

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Introduction

Today's Lesson

- You will learn how to determine if a given point is a solution to a linear inequality in two variables.
- You will use your knowledge of how to verify solutions of linear inequalities in one variable.



In this lesson, you will learn how to determine if a given point is a solution to a linear inequality in two variables. Your knowledge of how to verify solutions of linear inequalities in one variable, will be very useful as you move through this lesson. So before we begin our new material, let's jump back a bit and revisit how to verify solutions of inequalities in one variable.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Anticipatory Set

Is 5 a solution to the inequality, $2x - 3 > 1$?

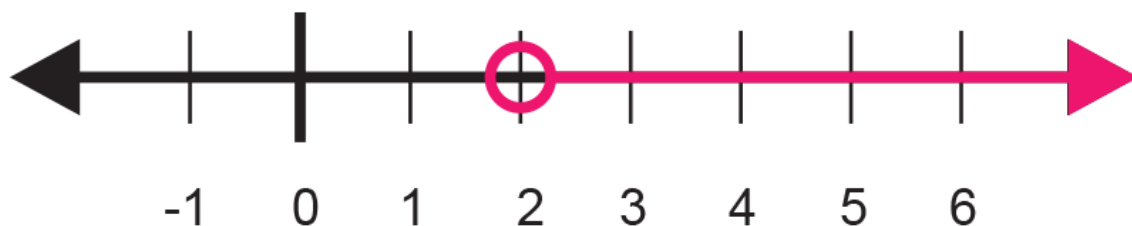
There are a few different ways you could determine the answer to this question. One method is to substitute five for x in the inequality and determine if the inequality still remains true.

$$\begin{array}{l} 2x - 3 > 1 \\ 2(5) - 3 \\ 10 - 3 \\ 7 > 1 \end{array} \quad \begin{array}{l} \text{Is } 2 \cdot 5 - 3 > 1 ? \\ \\ \\ 7 \text{ is less than } 1, \text{ so the inequality holds true. Therefore, } 5 \text{ is a solution to} \\ \text{the inequality.} \end{array}$$

Another strategy you could use to determine if 5 is a solution, is to solve the inequality for x . The inequality does not include any parentheses, and there are no like terms to combine, so you could begin using inverse operations to isolate x .

$$\begin{array}{l} 2x - 3 > 1 \\ + 3 \quad + 3 \\ \hline 2x > 4 \\ \hline 2 \quad 2 \\ \hline x > 2 \end{array} \quad \begin{array}{l} \text{Add 3 to both sides of the inequality.} \\ \\ \text{Divide each side by 2.} \\ \\ \text{Because 5 is less than 2, it would indeed be included in the solution set.} \end{array}$$

Graphing the solution set on a number line would also allow you to determine if 5 is a solution. The solution set includes all values greater than two. Graphically, you would represent this by sketching an open circle at two and highlighting all of the values greater than two, the values to the right. Because 5 is included among these values, it is a solution to the inequality.



Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Verifying Solutions

Verifying Solutions

- You are able to verify solutions to inequalities with two variables by analyzing the graph of the inequality.
- You can also verify solutions to inequalities with two variables by substituting values into the inequality.



When working with inequalities in two variables, you can also verify solutions graphically, by analyzing the graph of the inequality, and algebraically, by substituting values into the inequality. In the next example, you will determine if the given points are solutions to the given inequality.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Verifying Solutions to Multi-Step Linear Inequalities



The graphic features a dark background with faint chalkboard-style math symbols. At the top, a blue rounded rectangle contains the title "VERIFYING SOLUTIONS TO MULTI-STEP LINEAR INEQUALITIES" in white. Below this, a yellow text prompt reads "Click the Examples Below to Learn More". Three pink rounded rectangular buttons are arranged: "Example One" and "Self-Check" are in the top row, and "Example Two" is centered below them. In the bottom right corner, there is a 3D rendering of white rectangular blocks, some stacked and some separate.

Click the examples below to learn more.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Example 1

Given: $y + 3 < 2x$

A) Is the point (5, 1) a solution?

When verifying solutions to linear inequalities graphically, solutions can be found in two places:

1. on a solid line
2. in a shaded region

To use the graphing calculator to graph the inequality, $y + 3 < 2x$, you must first solve for y .

$$y + 3 < 2x$$

$$\underline{- 3 \quad - 3}$$

$$y < 2x - 3$$

There are no parentheses to eliminate and no like terms to combine, so begin using inverse operations to isolate y .

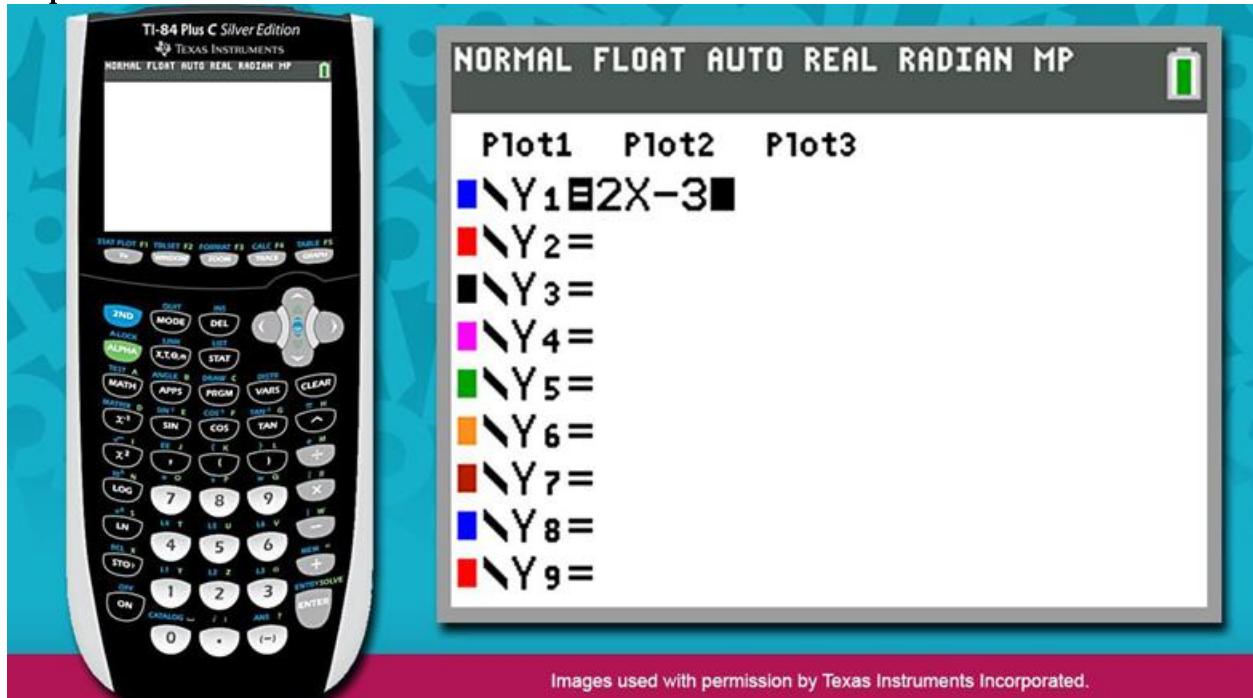
Subtract 3 from each side of the inequality.

The final inequality is $y < 2x - 3$.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 1



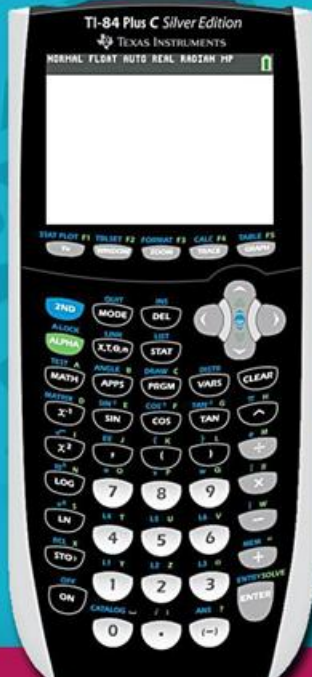
Now that the inequality is solved for y , you can use the graphing calculator to graph it on the coordinate plane. Clear the memory in the calculator.

Press the $Y=$ key. Then enter the right side of the inequality to the right of $Y1$.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Solid Line or Dashed Line



When graphing this inequality...

$$y < 2x - 3$$

which line would you use? Click your answer below.

Solid Line

Dashed Line

Images used with permission by Texas Instruments Incorporated.

