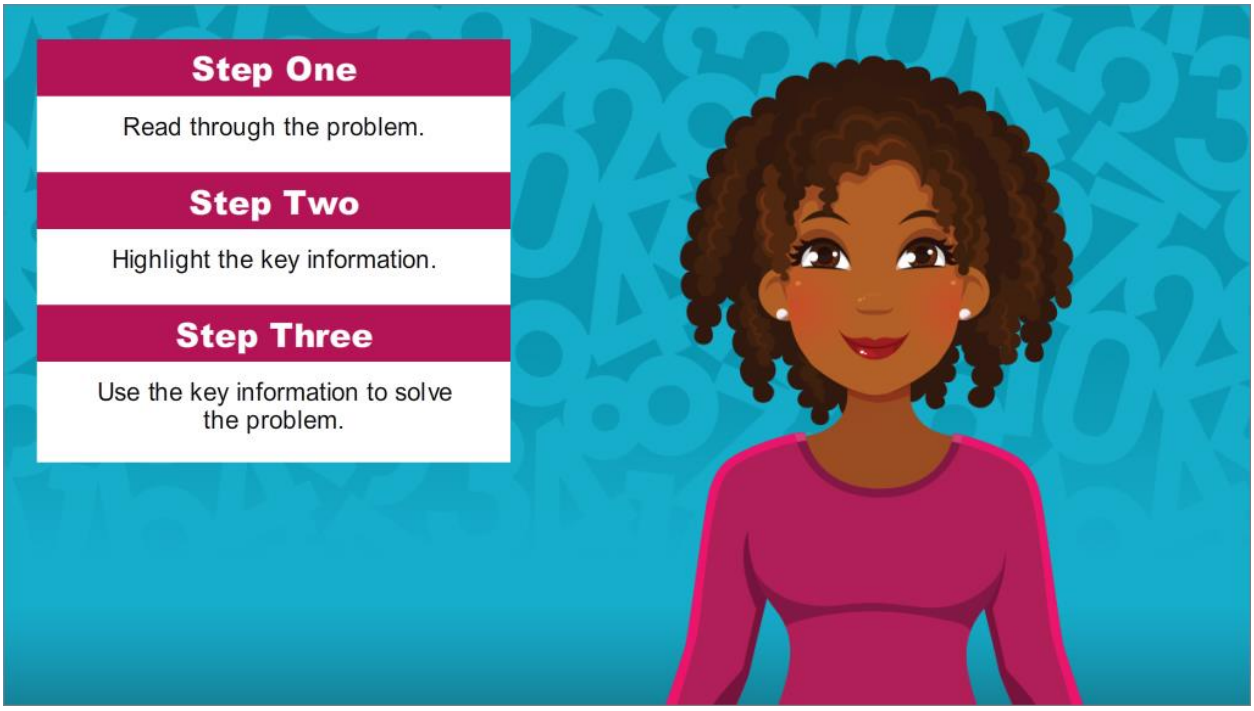
Module 7: Solving Linear Inequalities
Topic 4: Real-World Systems of Inequalities

Introduction

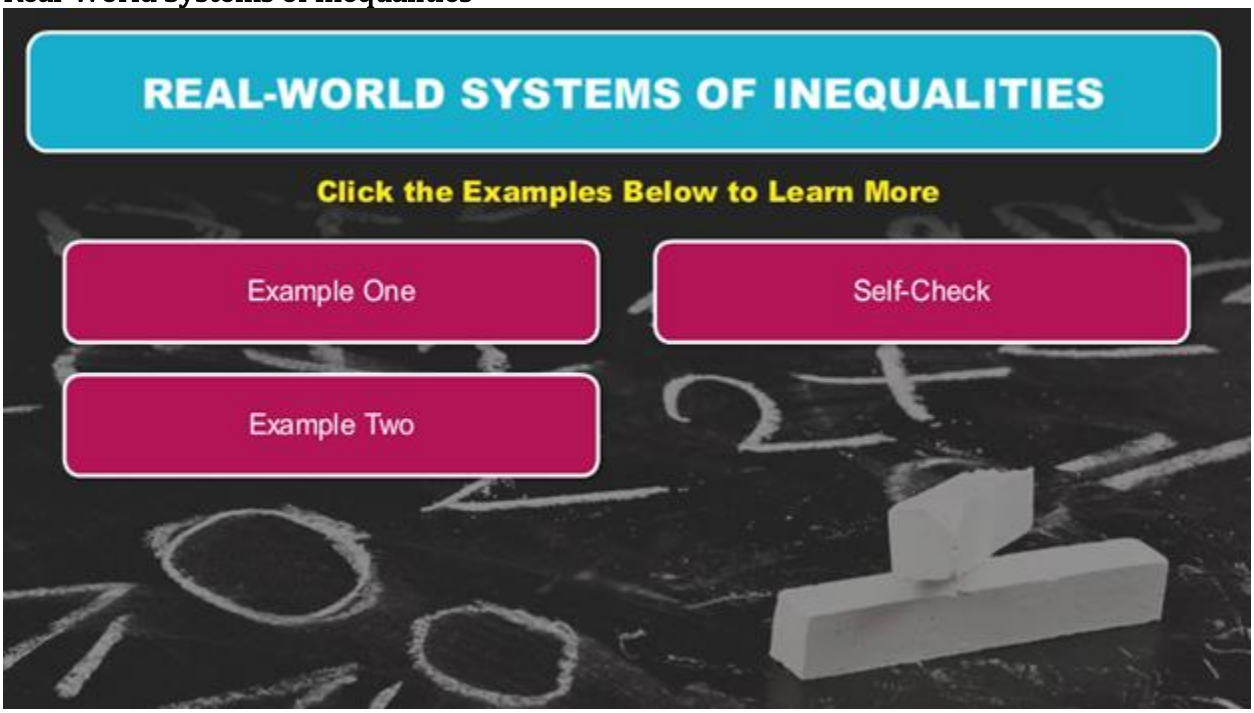


In this lesson you will learn how to solve practical problems involving systems of inequalities. Remember that although practical problems are packed with a lot of information to sort through, you have a three-step process to help you along:

Step 1: Read through the problem
Step 2: Highlight the key information
Step 3: Use the key information to solve the problem

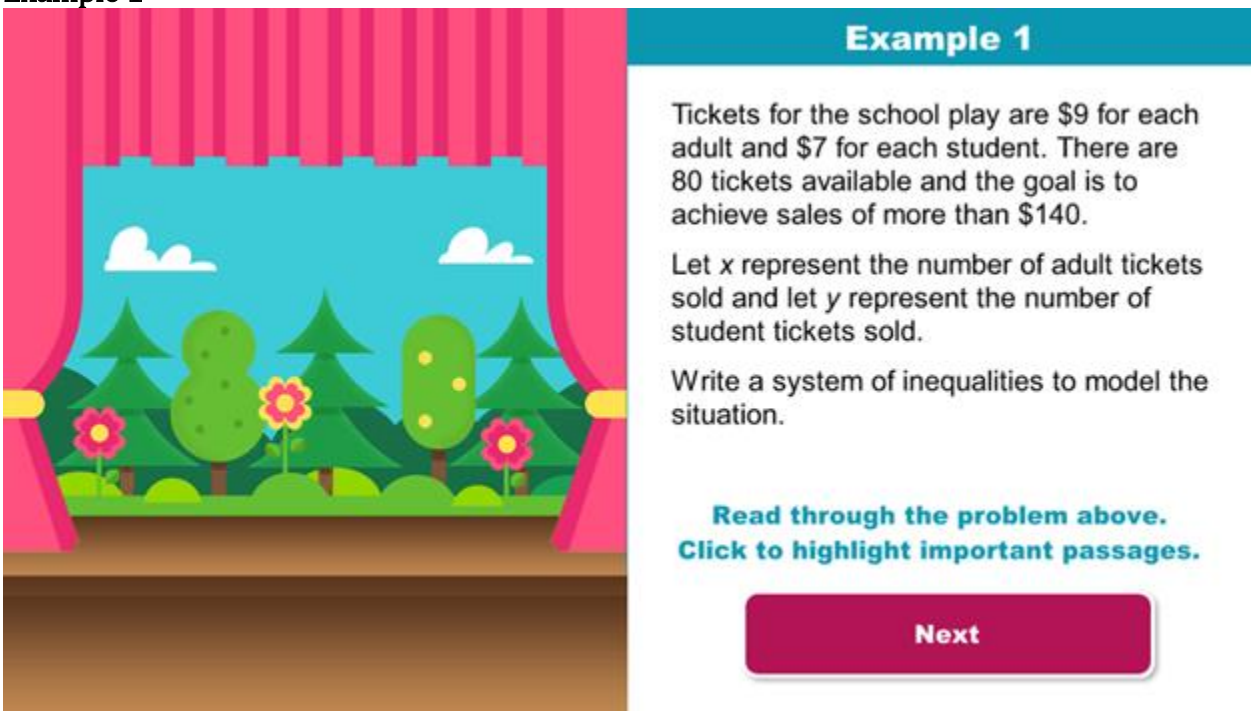
Remember your process as you prepare to start the first example.

Real-World Systems of Inequalities



Click the examples below to learn more.

Example 1



Take a few moments to read though Example 1. Highlight the information you think is necessary to solve the problem.

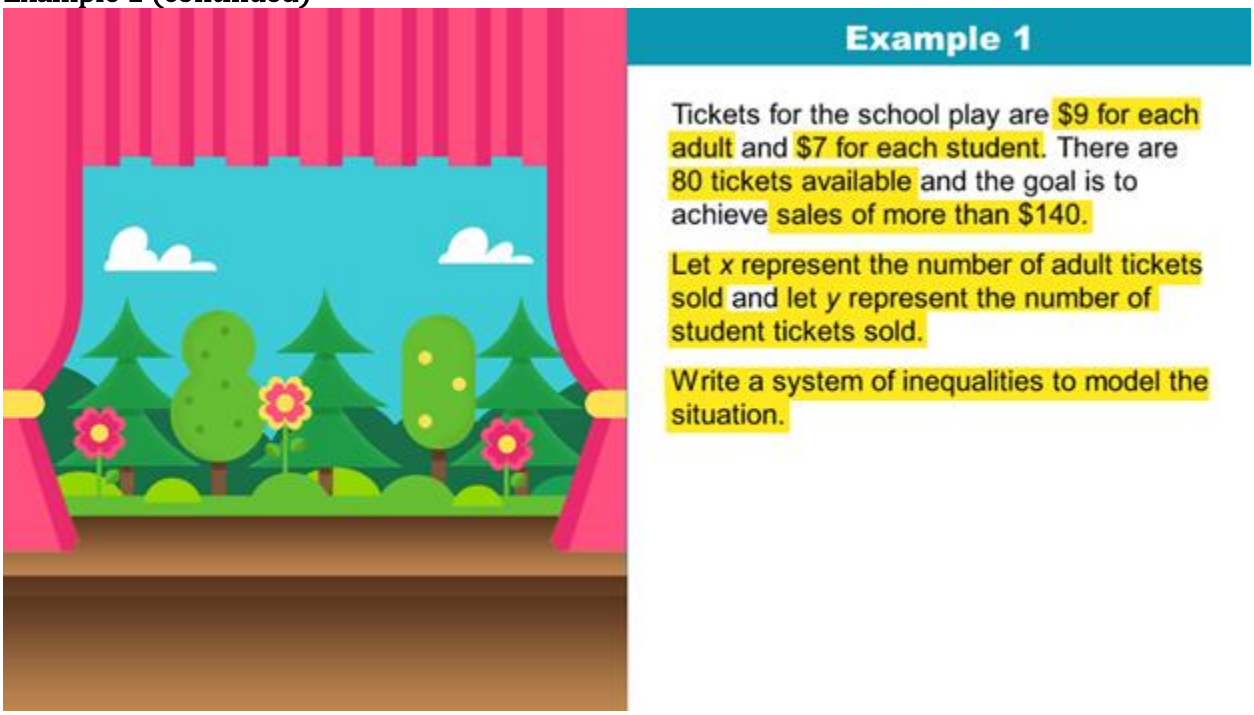
Tickets for the school play are \$9 *for each adult and* \$7 *for each student. There are 80 tickets available and the goal is to achieve sales of more than* \$140*.*

Let x represent the number of adult tickets sold and let y represent the number of student tickets sold.

Write a system of inequalities to model the situation.

Read through the problem above. Click to highlight important passages.

