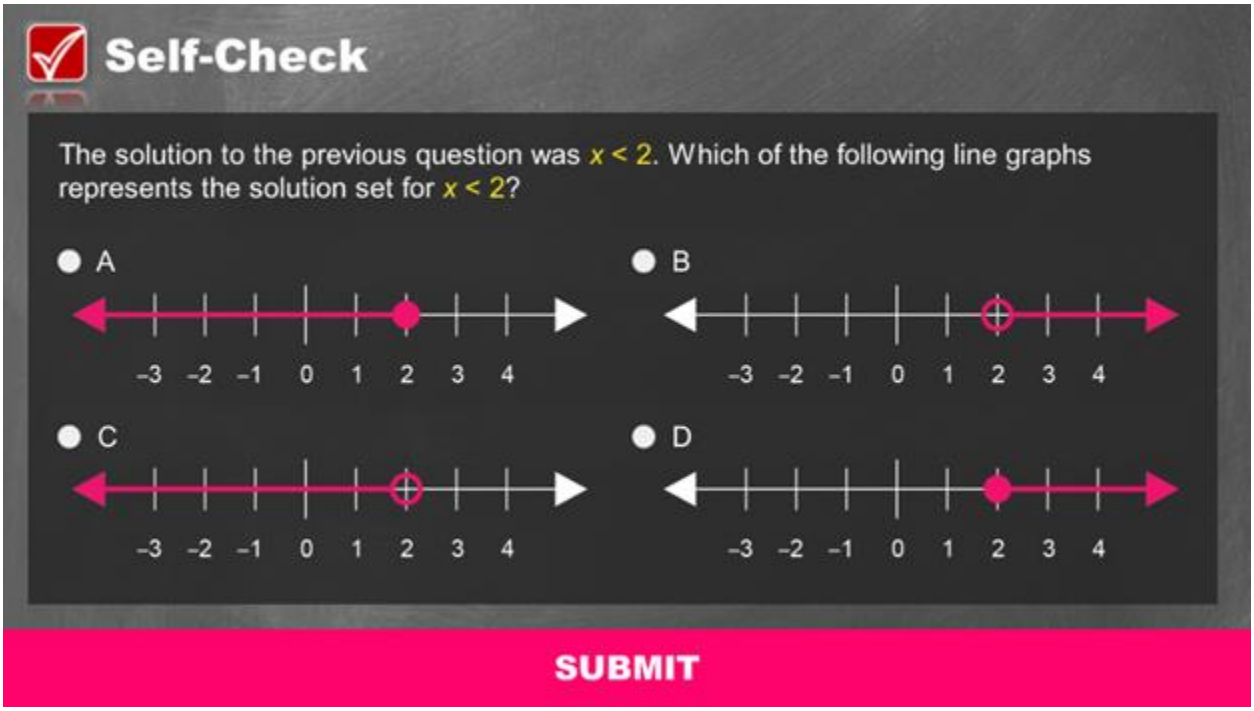
Solve the problem in the image above to check your understanding of the content.