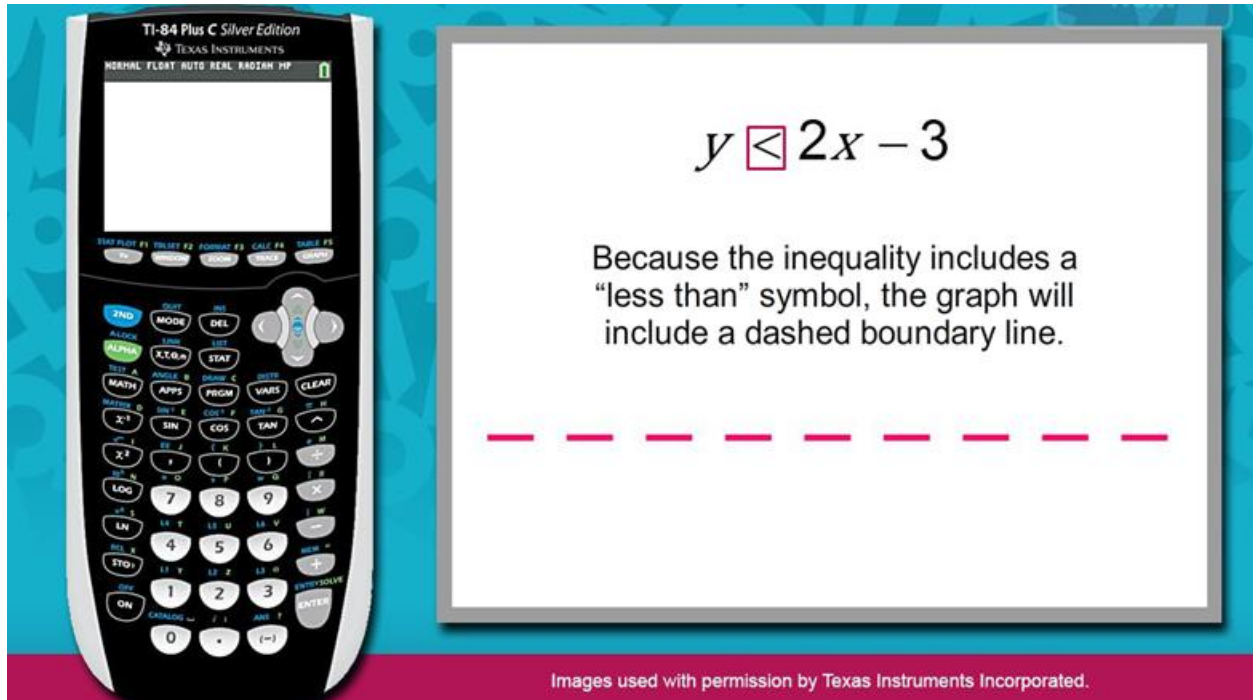
Recall that the graph of a strict inequality, an inequality that includes $<$ or $>$, will include a dashed boundary line. The graph of a non-strict inequality, an inequality that includes \leq or \geq , will include a solid boundary line.

Will the graph of $y < 2x - 3$ include a dashed boundary line or a solid boundary line?

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Feedback



The image shows a TI-84 Plus C Silver Edition calculator on the left. To its right is a whiteboard with a grey border. On the whiteboard, the inequality $y < 2x - 3$ is written in black. Below the inequality, a text box explains: "Because the inequality includes a 'less than' symbol, the graph will include a dashed boundary line." Below the text, a horizontal dashed pink line is drawn across the width of the whiteboard. At the bottom of the whiteboard area, there is a small caption: "Images used with permission by Texas Instruments Incorporated."

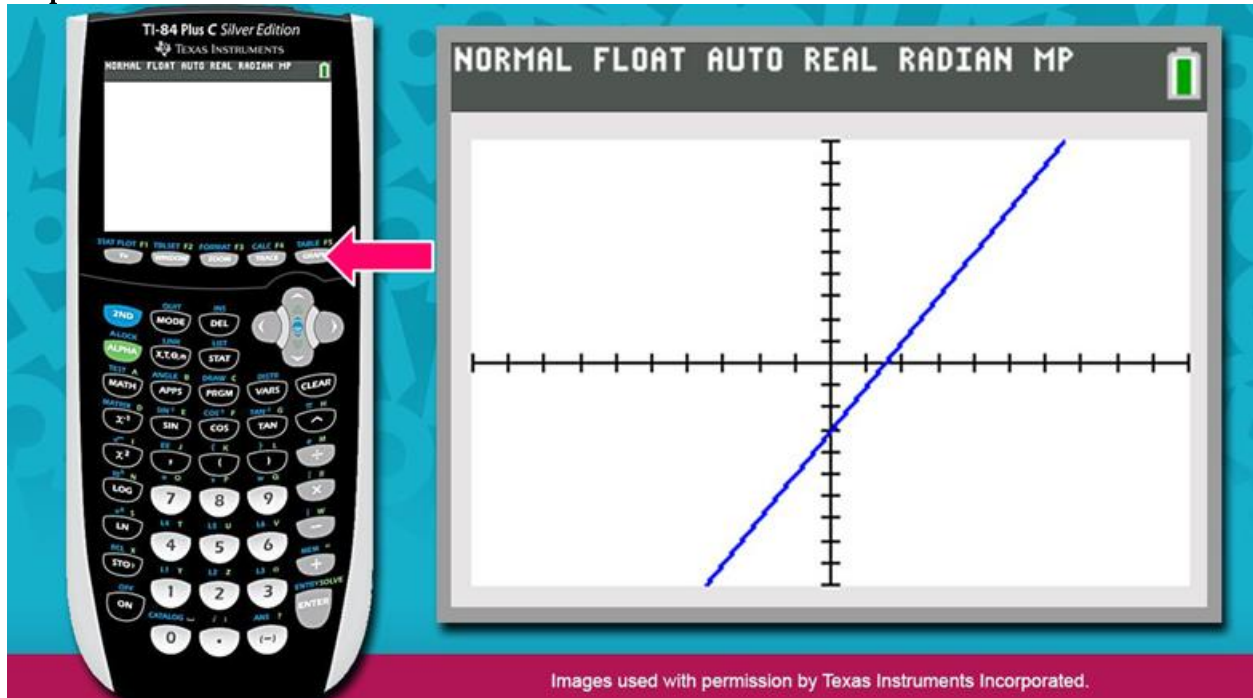
The inequality is strict. Therefore, its graph will include a dashed boundary line.

Remember that the calculator will only graph solid boundary lines. Therefore it is helpful to also prepare a rough sketch of the graph, so that you can remain mindful of whether the graph actually includes a dashed line or a solid line.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 2



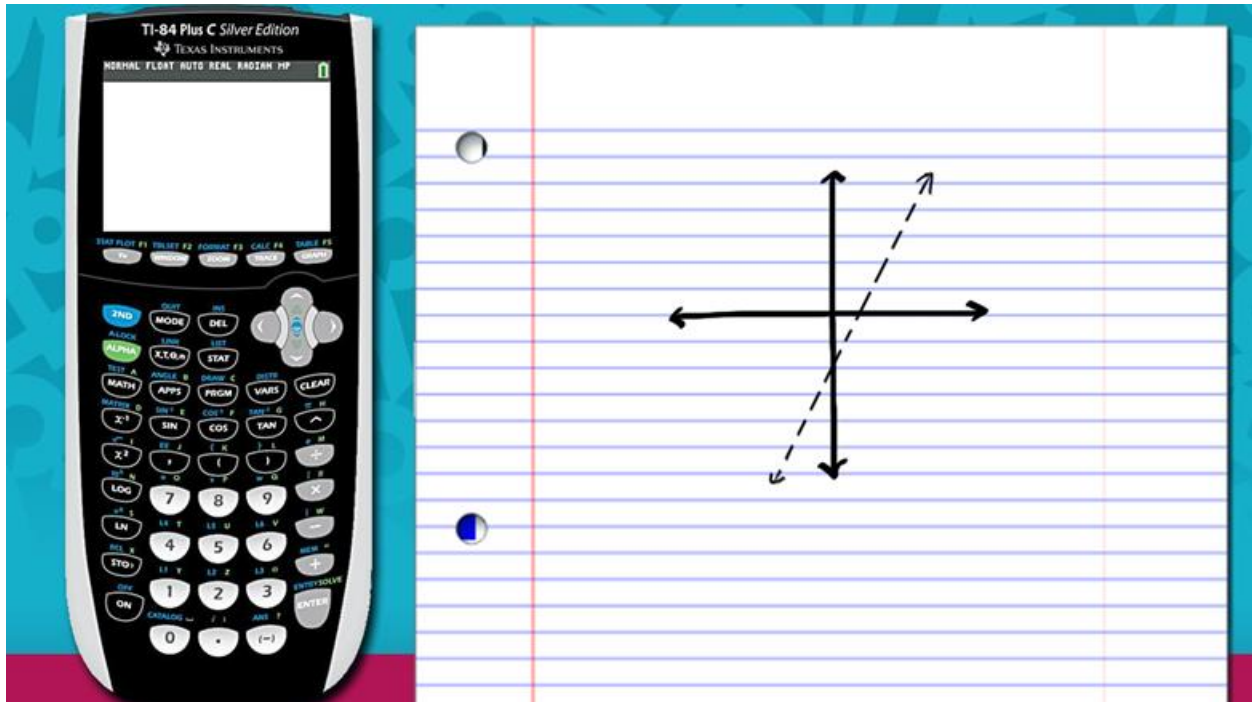
Now that you have determined the graph of $y < 2x - 3$ will include a dashed boundary line, press graph and begin preparing your rough sketch.

Press GRAPH.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Sketch

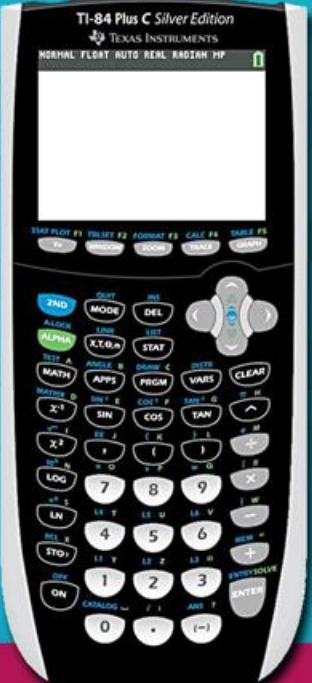


Make sure you include a dashed boundary line to represent the solid line shown on your calculator.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Above the Line or Below the Line



TI-84 Plus C Silver Edition
TEXAS INSTRUMENTS
NORMAL FLOAT AUTO REAL RADIAN MP

When graphing this inequality...

$$y < 2x - 3$$

would you shade above the line or below the line?
Click your answer below.