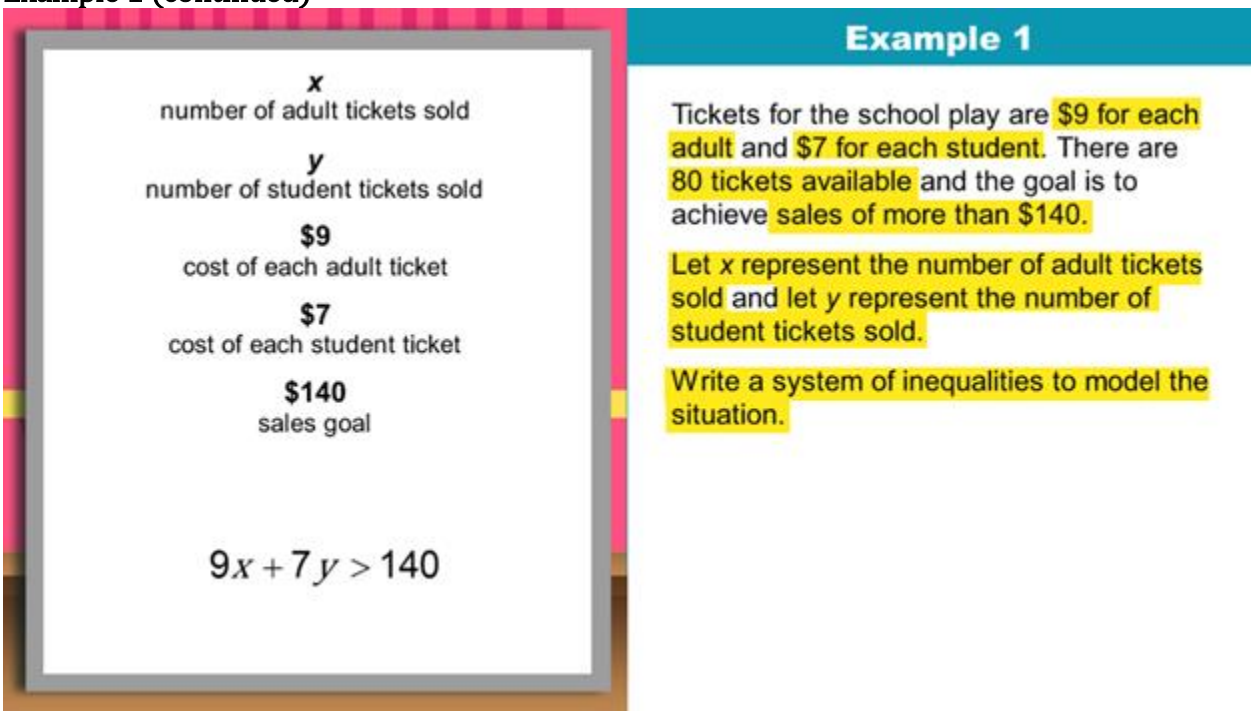
Example 1 (continued)



Did you highlight the important facts?

- Tickets are $9 for each adult and $7 for each student
- There are 80 tickets available
- The goal is to achieve sales of more than $140
- x represents the number of adult tickets sold and y represents the number of student tickets sold
- Write a system of inequalities to model the situation

Example 1 (continued)



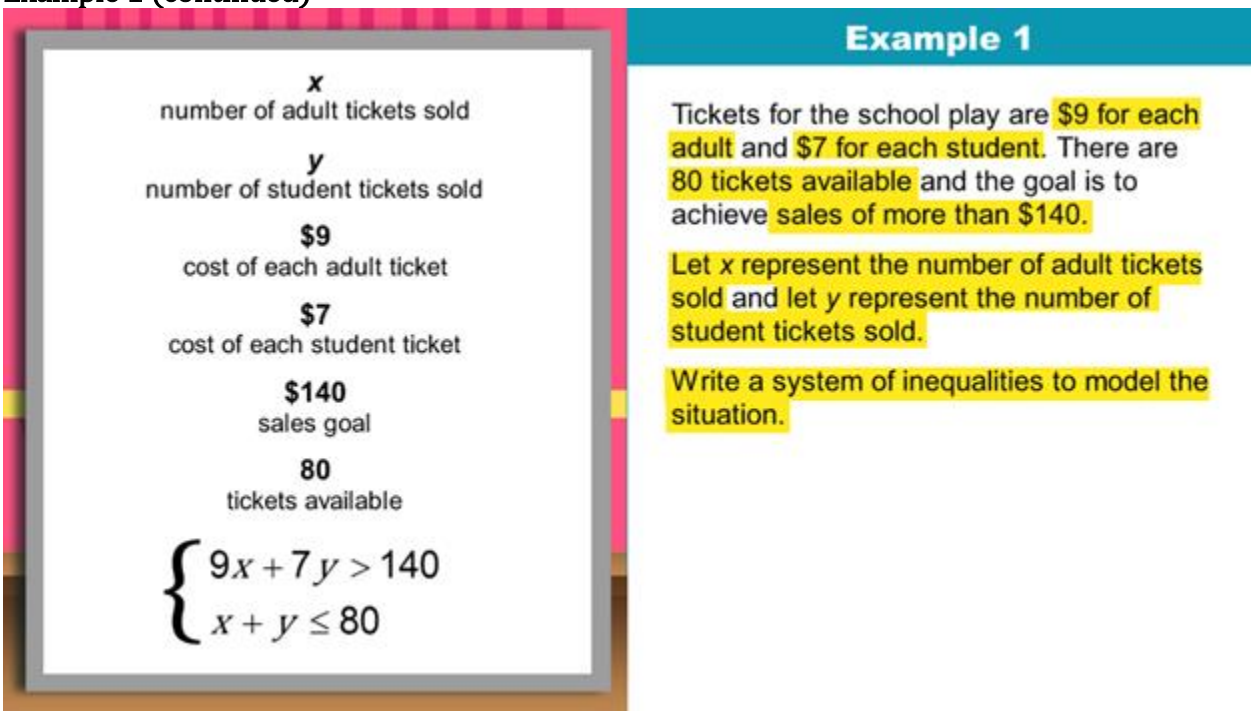
Remember that a system of inequalities is a group of inequalities. Therefore, you will write more than one inequality to model this situation. In this problem, x represents the number of adult tickets sold and y represents the number of student tickets sold. You know that adult tickets are \$9 each and student tickets are \$7 each. You can represent this part of the problem with the expression:

$$9x + 7y$$

You also know that the goal is to achieve sales of more than \$140. So:

$$9x + 7y > 140$$

Example 1 (continued)



Now you have written one inequality to represent part of the given information. In the problem, you are also informed that there are 80 tickets available for sale. So, the school can sell no more than 80 tickets. Therefore:

$$x + y \leq 80$$

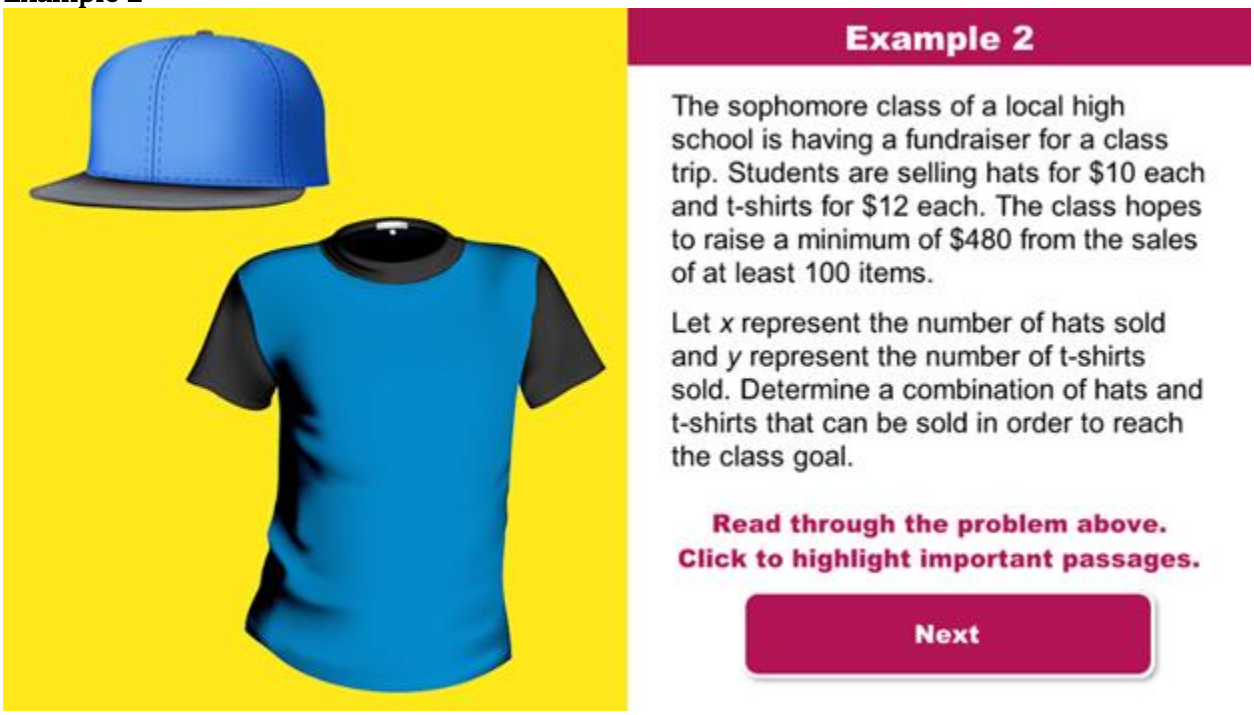
Now, you have written the two inequalities necessary to complete the system:

$$\begin{cases} 9x + 7y > 140 \\ x + y \leq 80 \end{cases}$$

It is worth it mention that the inequalities in a system can be written in any order. If you were to switch the order that the inequalities were written, the system would still correctly model the situation.

$$\begin{cases} x + y \leq 80 \\ 9x + 7y > 140 \end{cases}$$

Example 2



Now take a few moments to read though Example 2. Highlight the key information.

The sophomore class of a local high school is having a fundraiser for a class trip. Students are selling hats for \$10 *each and t-shirts for* \$12 *each. The class hopes to raise a minimum of* \$480 *from the sales of at least 100 items.*

Let x represent the number of hats sold and y represent the number of t-shirts sold. Determine a combination of hats and t-shirts that can be sold in order to reach the class goal.

Read through the problem above. Click to highlight important passages.

Example 2 (continued)

Example 2

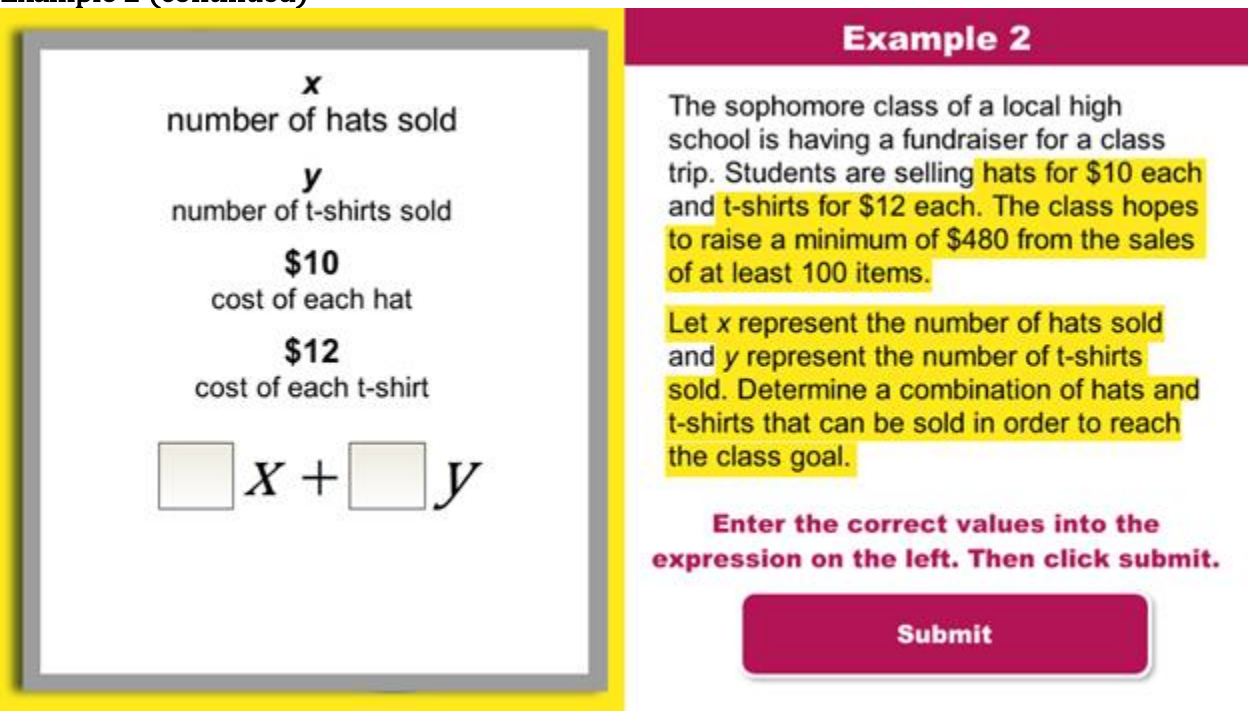
The sophomore class of a local high school is having a fundraiser for a class trip. Students are selling hats for $10 each and t-shirts for $12 each. The class hopes to raise a minimum of $480 from the sales of at least 100 items.