Self-Check 1: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Self-Check 2



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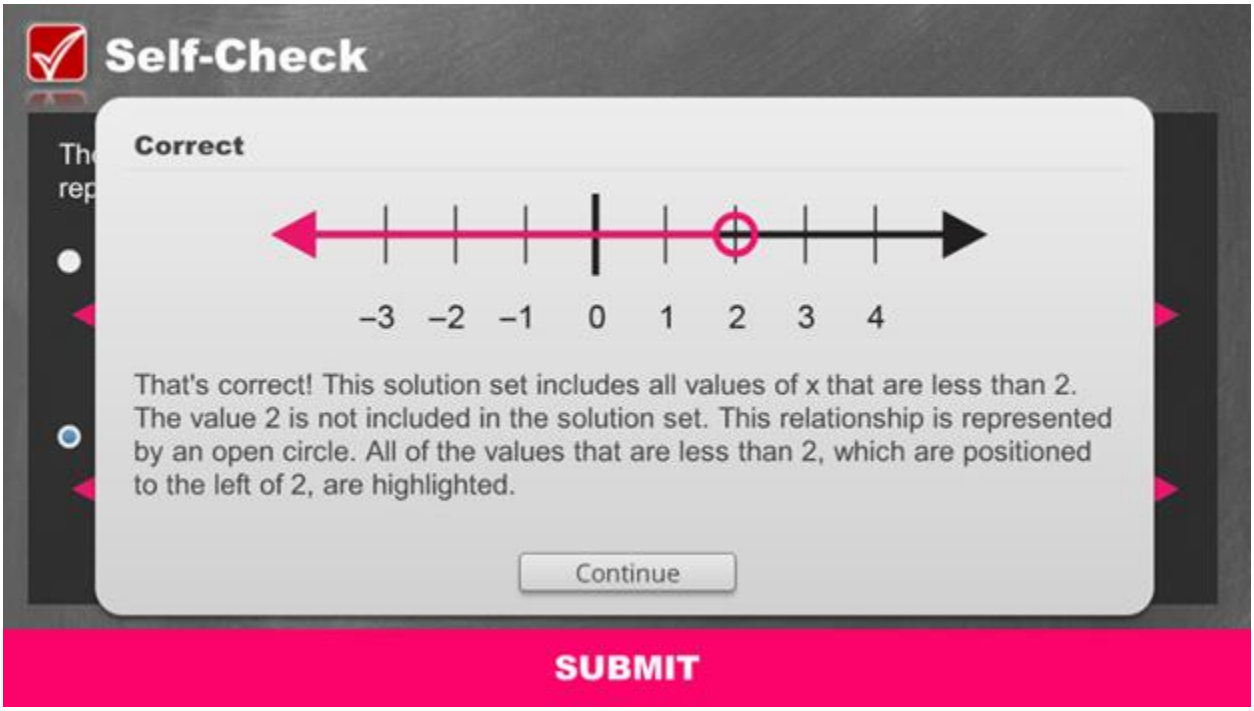
Self-Check 2: Answer



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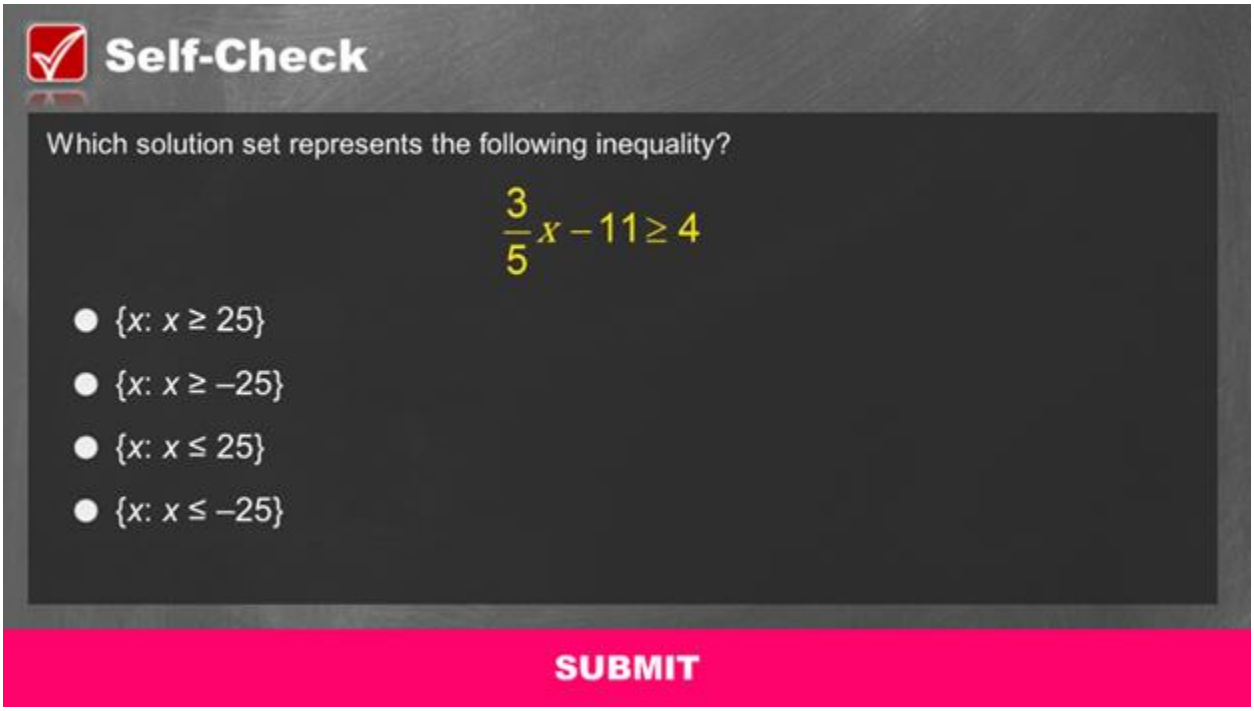