Above the Line

Below the Line

Images used with permission by Texas Instruments Incorporated.


Now it's time to determine which region of the graph should be shaded.

Should the graph of $y < 2x - 3$ include shading above the boundary line or below the boundary line?

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Feedback



A TI-84 Plus C Silver Edition calculator is shown on the left side of the image. The screen is blank.

$$y \leq 2x - 3$$

Because the inequality includes a “less than” symbol, you will need to shade below the line.

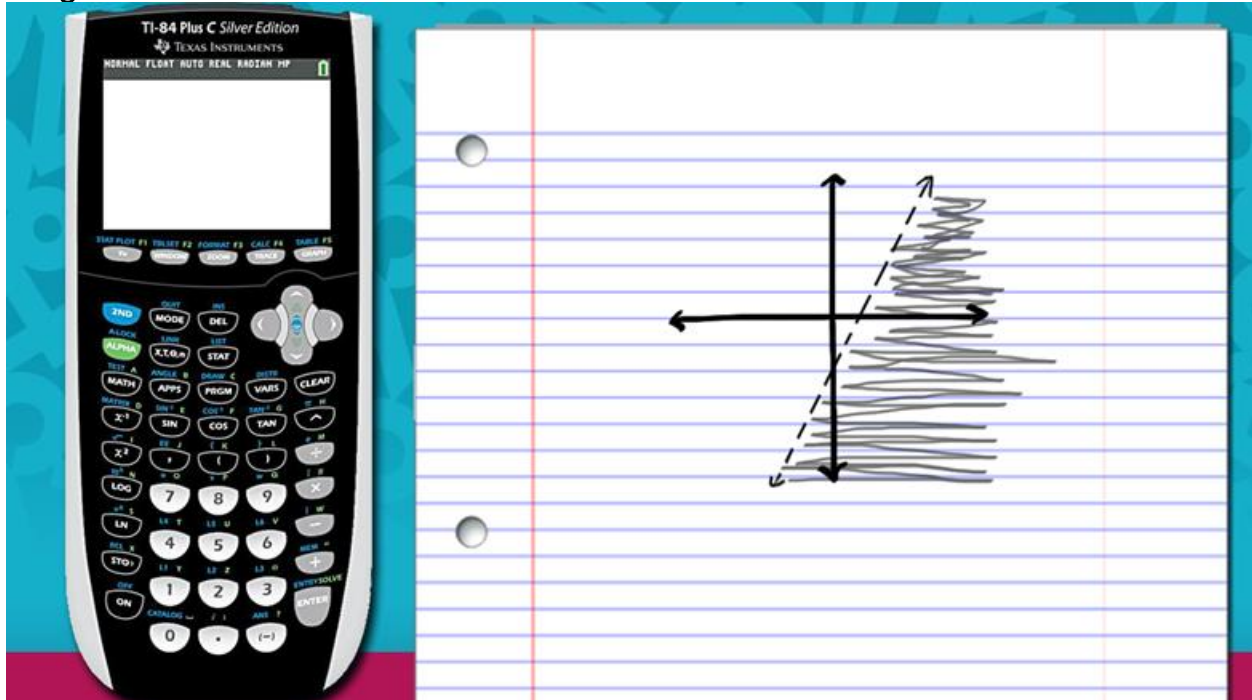
Images used with permission by Texas Instruments Incorporated.

Because the inequality includes “less than,” the region below the boundary line should be shaded.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Rough Sketch

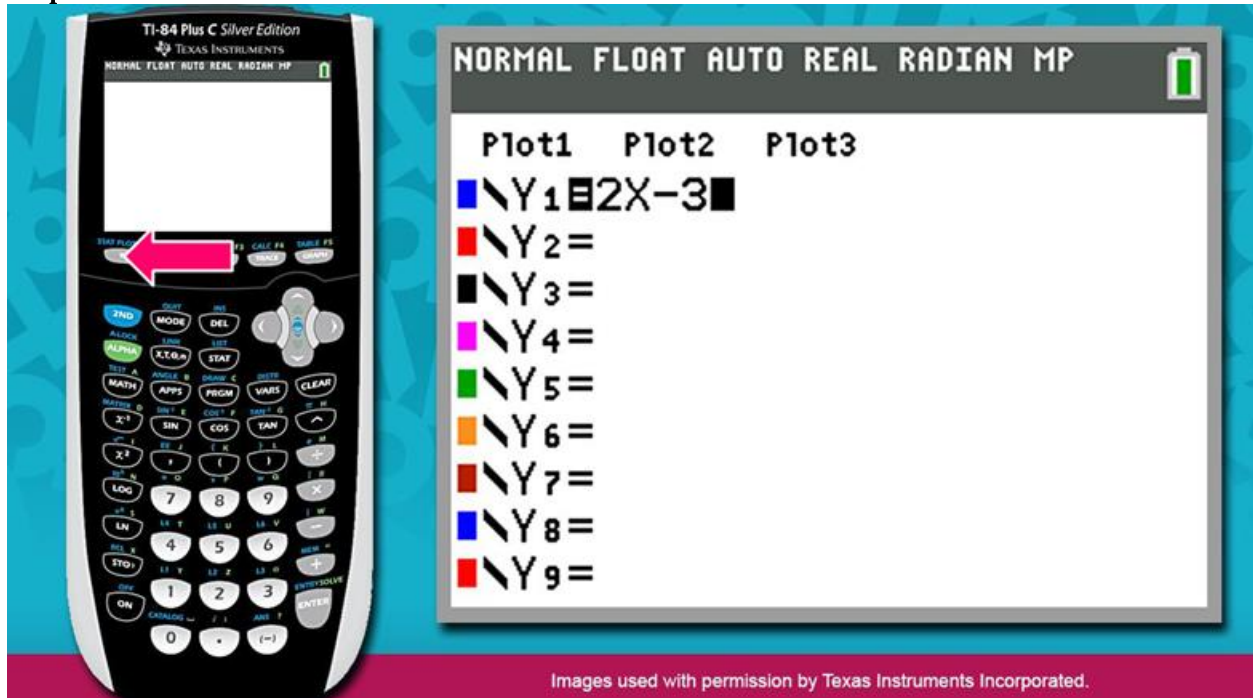


Now that you have determined that the region below the boundary line should be shaded, add the shading to your rough sketch.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 3

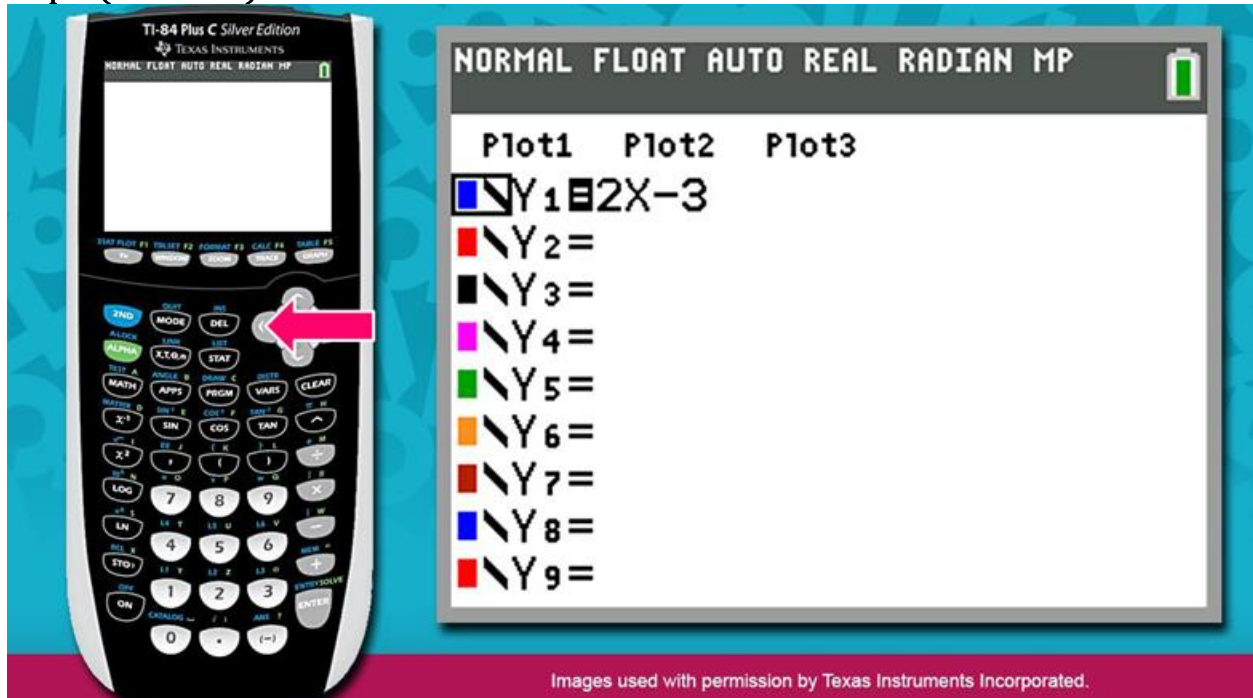


Now instruct the calculator to shade the appropriate region. Press the $Y=$ key. Then press the left arrow until the cursor is blinking to the left of Y_1 .