Let x represent the number of hats sold and y represent the number of t-shirts sold. Determine a combination of hats and t-shirts that can be sold in order to reach the class goal.

Did you highlight these important facts?

- The sophomore class is selling hats for $10 each and t-shirts for $12 each
- The goal is to raise a minimum of $480 from the sales of at least 100 items
- Determine a combination of hats and t-shirts that can be sold in order to reach the goal

Example 2 (continued)



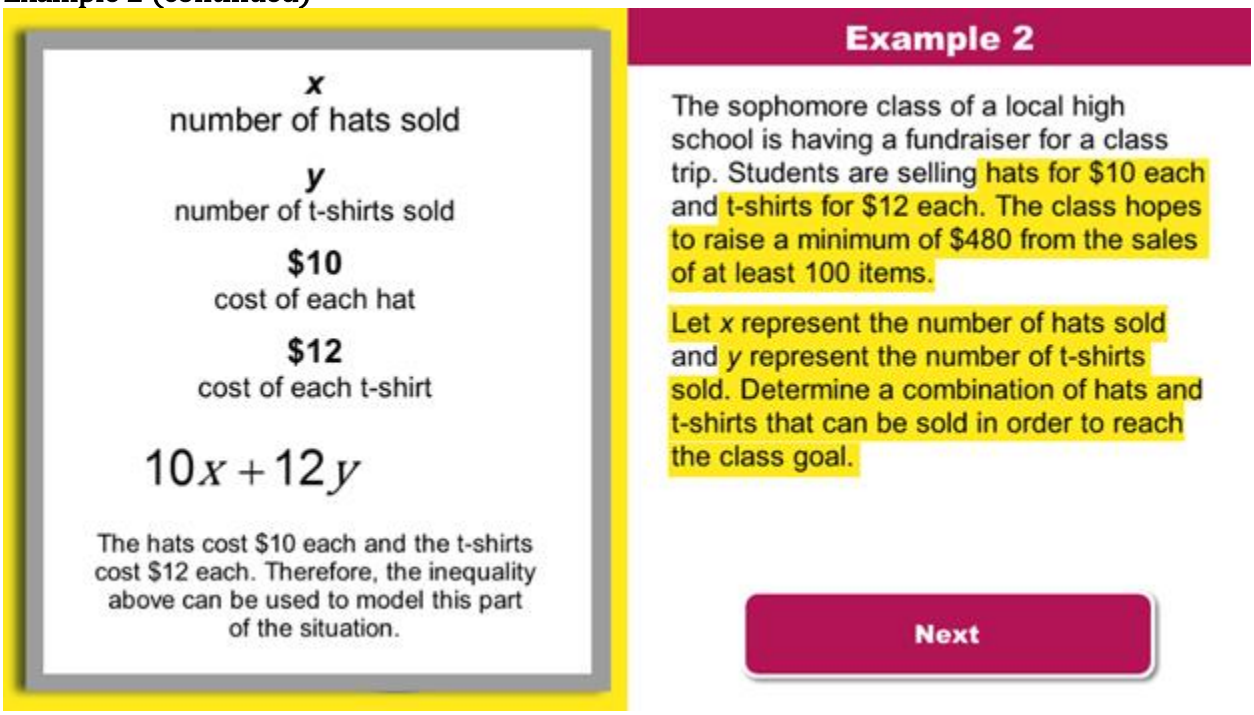
In order to determine a combination of hats and t-shirts that can be sold to reach the goal, you will need to begin by writing a system to model the situation. Then, you can use the graphing calculator to identify a solution.

You know that x represents the number of hats sold and y represents the number of t-shirts sold. You also know that the hats are sold for $10 each and t-shirts for $12 each. So you can model this part of the situation by using the expression:

$$?x+?y$$

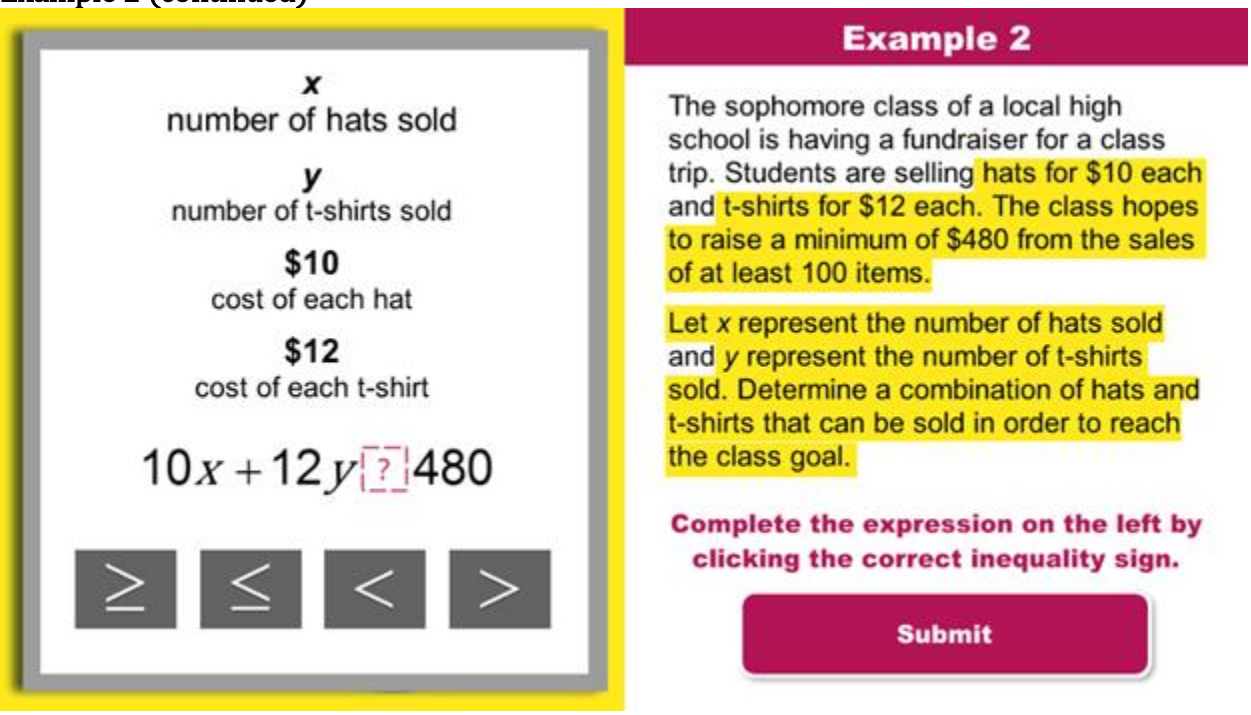
Enter the correct values into the expression on the left. Then click submit,

Example 2 (continued)



The hats cost \$10 each and the t-shirts cost \$12 each. Therefore, $10x + 12y$ can be used to model this part of the situation.

Example 2 (continued)

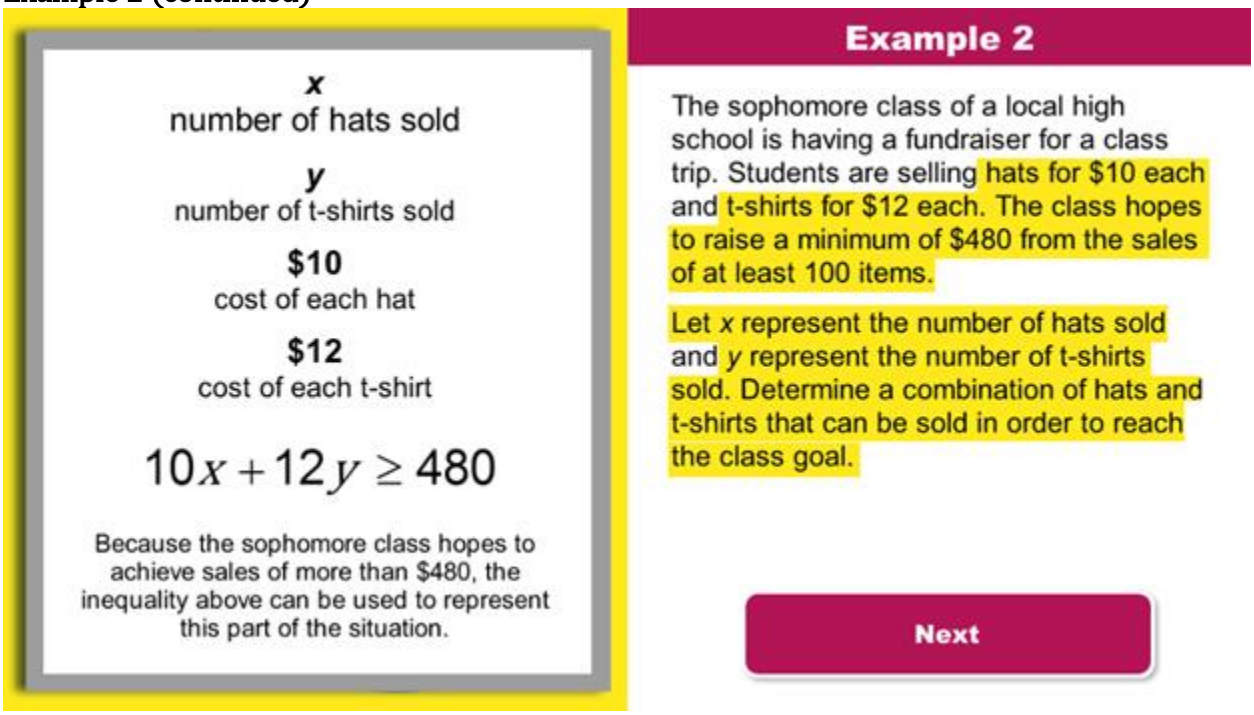


You also know that the sophomore class hopes to achieve a minimum of $480 in sales. Therefore?

$$10x + 12y \quad ? \quad 480$$

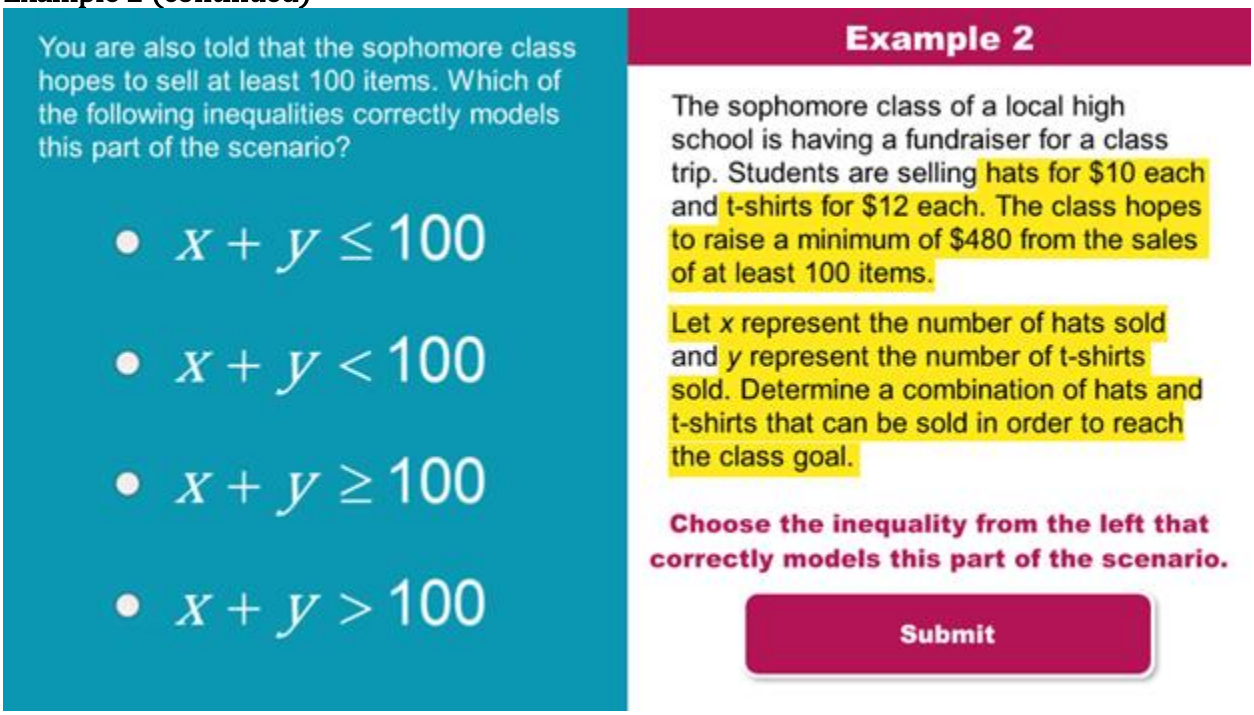
Complete the expression on the left by click the correct inequality sign.