Self-Check 3



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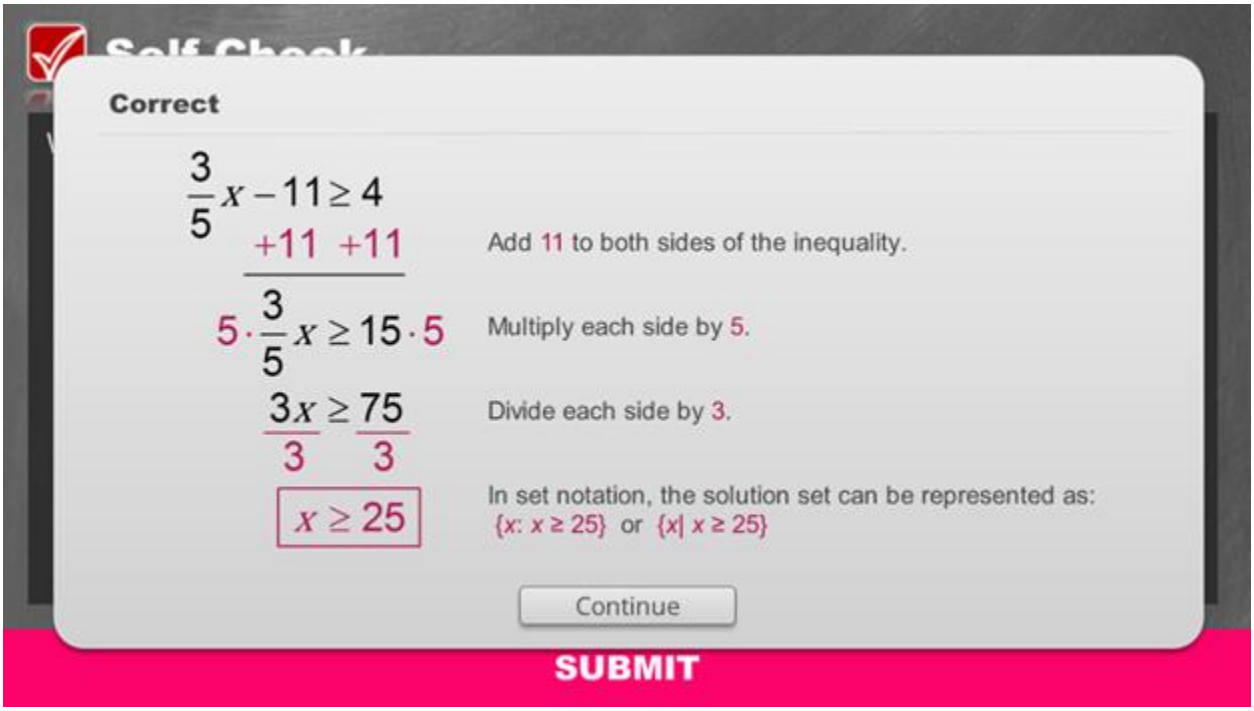
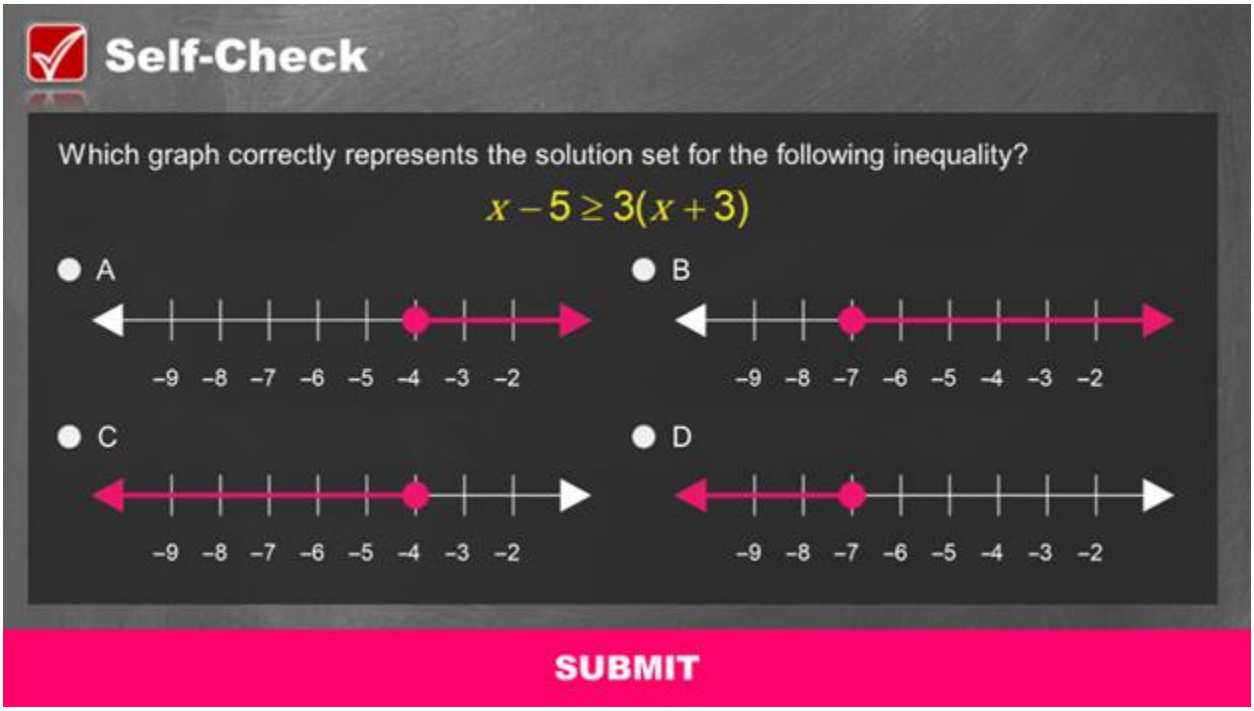
Self-Check 3: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Self-Check 4