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 3 (continued)



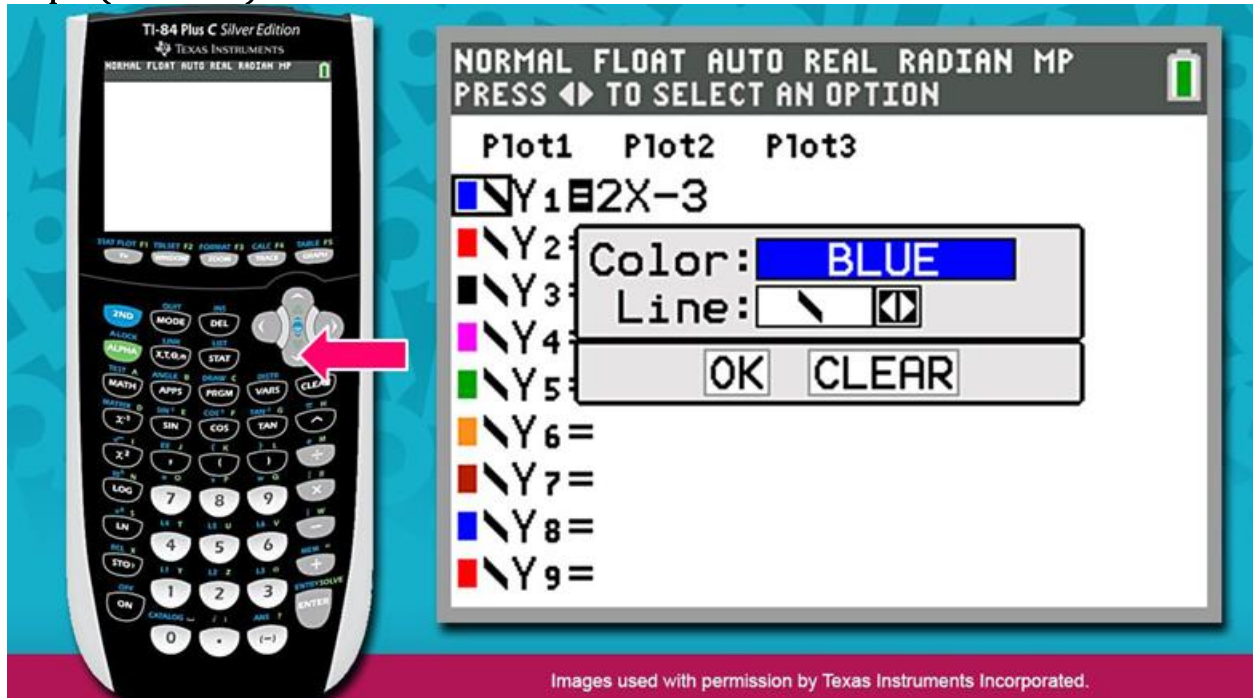
Images used with permission by Texas Instruments Incorporated.

Now press enter. Then press the down arrow key until the cursor is blinking to the right of the “Line” prompt.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 3 (continued)

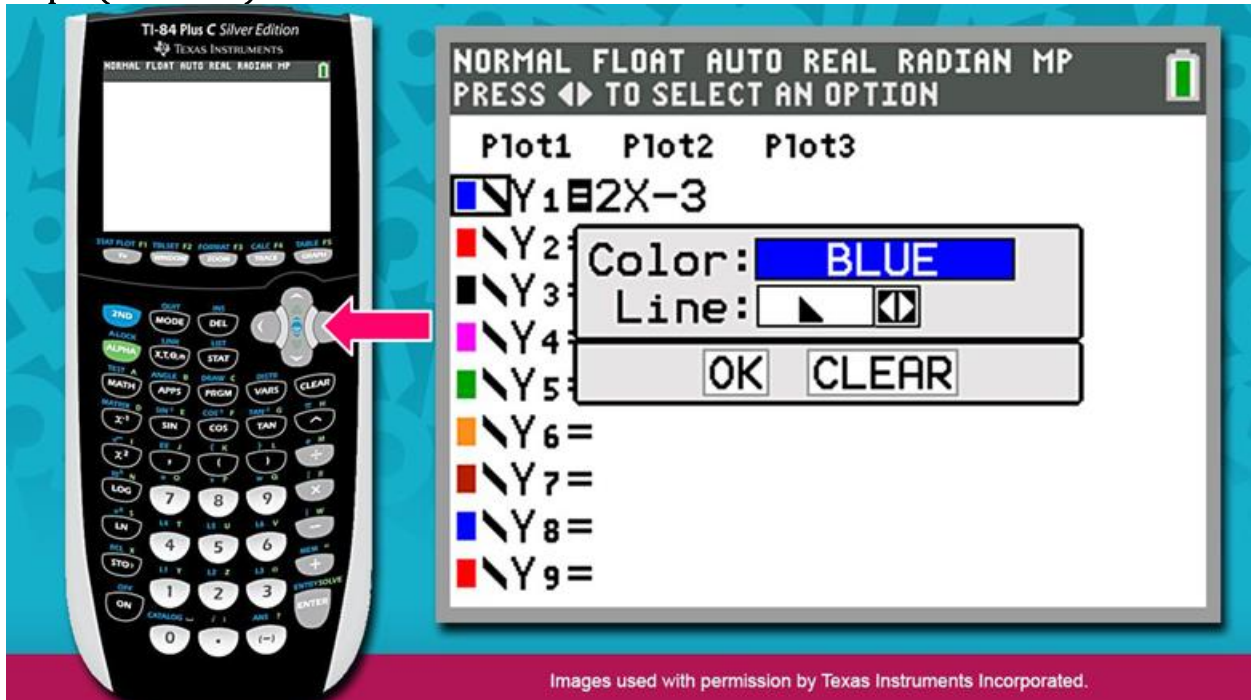


Press the right arrow key until the option for shading below the line appears. Then press enter.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 3 (continued)



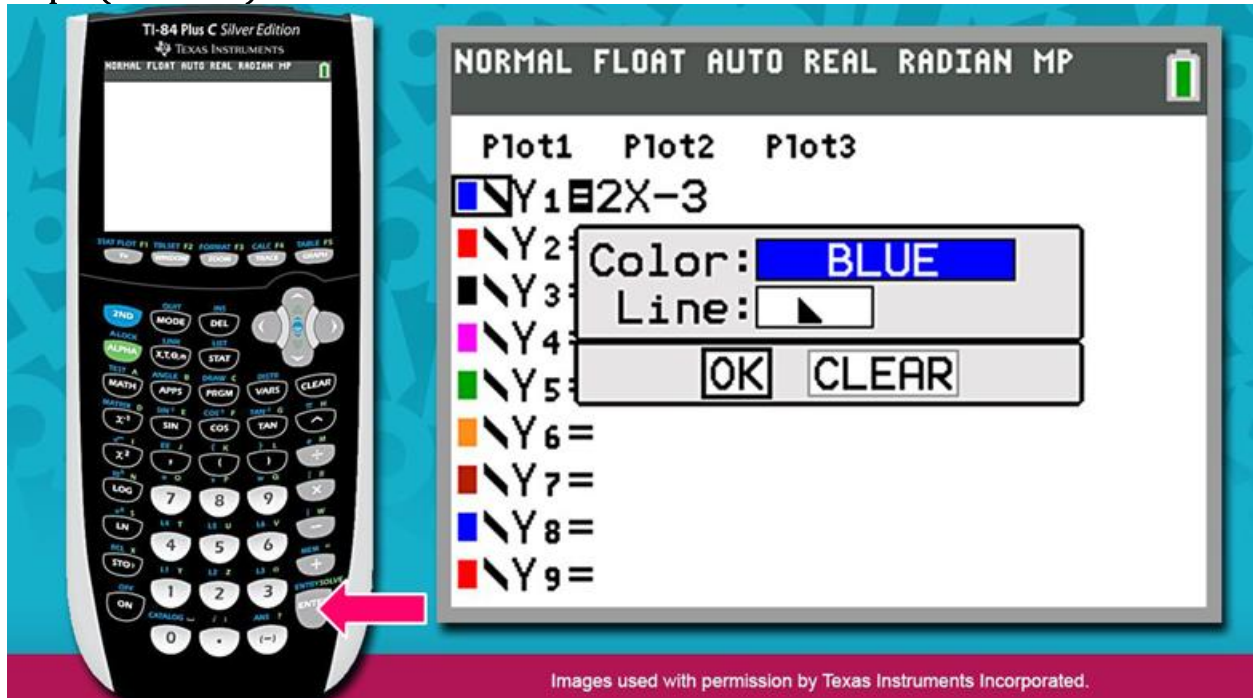
Images used with permission by Texas Instruments Incorporated.