Example 2 (continued)



Because the sophomore class hopes to achieve sales of more than \$480, the inequality $10x + 12y \geq 480$ can be used to represent this part of the situation.

Example 2 (continued)

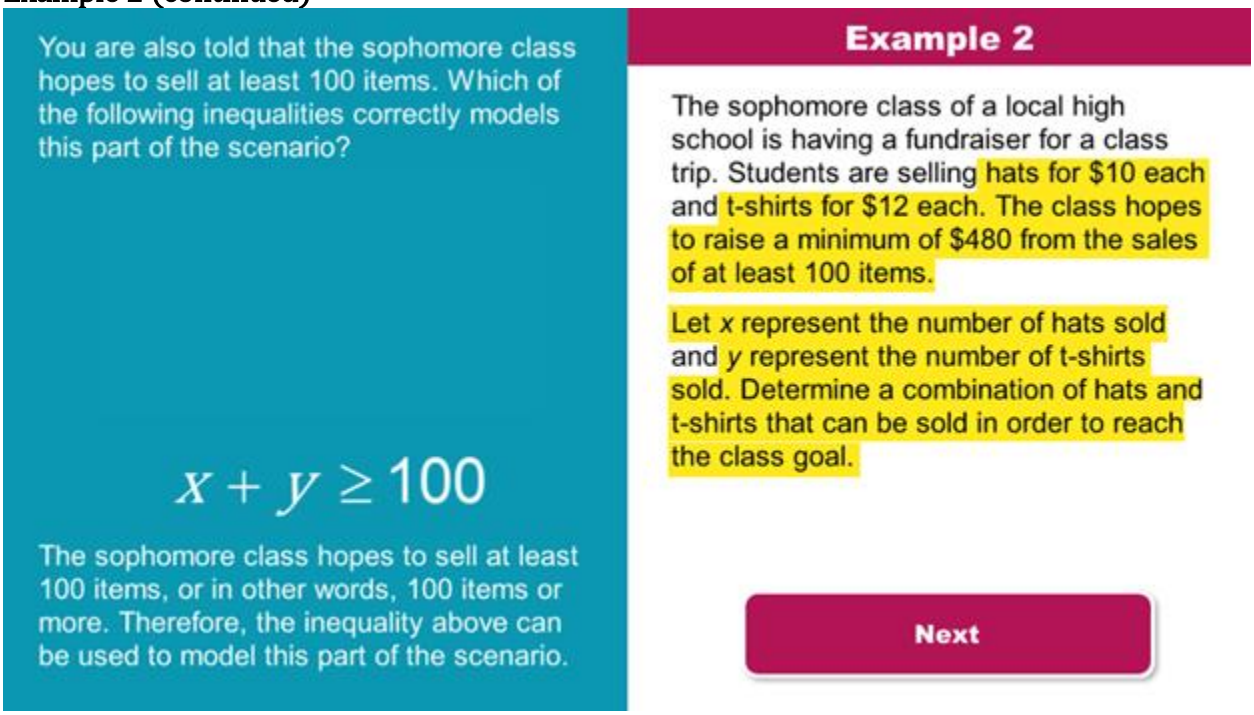


You are also told that the sophomore class hopes to sell at least 100 items. Which of the following inequalities correctly models this part of the scenario?

A) $x + y \leq 100$
B) $x + y < 100$
C) $x + y \geq 100$
D) $x + y > 100$

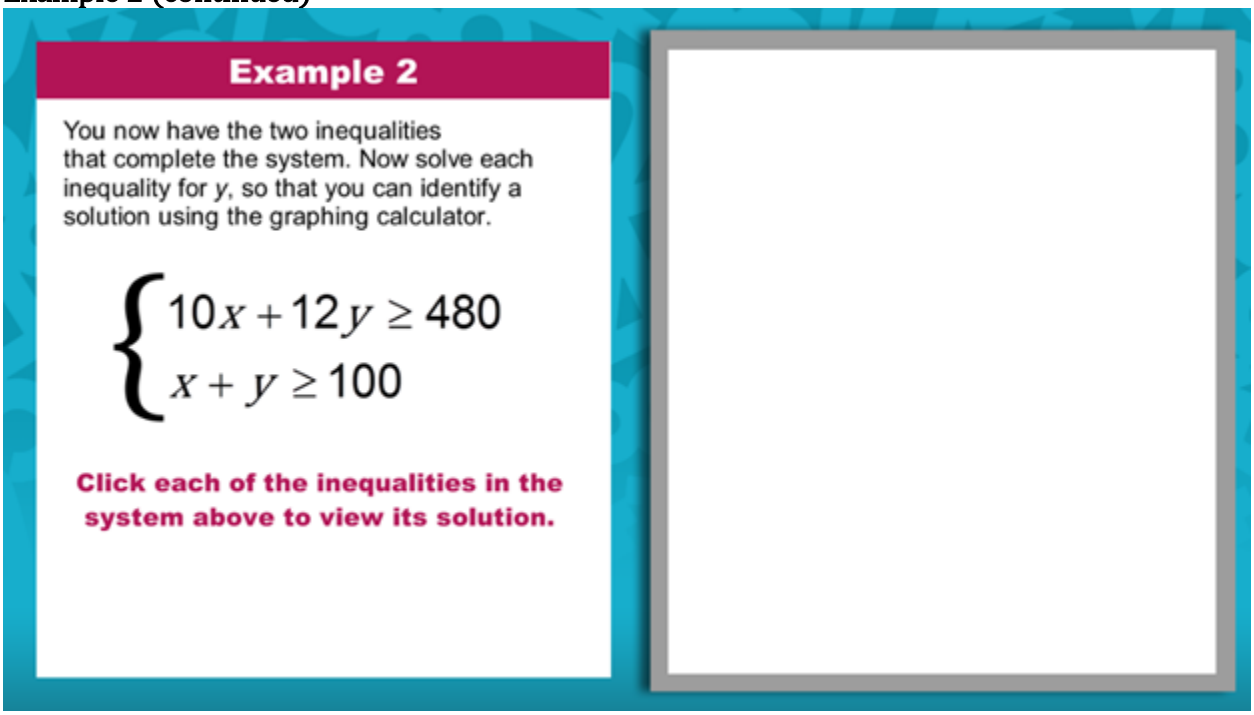
Choose the inequality from the left that correctly models this part of the scenario.

Example 2 (continued)



The sophomore class hopes to sell at 100 items, or in other words 100 items or more. Therefore, $x + y \geq 100$ can be used to model this part of the scenario.

Example 2 (continued)



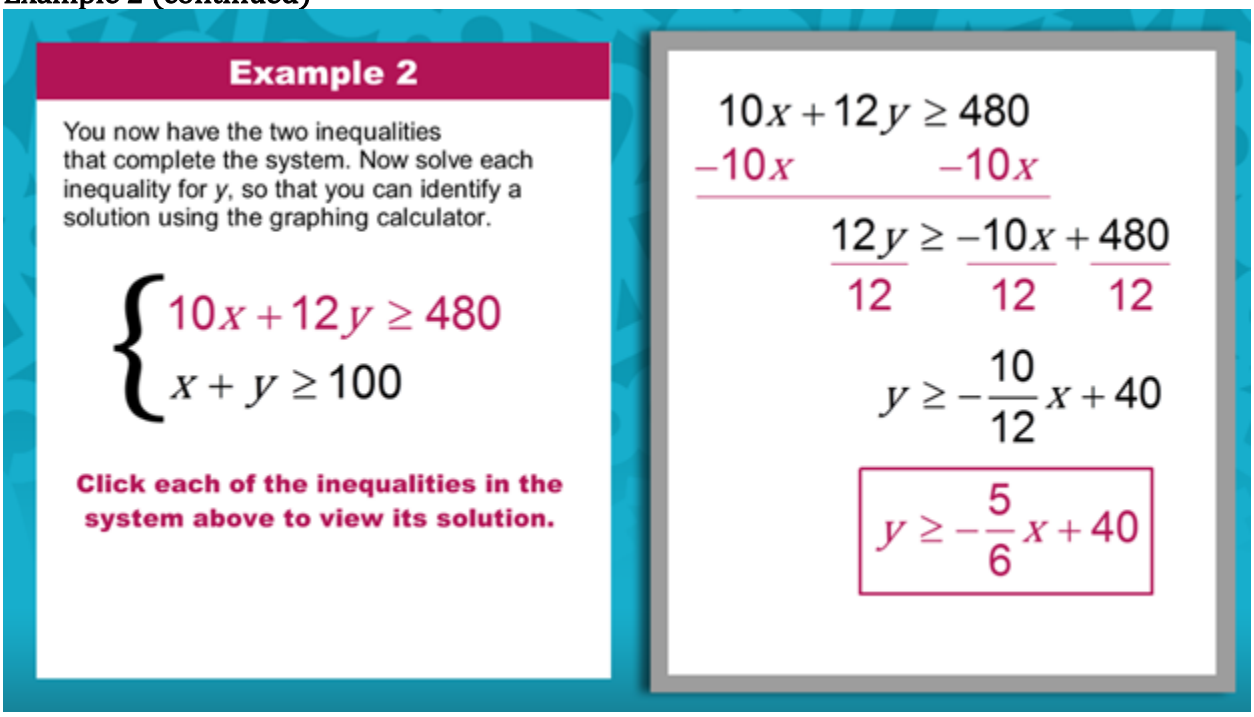
You now have the two inequalities that complete the system:

$$\begin{cases} 10x + 12y \geq 480 \\ x + y \geq 100 \end{cases}$$

Now solve each inequality for y, so that you can identify a solution using the graphing calculator.

Click each of the inequalities in the system above to view its solution.

Example 2 (continued)



$10x + 12y \geq 480$

$-10x \qquad -10x$

Subtract $10x$ from both sides.

$\frac{12y}{12} \geq \frac{-10x}{12} + \frac{480}{12}$

Divide each term by 12.

$y \geq -\frac{10}{12}x + 40$

$-\frac{10}{12}$ can be reduced to $-\frac{5}{6}$

$y \geq -\frac{5}{6}x + 40$

$x + y \geq 100$

$-x \qquad -x$

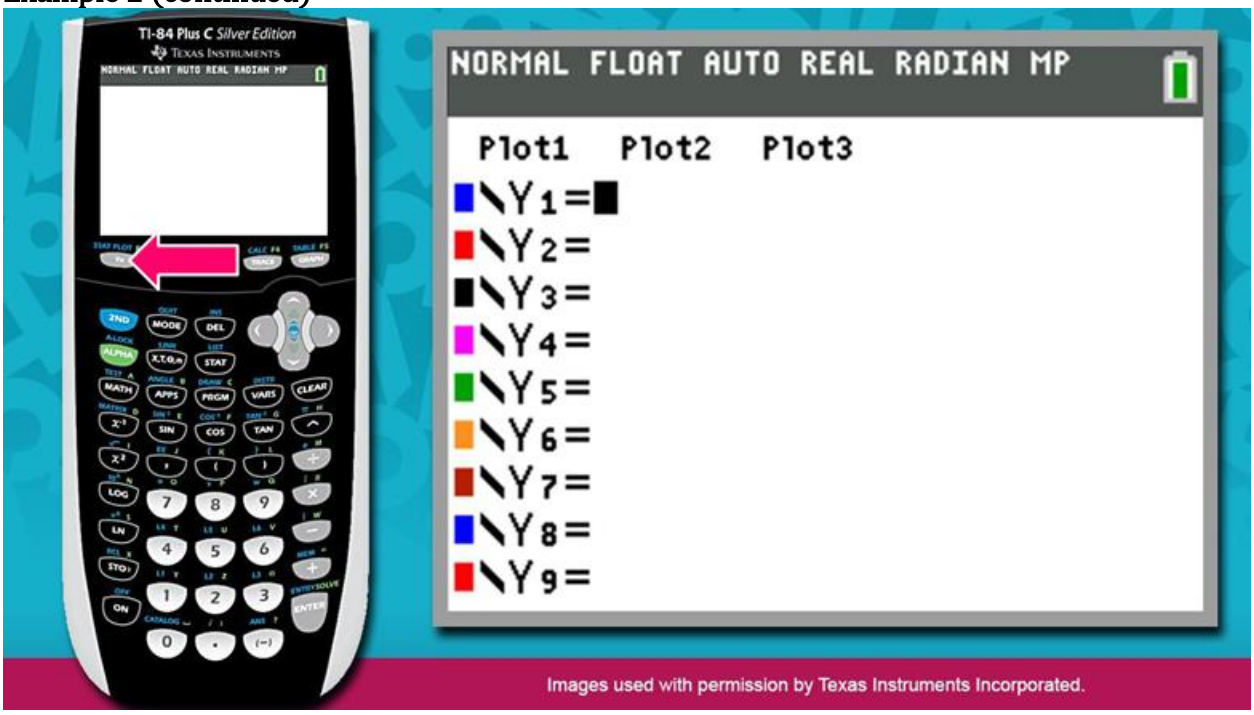
Subtract x from both sides.