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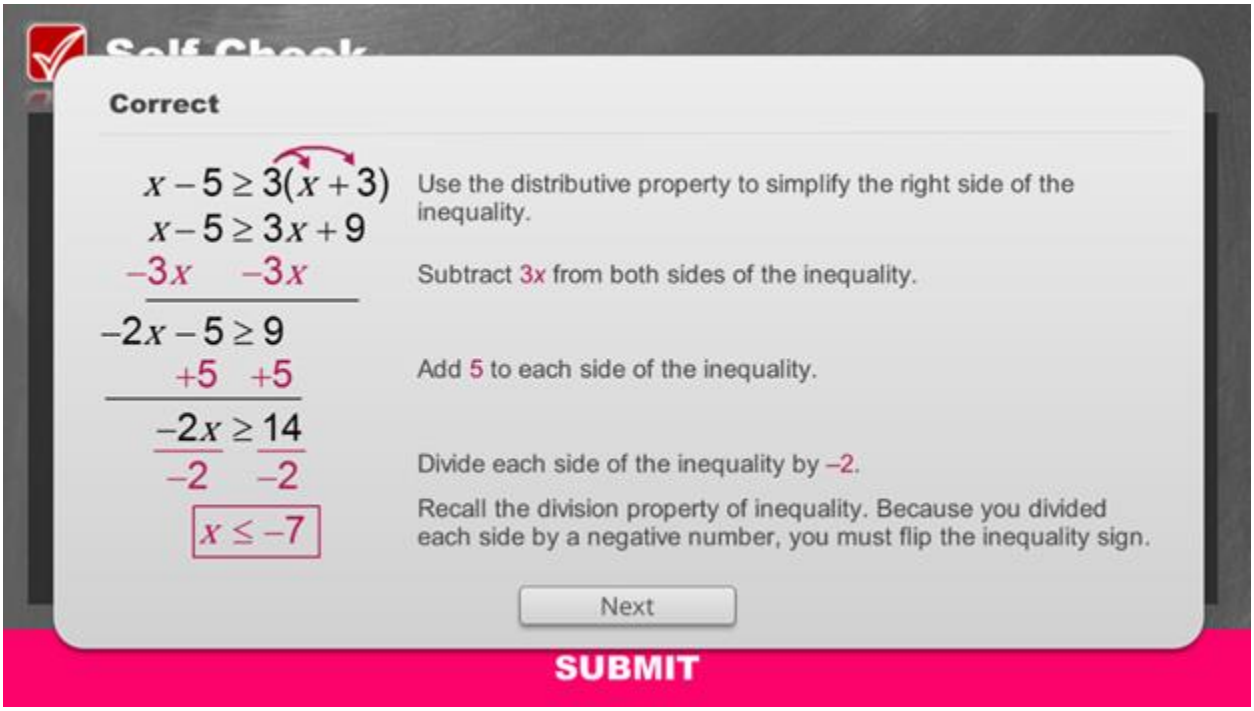
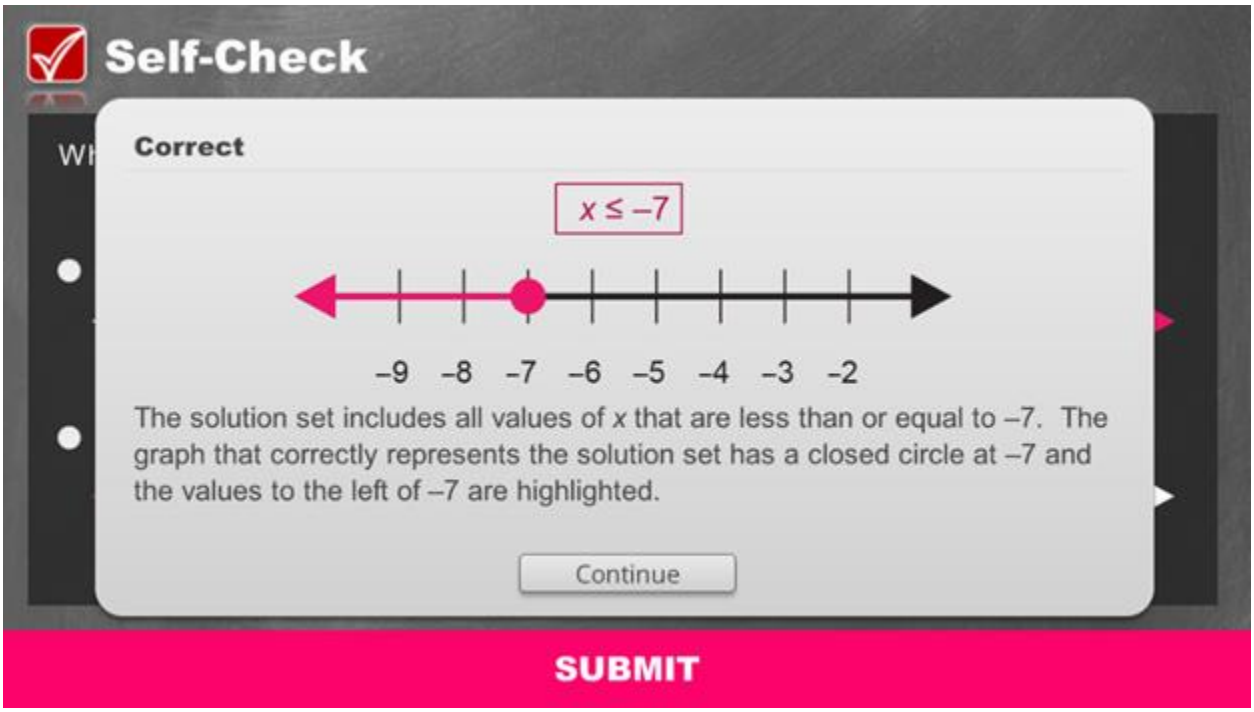
Self-Check 4: Answer





For your reference, the images above shows the correct solution to the self-check problem.

Conclusion

Well done! You have reached the end of your lesson on solving multistep linear inequalities. During this lesson, you used your prior knowledge of solving linear equations, as well as your knowledge of the properties of inequality, to help you successfully solve multistep linear inequalities.