Press enter again once the cursor moves to OK.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 3 (continued)

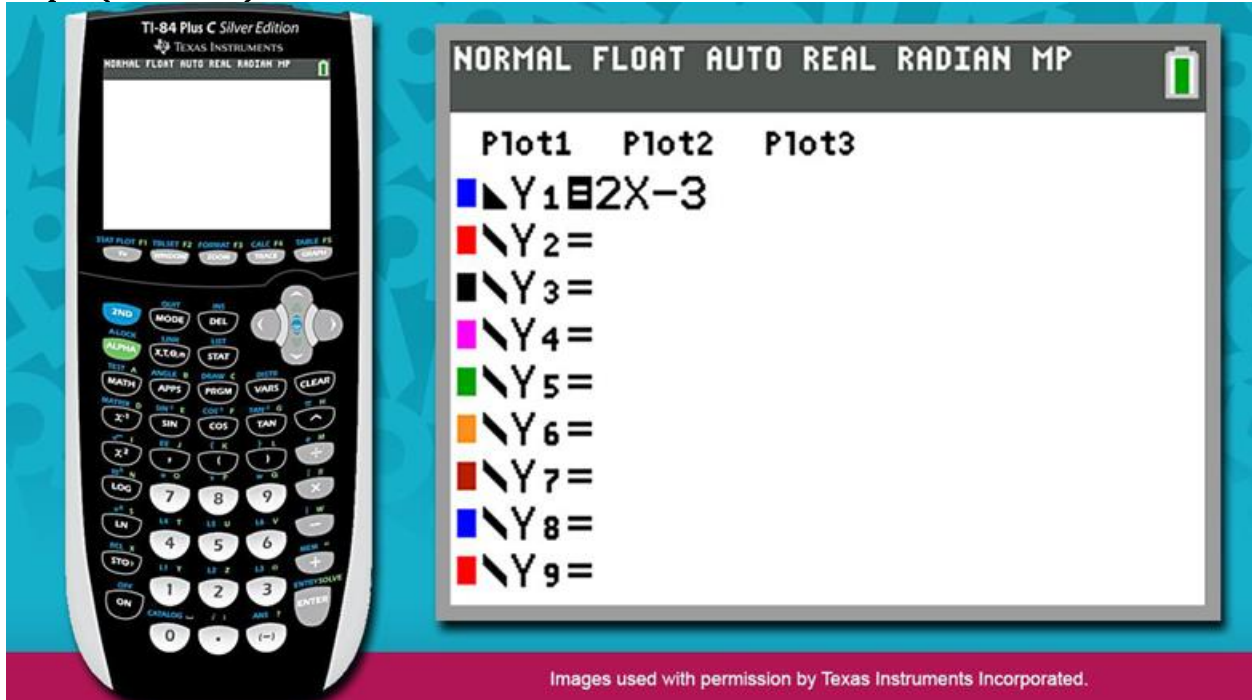


The screen will then display the inequality to the right of Y1, with the appropriate shading option to the left of Y1.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 3 (continued)

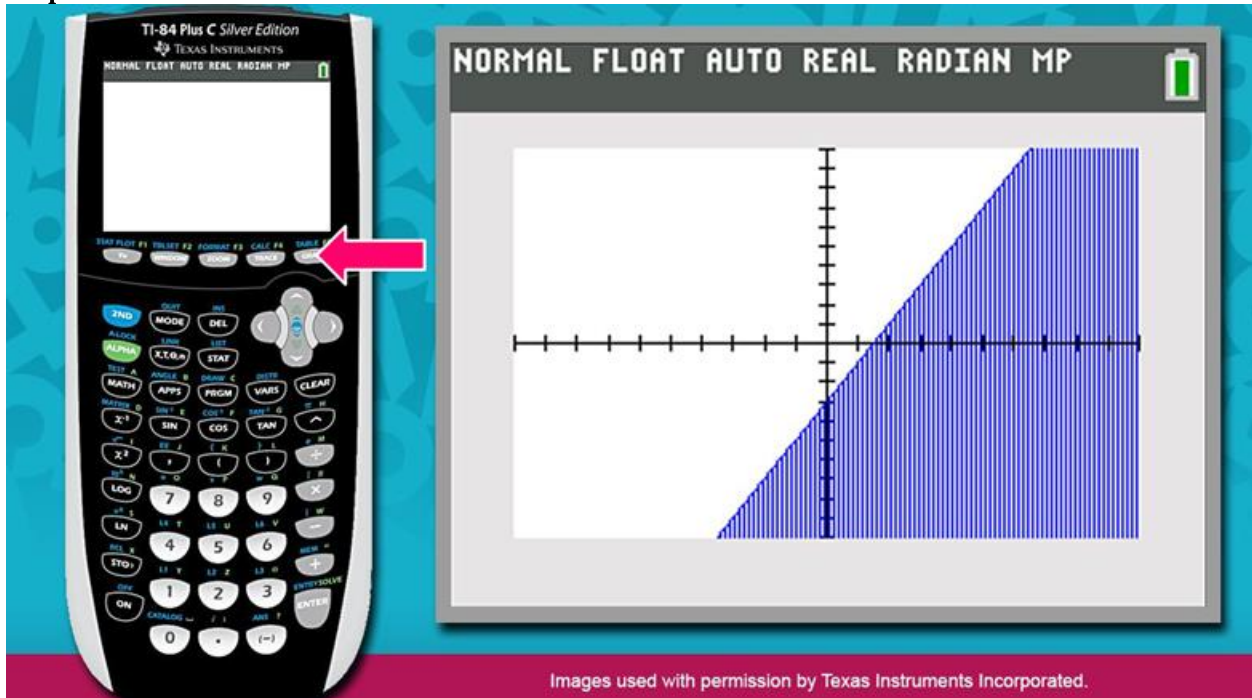


Now press GRAPH.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 4

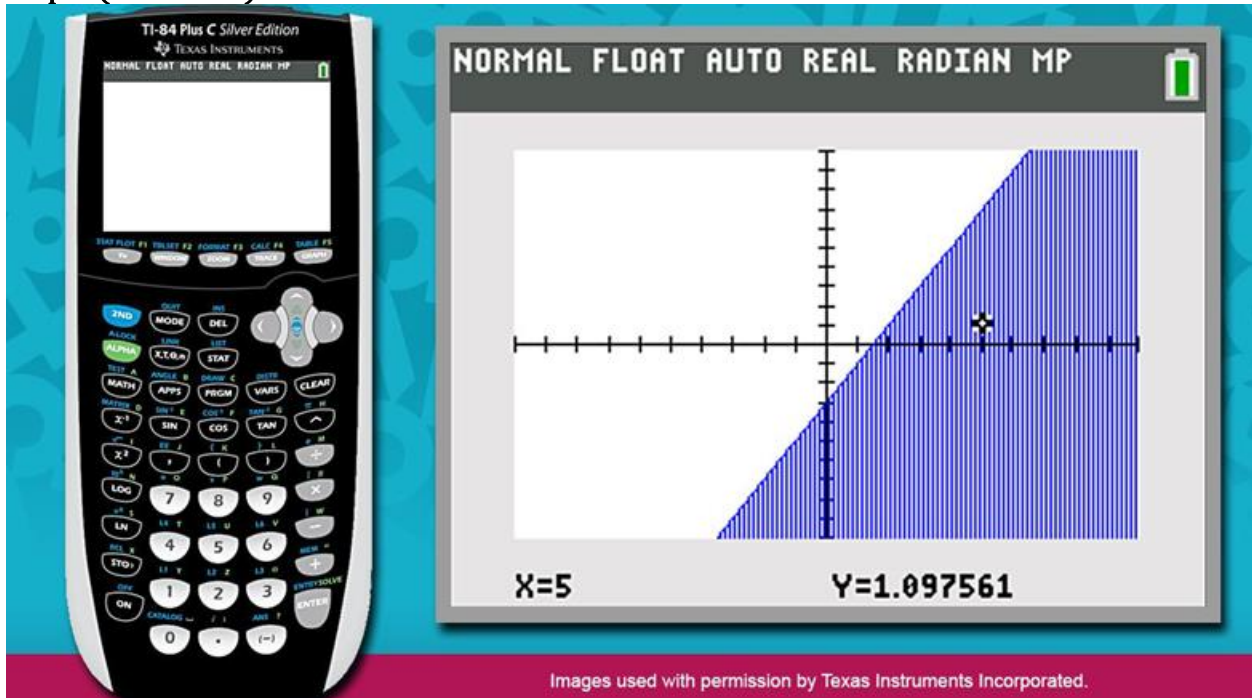


Now that you have graphed the inequality, you can analyze the graph to determine if the point (5, 1) is a solution.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 4 (continued)