$y \geq -x + 100$

After solving each inequality for y, the results are:

$$y \geq -\frac{5}{6}x + 40$$
$$y \geq -x + 100$$

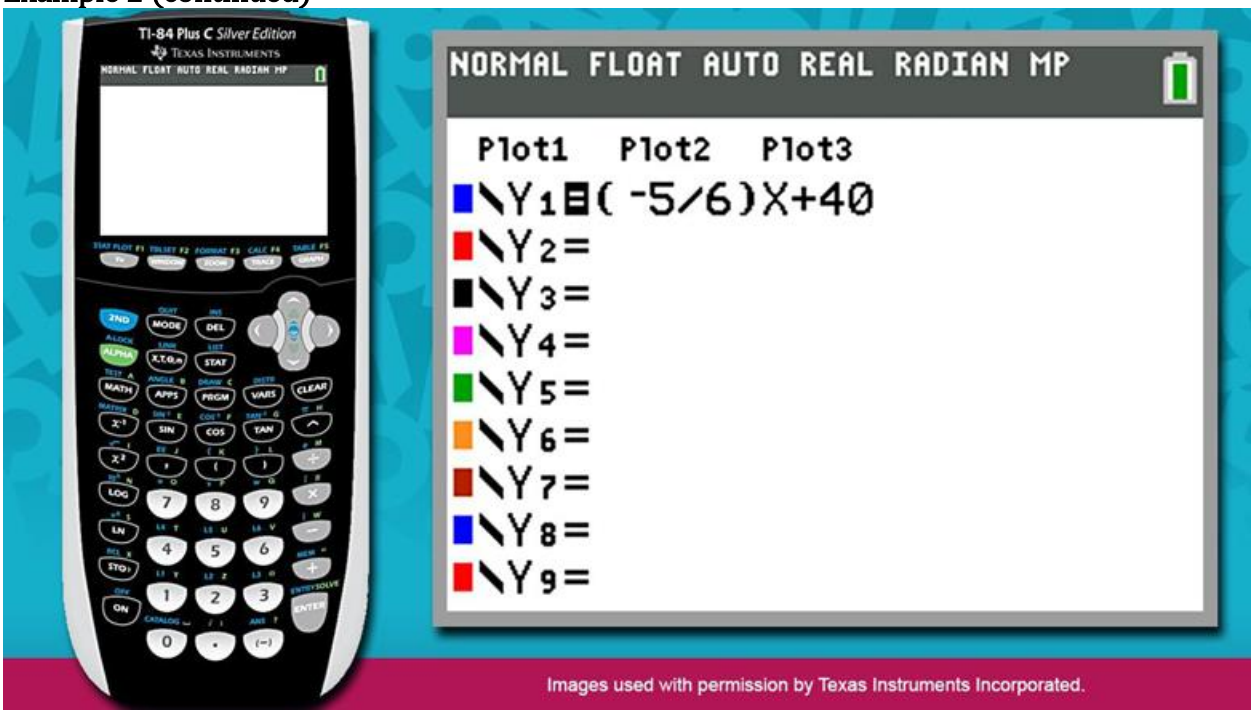
Example 2 (continued)



Now that you have solved each inequality for y, you can use the graphing calculator to identify a solution. Begin by clearing the calculator's memory.

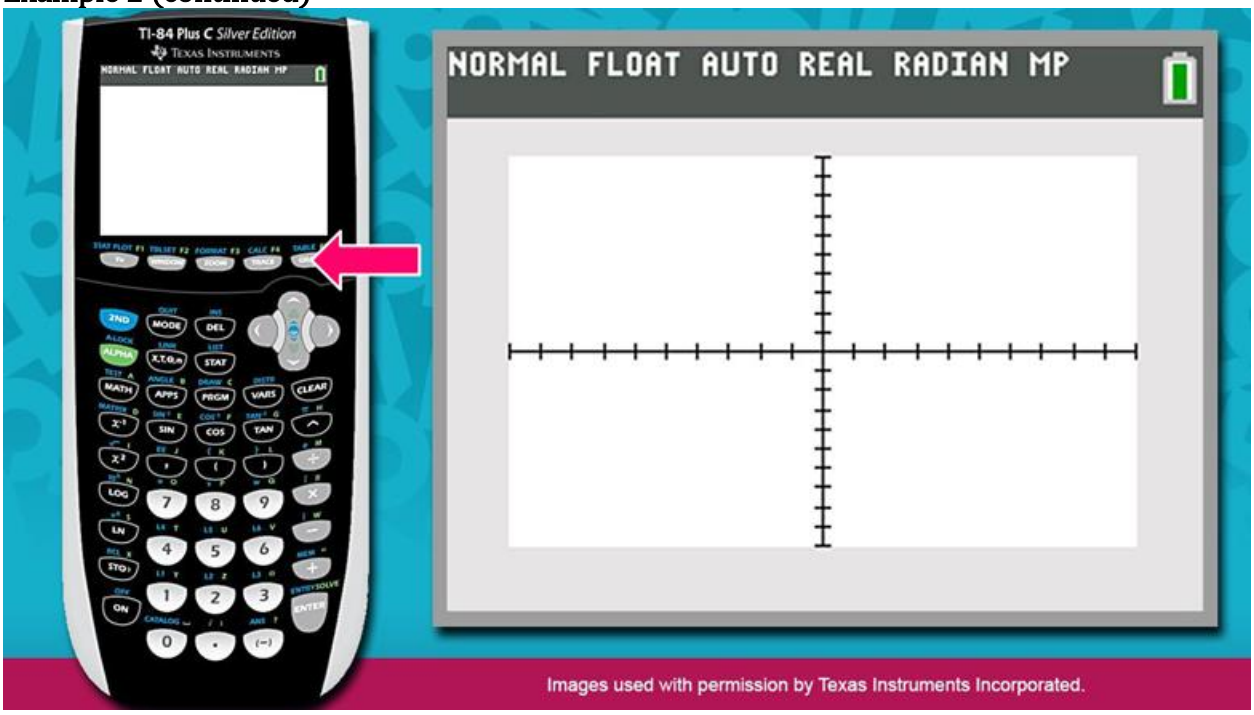
Press Y=

Example 2 (continued)



Now enter the right side of the first inequality to the right of Y_1. Remember when entering a fraction, you should use parentheses.

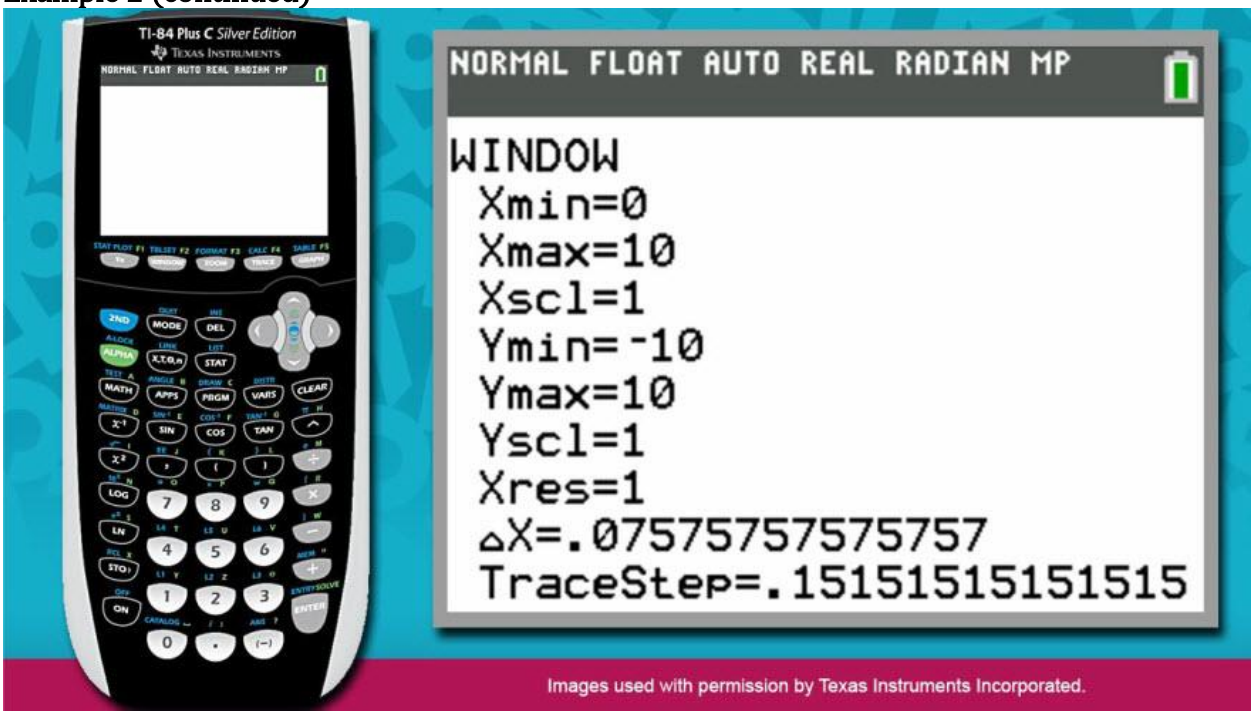
Example 2 (continued)



Press GRAPH

You will immediately notice that you don't see a line appear on the calculator's screen. This is a signal that the values along the x- and y-axes are not large enough for the line to appear. You will need to adjust the scale on the x- and y-axes. This is the time for you to consider what values of x and y make sense for this real-world scenario.

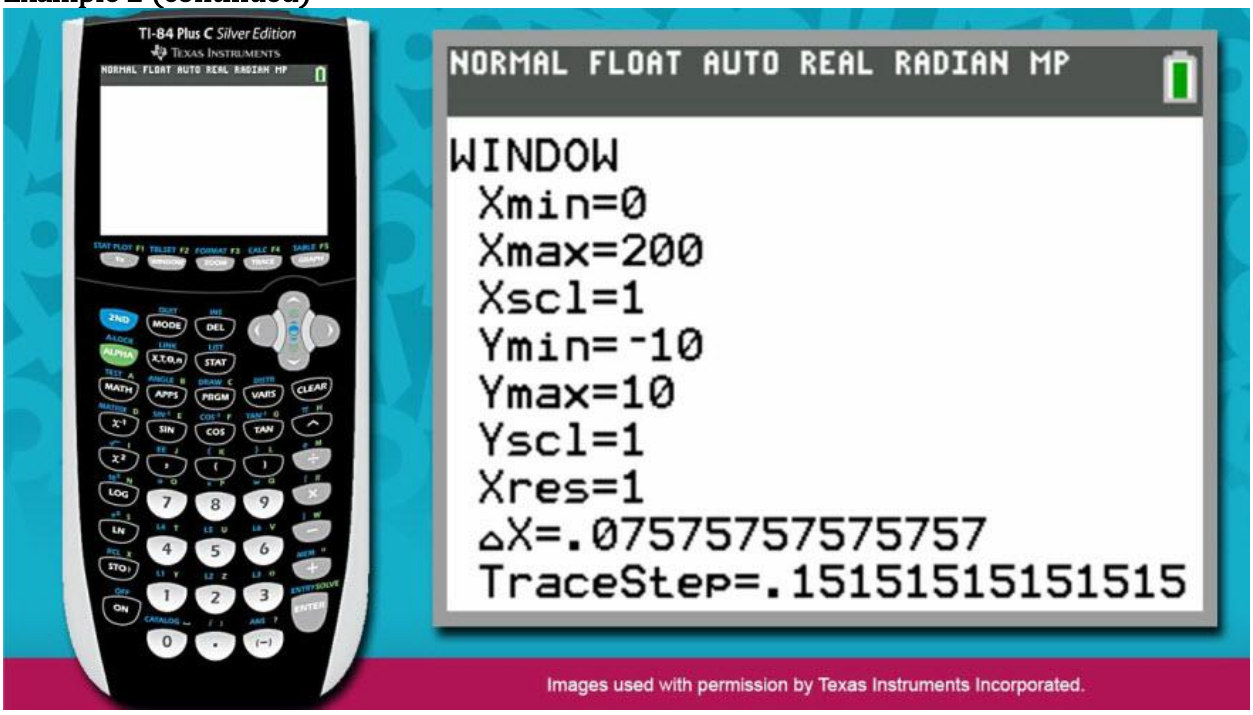
Example 2 (continued)