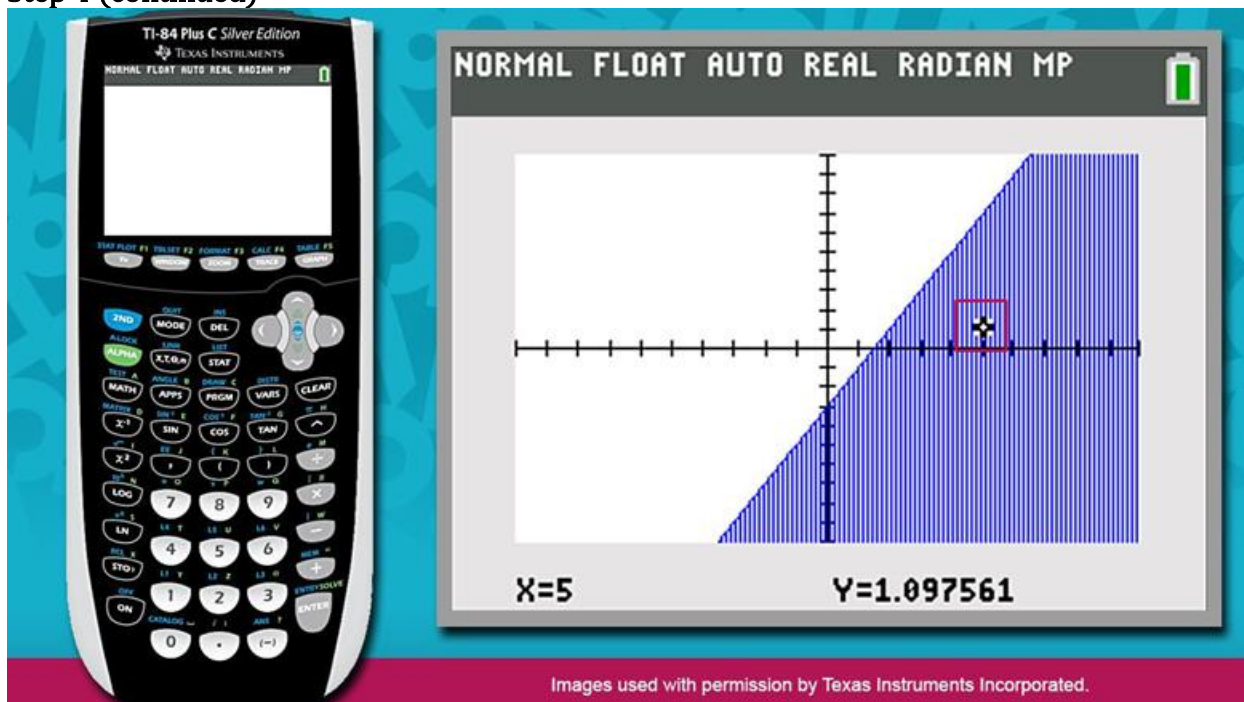


Press the right arrow key and the key up arrow key to move the cursor around the coordinate plane. You may not be able to land exactly on the point (5, 1), but get as close to it as you can.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Step 4 (continued)



It appears that the point (5, 1) is included in the shaded region of the inequality. Therefore, it is a solution.

You can also verify that the point (5, 1) is a solution, algebraically. The point (5, 1) can also be referred to as an ordered pair. It consists of an x value of 5 and a y value of 1. By substituting 5 for x and 1 for y , into the original inequality, you can verify that it is a solution.

$$\begin{pmatrix} 5, & -1 \\ x & y \end{pmatrix}$$

$$y + 3 < 2x$$

$$(1) + 3 < 2(5)$$

$$4 < 10$$

$$\text{Is } 1 + 3 < 2 \cdot 5?$$

Is 4 less than 10? Yes it is. Therefore the point (5, 1) is a solution to the inequality.

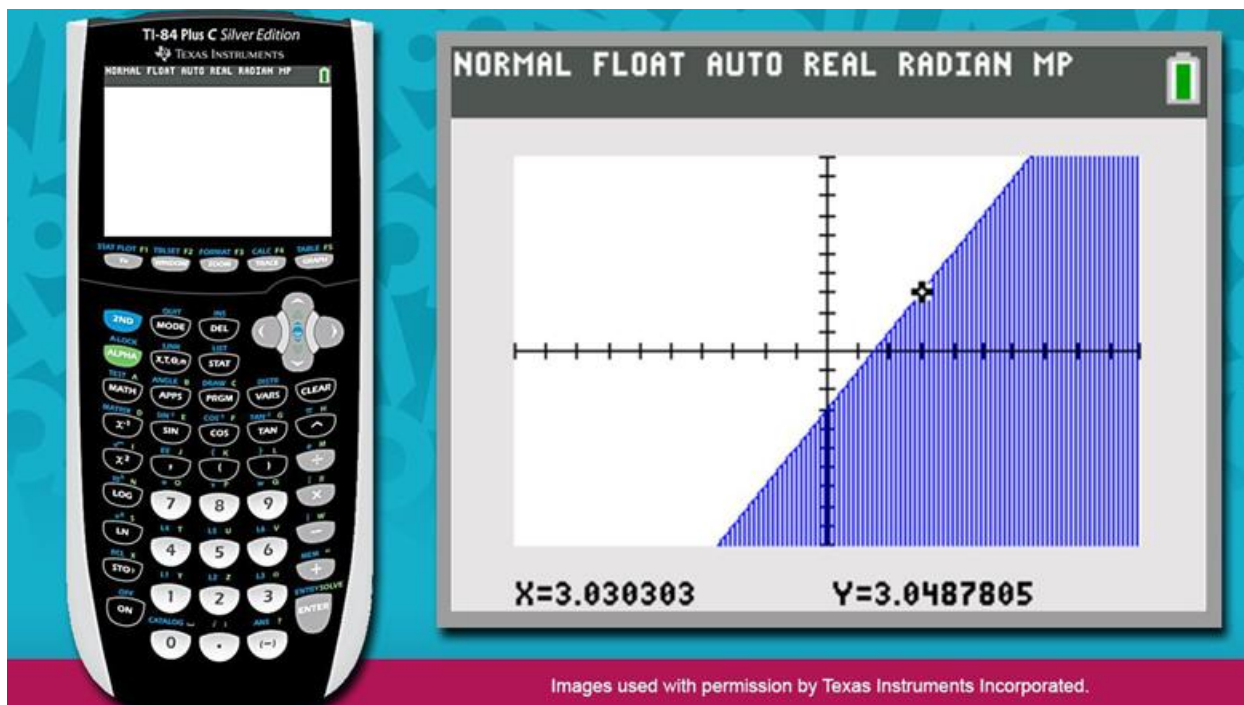
Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

B) Is $(3, 3)$ a solution?

For this example, you will continue working with the inequality, $y + 3 < 2x$. This time you will need to determine if $(3, 3)$ is a solution. Remember, you can verify solutions graphically and algebraically.

You have already determined what the graph of $y + 3 < 2x$ looks like. Press the GRAPH key on the calculator to recall the graph.

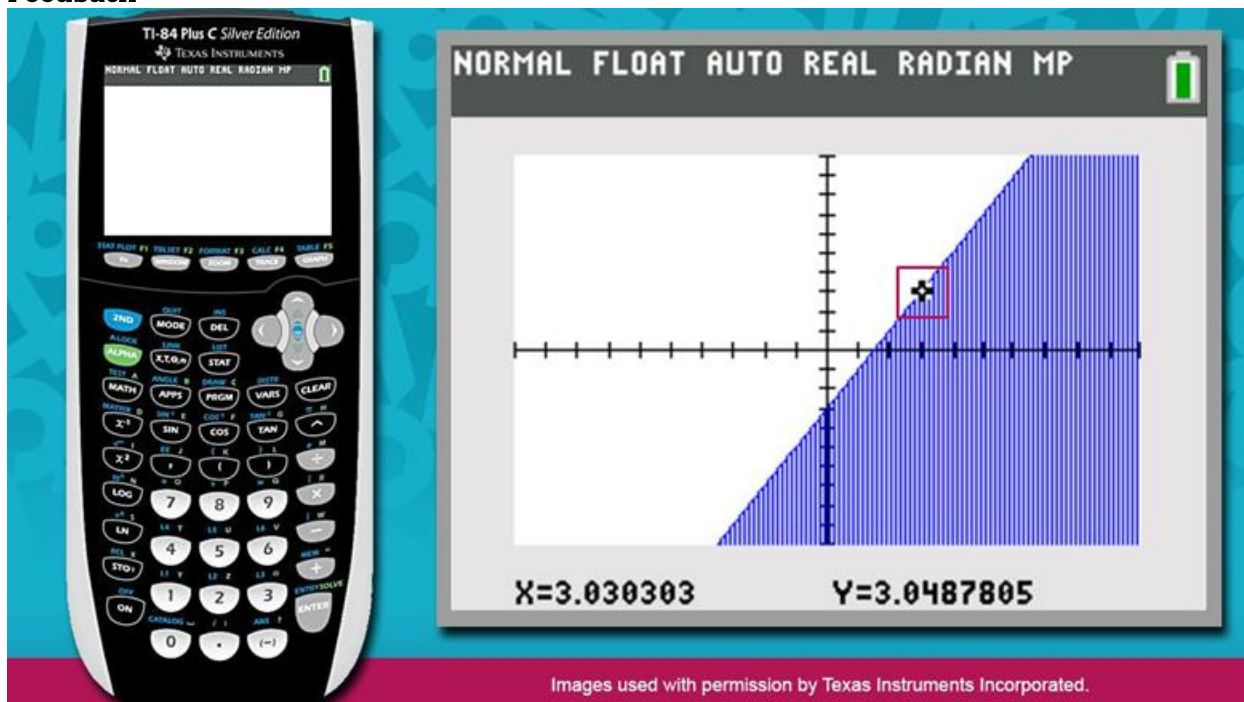


Because you want to determine if the point $(3, 3)$ is a solution, you will need to press the left arrow key and the up arrow key until you locate the point $(3, 3)$. If you are not able to locate the point exactly, get as close to it as you can.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Feedback



The point (3, 3) appears to be located on the boundary line. And although the calculator graphed a solid line, you know from your rough draft that the graph is a dashed line. Therefore, (3, 3) is not a solution to the inequality.

Because you were not able to land exactly on the point (3, 3), however, it is a good idea to confirm your answer algebraically, by substituting (3, 3) into the original inequality.

$$\begin{matrix} (3, 3) \\ x \quad y \end{matrix}$$

$$y + 3 < 2x$$

$$(3) + 3 \quad 2(3)$$

$$6 < 6$$

$$\text{Is } 3 + 3 < 2 \cdot 3?$$

Is 6 less than 6? No, it is not. Therefore the point (3, 3) is not a solution to the inequality.

Module 7: Solving Linear Inequalities

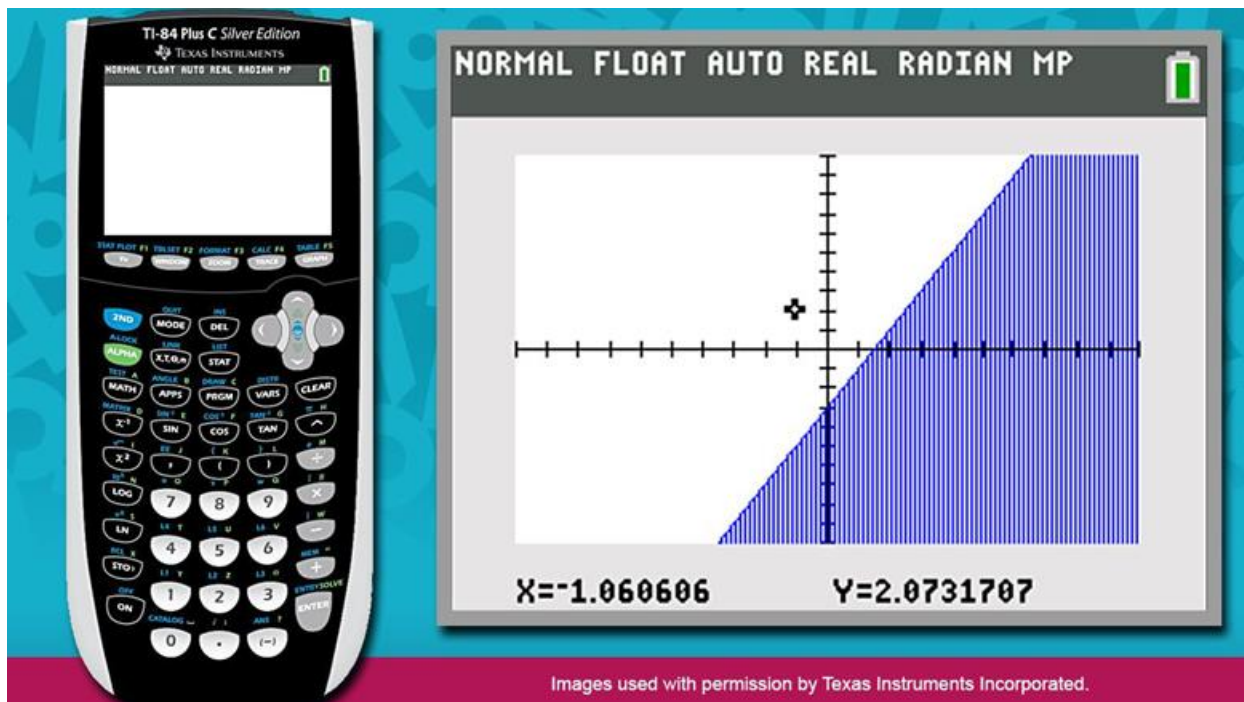
Topic 2: Verifying Solutions

C) Is $(-1, 2)$ a solution?

For this example, you will continue working with the inequality, $y + 3 < 2x$. This time you will need to determine if $(-1, 2)$ is a solution. Remember, you can verify solutions graphically and algebraically.

You have already determined what the graph of $y + 3 < 2x$ looks like.

Press the GRAPH key to recall the graph of the inequality.



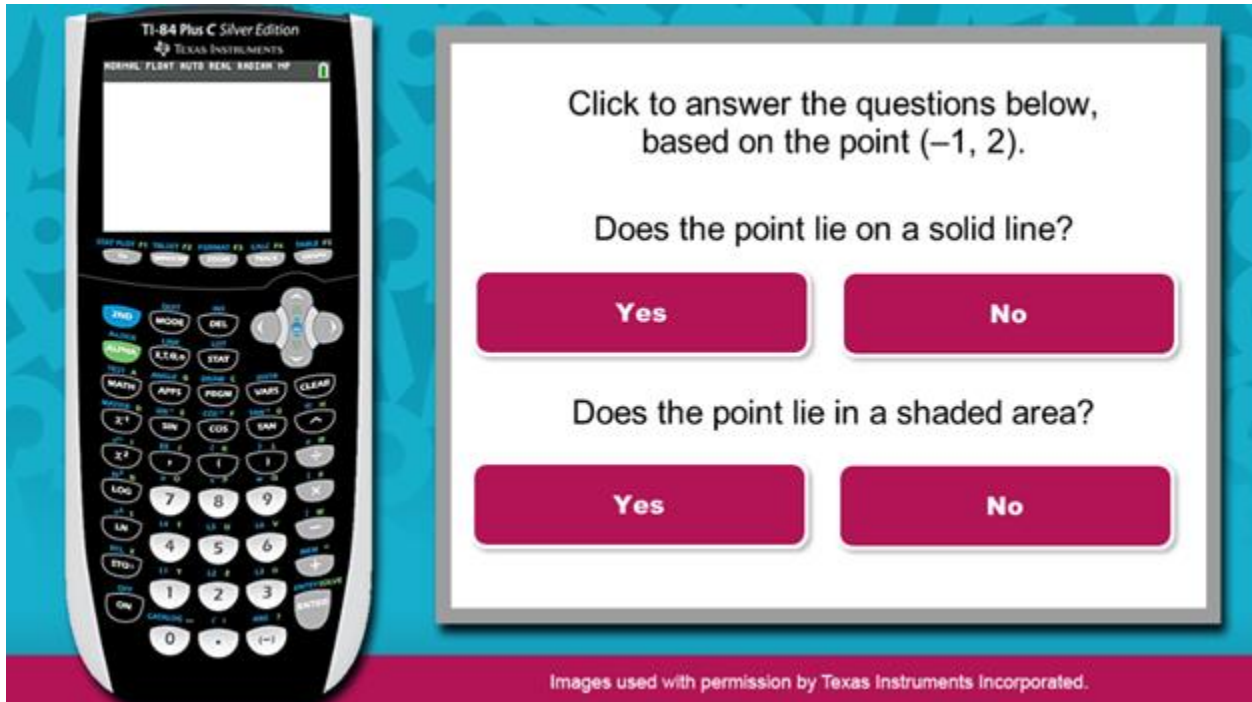
Images used with permission by Texas Instruments Incorporated.

Because you want to determine if the point $(-1, 2)$ is a solution, you will need to press the left arrow key and the down arrow key until you locate the point $(-1, 2)$. If you are not able to locate the point exactly, get as close to it as you can.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Solid Line or Shaded Area



The image shows a TI-84 Plus C Silver Edition calculator on the left and a digital interface on the right. The interface contains the following text and buttons:

Click to answer the questions below, based on the point $(-1, 2)$.

Does the point lie on a solid line?

Yes **No**

Does the point lie in a shaded area?

Yes **No**

Images used with permission by Texas Instruments Incorporated.

Although you are not able to land on the point exactly, you are still able to answer the following questions with certainty.

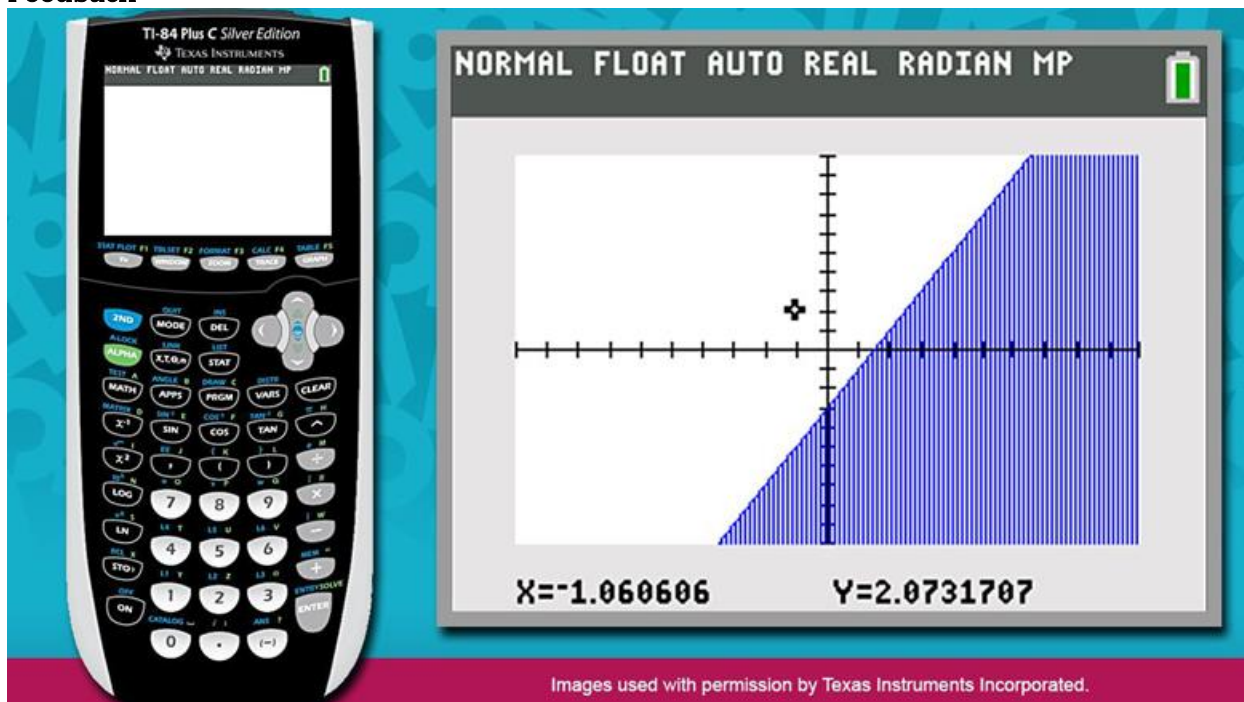
Does the point lie on a solid line? No. The point does not lie on a line at all.