Press WINDOW

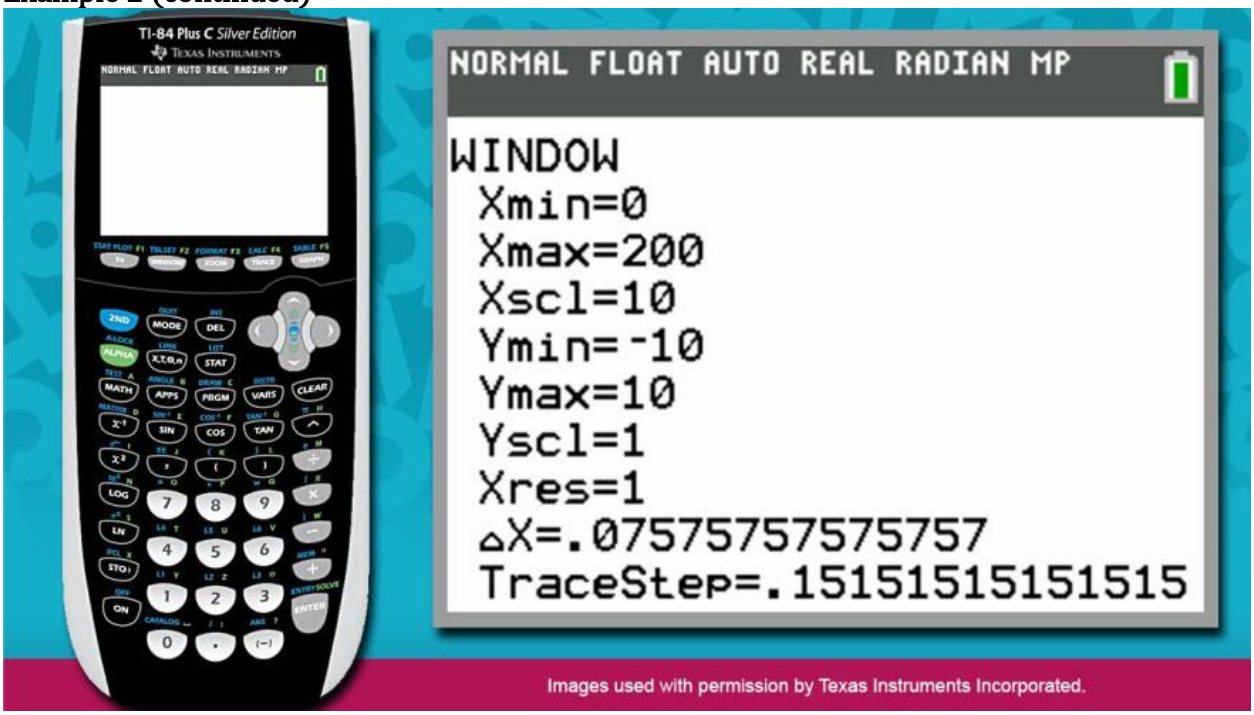
Because x represents the number of hats sold, negative x-values will not make sense. It is impossible to sell a negative number of hats. The smallest number of hats that can be sold is 0. So set the minimum x-value to 0.

Example 2 (continued)



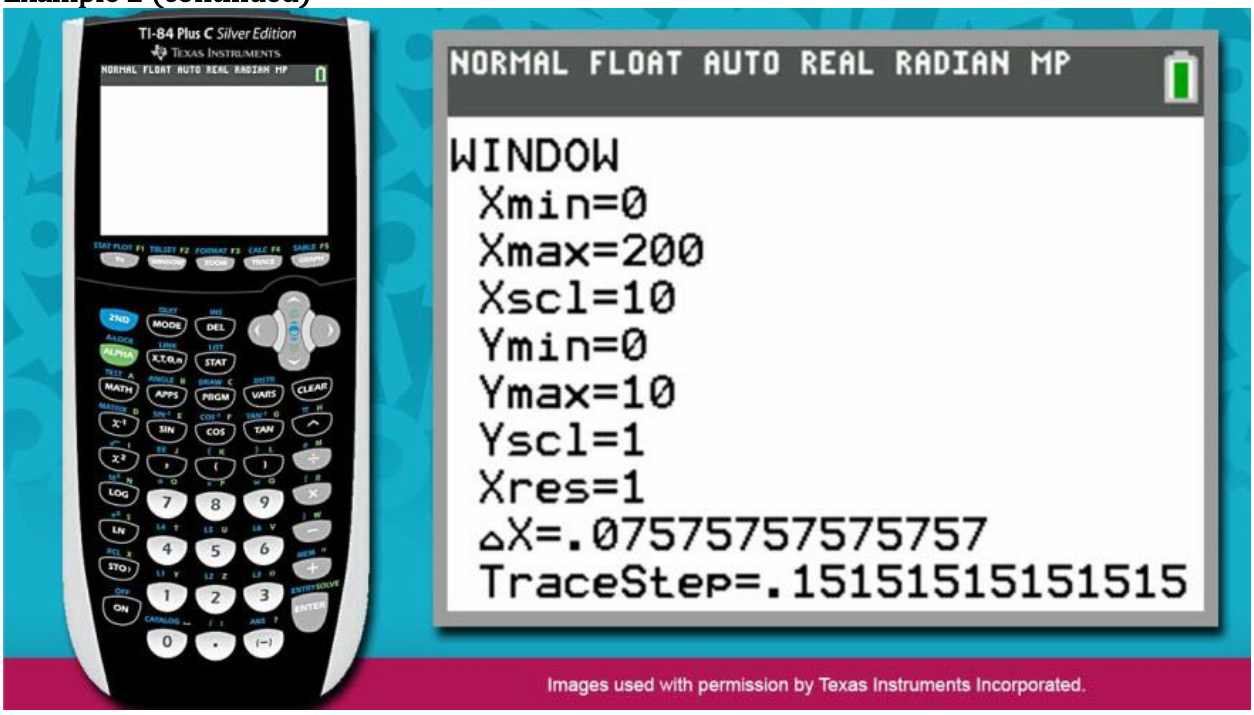
You aren't told how many hats, in total, are available for sale. So you can't be certain exactly what the maximum x-value should be. You will have to reason your way through this part, a little. You know that the goal is to sell at least 100 hats and t-shirts, in total. So set the x-max to a value more than 100; say 200. If you notice that this value is too small or too large, you can change it later.

Example 2 (continued)



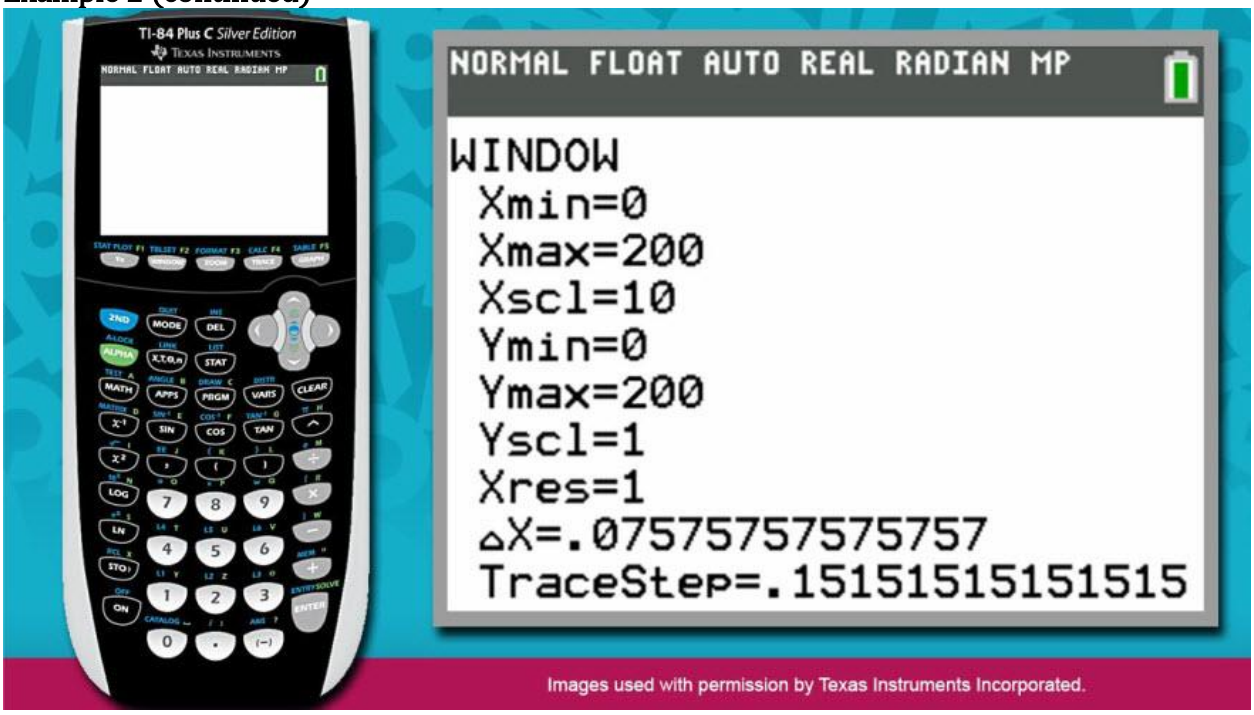
Because the x-min is set to 0 and the x-max is set to 200, it is appropriate to increase the x-scale, so that you will have a good view of the line once it is graphed. Change the x-scale from 1 to 10. Now, there will be 10 units between consecutive tick marks along the x-axis.

Example 2 (continued)



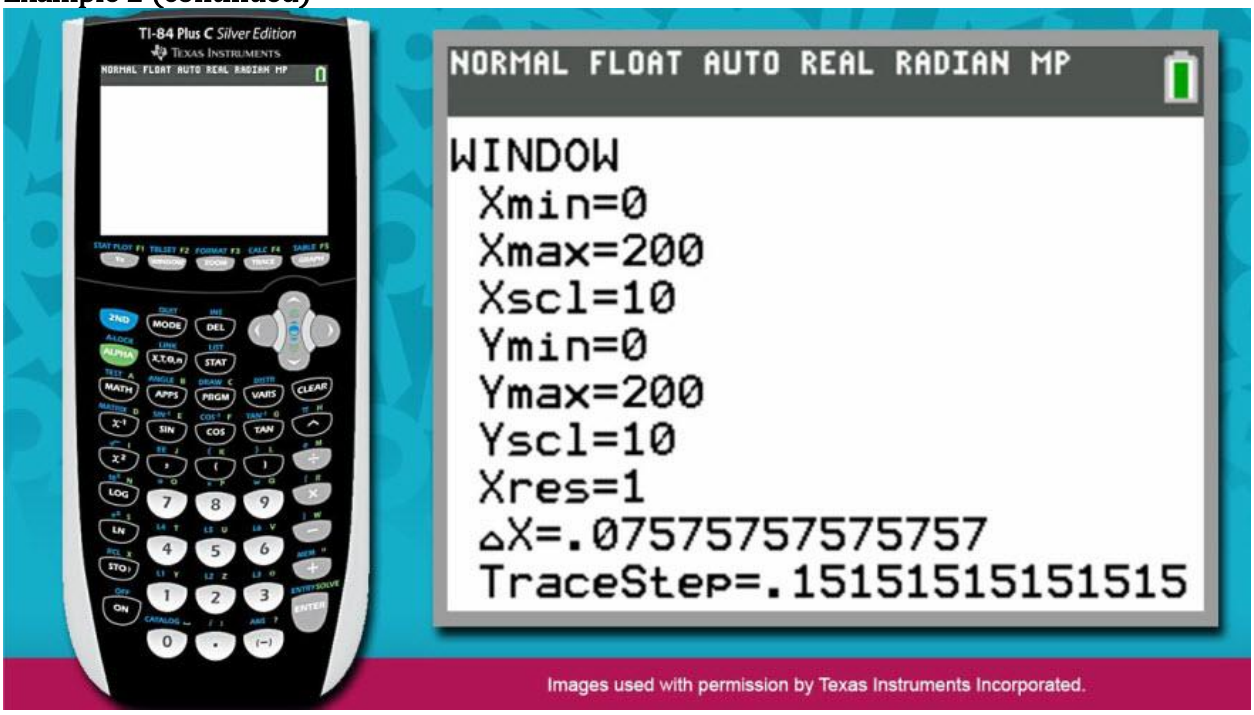
Now move on to adjust the values on the y-axis. Because y represents the number of t-shirts sold, negative y-values will not make sense. It is impossible to sell a negative number of t-shirts. The smallest number of t-shirts that can be sold is 0. So set the y-min to 0.

Example 2 (continued)



You aren't told how many t-shirts, in total, are available for sale. So you can't be certain exactly what the maximum y-value should be. You do know, however, that the goal is to sell at least 100 hats and t-shirts, in total. So set the y-max to a value more than 100; say 200. If you notice that this value is too small or too large, you can change it later.

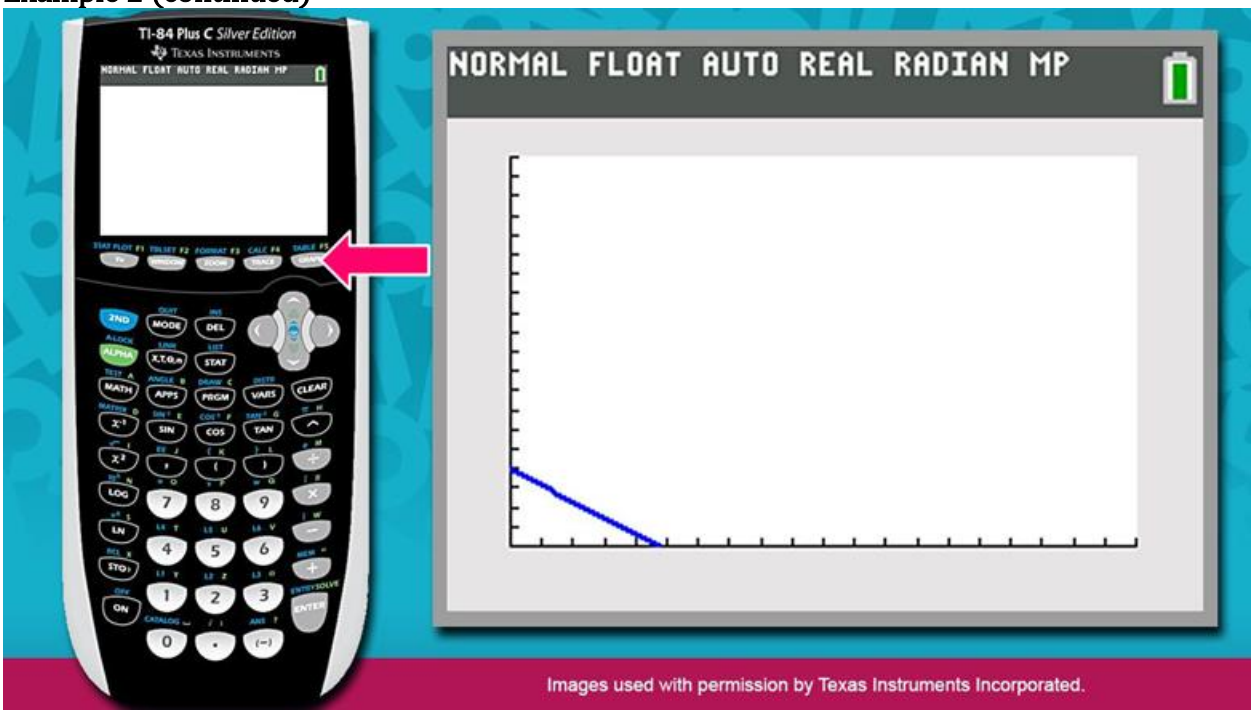
Example 2 (continued)



Because the y-min is set to 0 and the y-max is set to 200, it is appropriate to increase the y-scale, so that you will have a good view of the line once it is graphed. Change the y-scale from 1 to 10. Now, the y-axis will count off by tens, just as the x-axis.

Leave the remaining values as they are.

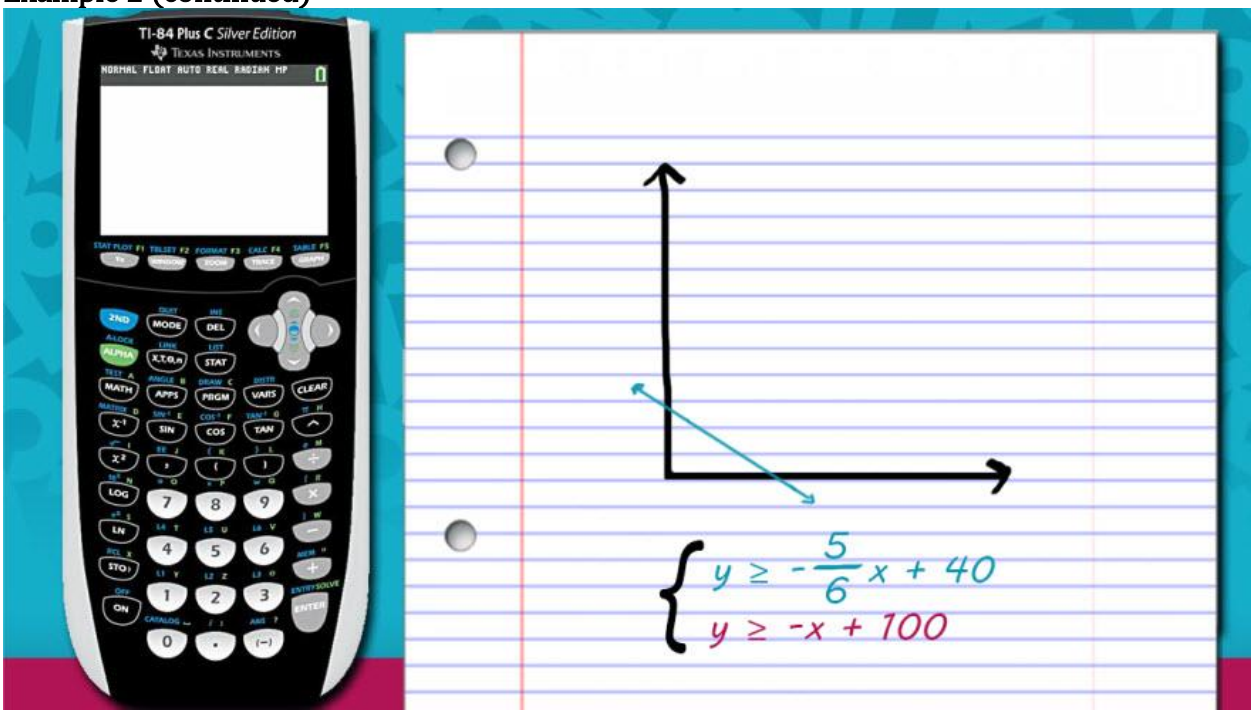
Example 2 (continued)



Press GRAPH

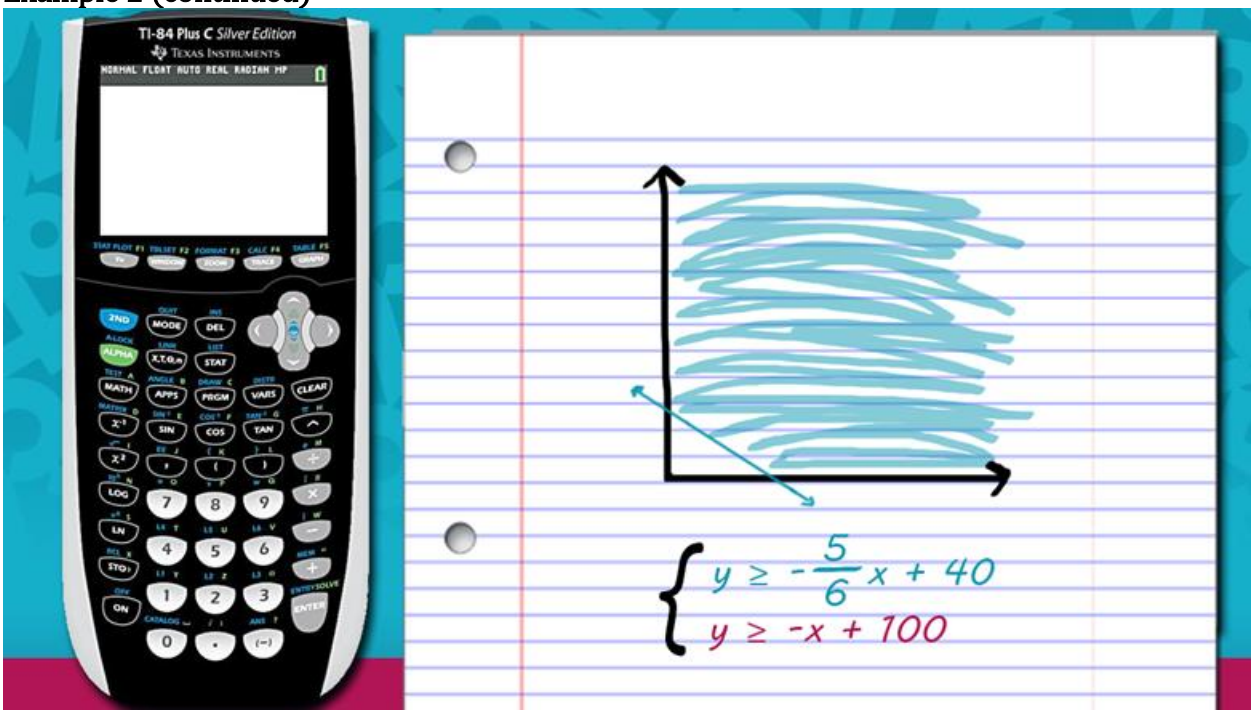
Notice that the calculator only shows the portion of the line included in the first quadrant. The minimum x-value is 0 and the maximum x-value is 200. The x-axis counts off by tens, beginning at 0 and ending at 200. The same pattern is true of the values along the y-axis.

Example 2 (continued)



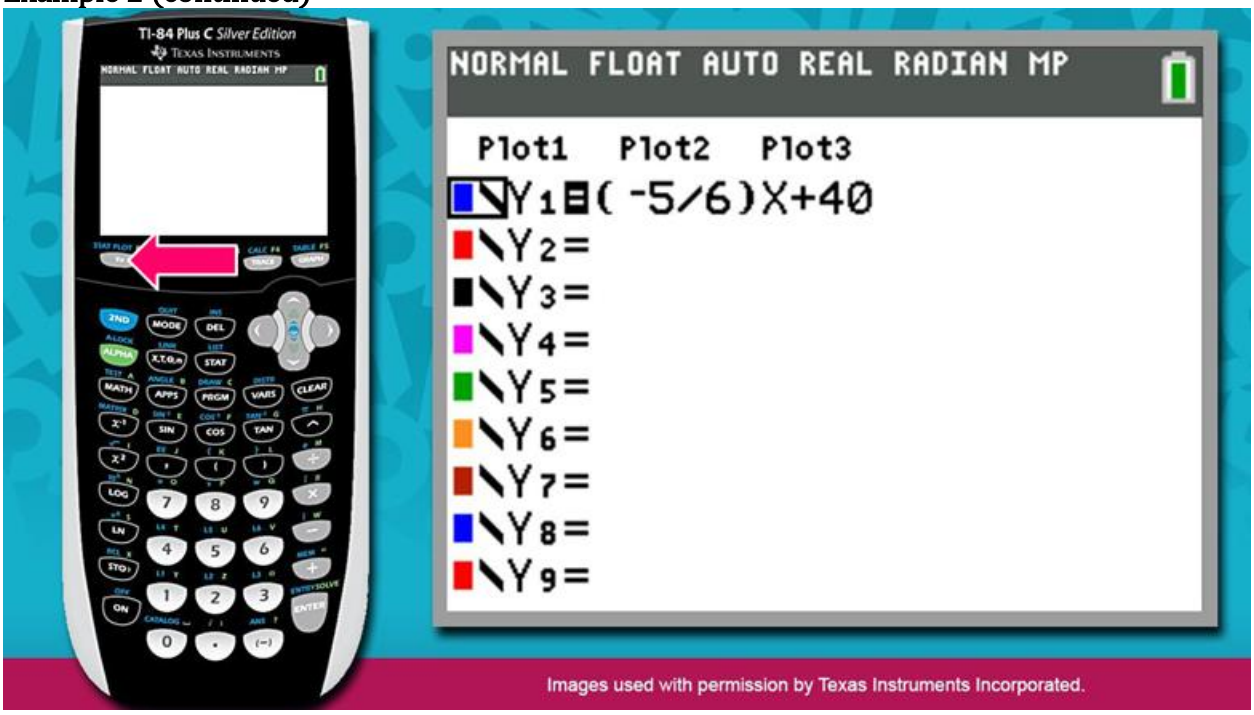
On a piece of paper, prepare a rough sketch of the graph. When roughly sketching a system of inequalities, you may find it helpful to use a different colored pencil for each inequality. Remember that the inequality $y \geq -\frac{5}{6}x + 40$ is a non-strict inequality. So your rough sketch should include a solid boundary line.