Does the point lie in the shaded area? No. The point lies in the unshaded area.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Feedback



Images used with permission by Texas Instruments Incorporated.

It is safe to assume that the point $(-1, 2)$ is located in the unshaded region of the graph. Therefore it is not a solution to the inequality. You can also verify this algebraically, by substituting $(-1, 2)$ into the original inequality.

$$\begin{matrix} (-1, 2) \\ x \quad y \end{matrix}$$

$$y + 3 < 2x$$

$$(2) + 3 \quad 2(-1)$$

$$5 < -2$$


$$\text{Is } 2 + 3 < 2 \cdot -1?$$

Is 5 less than -2 ? No, it is not. Therefore the point $(-1, 2)$ is not a solution to the inequality.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Self-Check 1



Self-Check

Given the inequality below...

$$2y + 8 \geq x$$

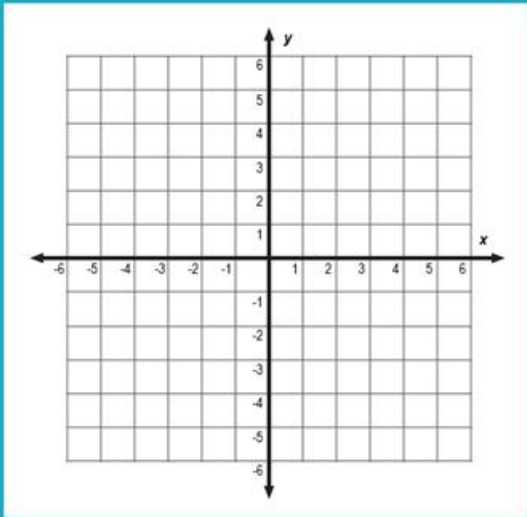
Is $(1, 5)$ a solution? Solve the inequality for y first. Then, use the graphing calculator to graph the inequality.

Yes

No

SUBMIT

Graph



Click the graph above to reveal how the graph of the inequality should appear.

Solve the problem in the image above to check your understanding of the content.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Self-Check 1: Answer

Self-Check

Given the inequality below...

$$2y + 8 \geq x$$

Is $(1, 5)$ a solution? Solve the inequality for y first. Then, use the graphing calculator to graph the inequality.

Yes

No

SUBMIT

Graph

Click the graph above to reveal how the graph of the inequality should appear.

Self-Check

Correct

That's correct! When you solve the inequality $2y + 8 \geq x$ for y , you end up with the result below.

$$\begin{array}{r} 2y + 8 \geq x \\ -8 \quad -8 \\ \hline 2y \geq x - 8 \\ \frac{2}{2} \quad \frac{2}{2} \quad \frac{2}{2} \\ \boxed{y \geq \frac{1}{2}x - 4} \end{array}$$

After graphing the result, you can see that the point $(1, 5)$ is included in the solution set.

SUBMIT

Graph

the graph of the inequality should appear.

For your reference, the image above shows the correct solution to the self-check problem.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Self-Check 2



Self-Check

Given the same inequality from the previous problem...

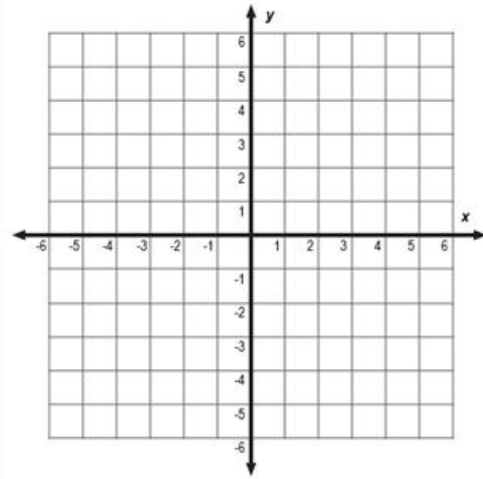
$$2y + 8 \geq x$$

Is $(2, -3)$ a solution? Justify your answer graphically.

- Yes
- No

SUBMIT

Graph



Click the graph above to reveal how the graph of the inequality should appear.

Solve the problem in the image above to check your understanding of the content.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Self-Check 2: Answer

Self-Check

Given the same inequality from the previous problem...

$2y + 8 \geq x$

Is $(2, -3)$ a solution? Justify your answer graphically.

Yes
 No

SUBMIT

Graph

Click the graph above to reveal how the graph of the inequality should appear.

Self-Check

Correct

That's correct! When you solve the inequality $2y + 8 \geq x$ for y , you end up with the result below.

$$\begin{array}{r}
 2y + 8 \geq x \\
 \underline{-8 \quad -8} \\
 2y \geq x - 8 \\
 \underline{2 \quad 2 \quad 2} \\
 y \geq \frac{1}{2}x - 4
 \end{array}$$

After graphing the result, you can see that the point $(2, -3)$ is included in the solution set.

SUBMIT

Graph


the graph of the inequality should appear.

For your reference, the image above shows the correct solution to the self-check problem.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Self-Check 3

**Self-Check**

Given the inequality below, is $(3, -4)$ a solution? Justify your answer algebraically.

$$2y + 8 \geq x$$

Yes

No

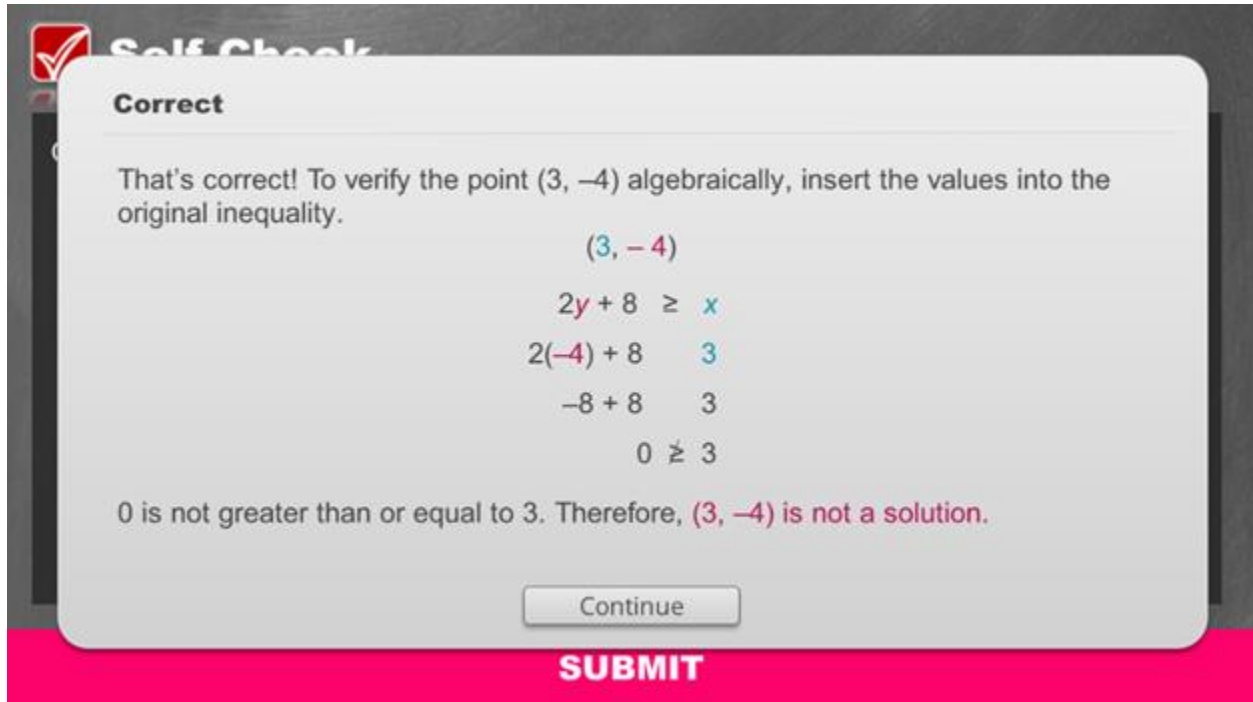
SUBMIT

Solve the problem in the image above to check your understanding of the content.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Self-Check 3: Answer



Correct

That's correct! To verify the point $(3, -4)$ algebraically, insert the values into the original inequality.

$$(3, -4)$$
$$2y + 8 \geq x$$
$$2(-4) + 8 \geq 3$$
$$-8 + 8 \geq 3$$
$$0 \geq 3$$

0 is not greater than or equal to 3. Therefore, $(3, -4)$ is not a solution.

Continue

SUBMIT

For your reference, the image above shows the correct solution to the self-check problem.

Module 7: Solving Linear Inequalities

Topic 2: Verifying Solutions

Conclusion

A digital interface for a lesson conclusion. On the left, a white box with a pink header titled "Today's Lesson" contains two checked items: "Verified solution of inequalities with two variables algebraically" and "Verified solutions of inequalities with two variables graphically". Below these are two pink buttons: "Exit Lesson" and "Restart Lesson". On the right, a cartoon illustration of a smiling woman with dark curly hair and a pink top is set against a blue background with faint mathematical symbols like pi, infinity, and numbers.

Great job! Your knowledge of how to verify solutions of inequalities in one variable, served as the foundation, for you to build your knowledge on verifying the solutions of inequalities in two variables. You are now well-equipped to use graphic or algebraic techniques in order to verify a solution.