Example 2 (continued)



Now that you have roughly sketched the boundary line, you can move on to handle the shading. Because the inequality includes, ≥, you will need to shade above the boundary line. On your rough sketch, shade the region above the boundary line.

Example 2 (continued)

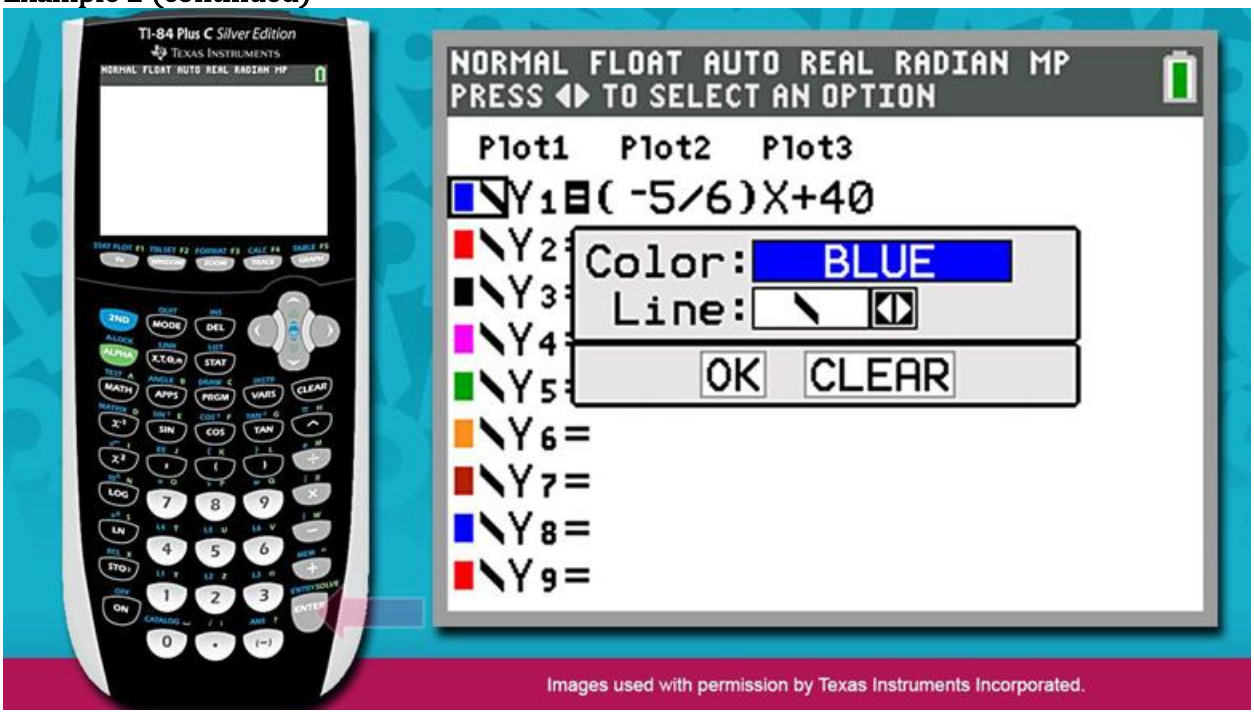


To include the shading on the calculator, press the following keys:

Press Y=

Then press the left arrow until the cursor is blinking on the line beside Y_1.

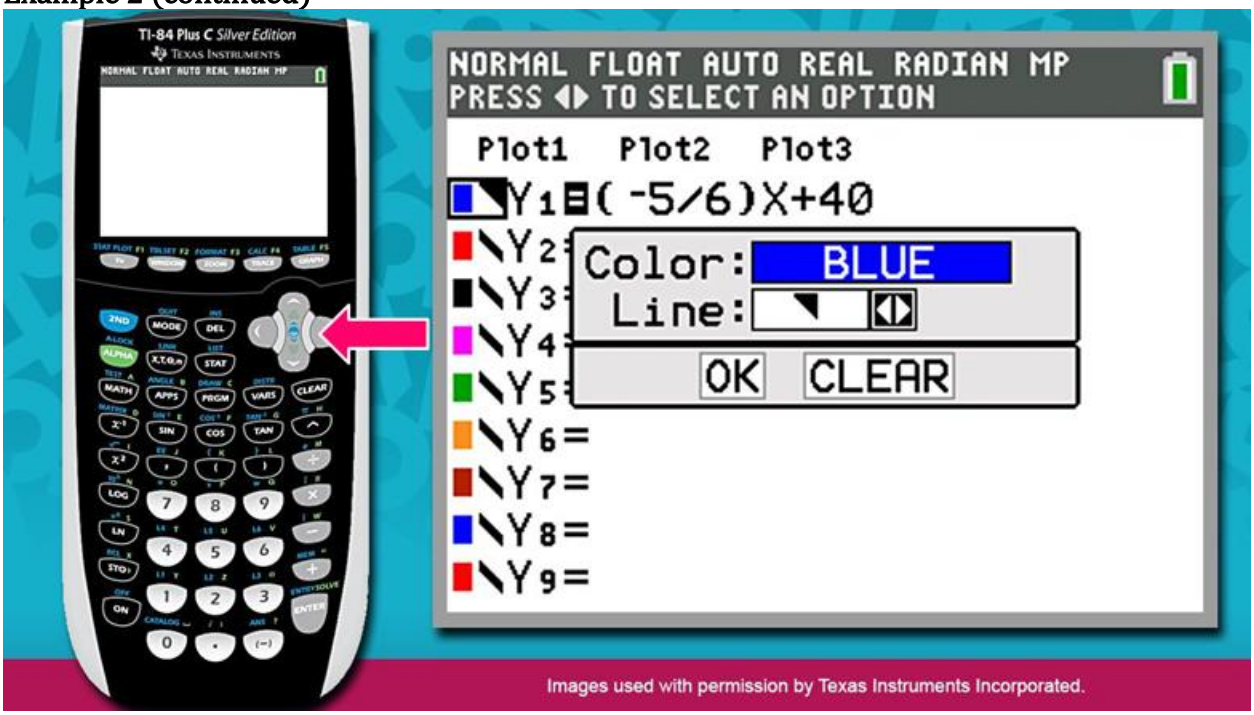
Example 2 (continued)



Press enter

Then press the down arrow until the cursor is blinking beside the “Line” prompt.

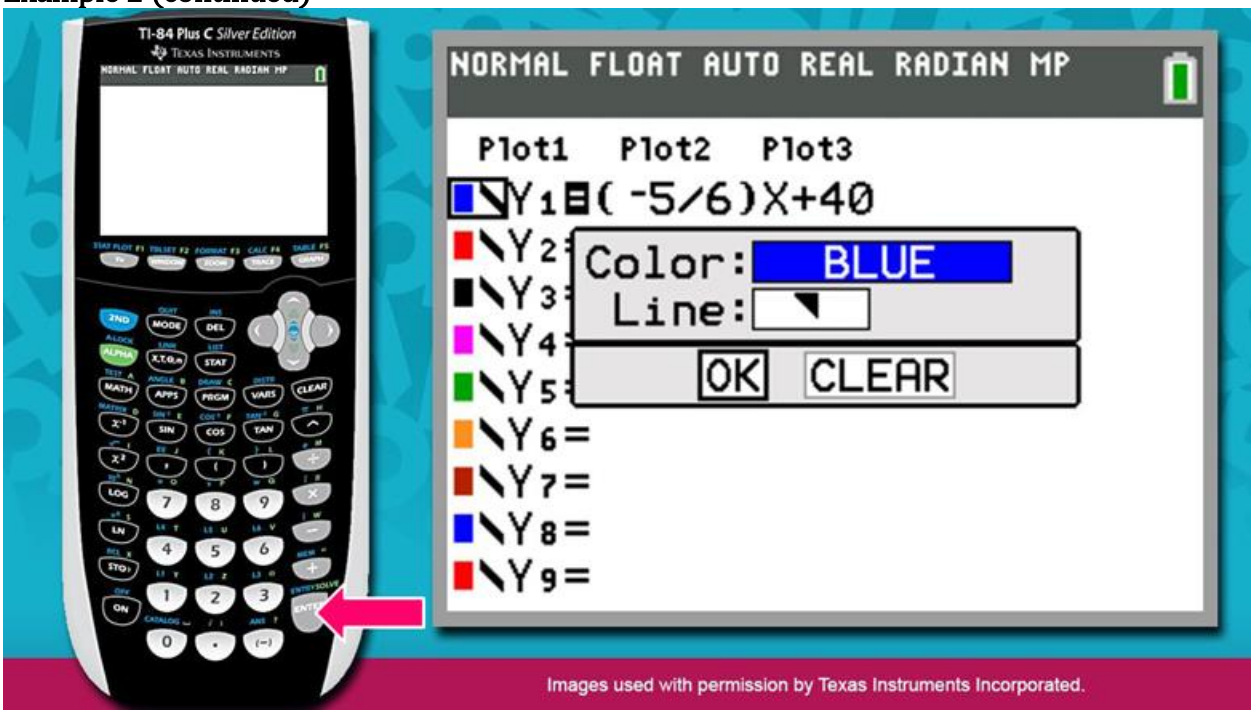
Example 2 (continued)



Press the right arrow until the option for shading above the line appears.

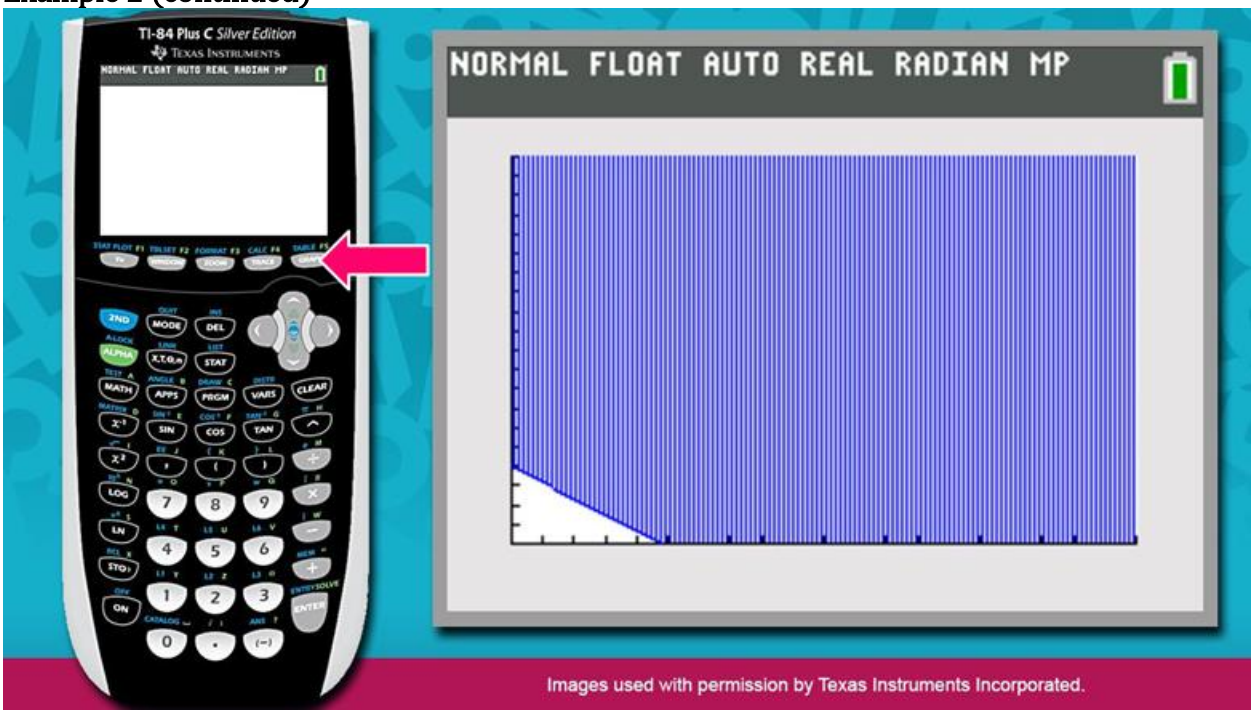
Press enter to select the shading option.

Example 2 (continued)



Then press enter again once the cursor moves to OK

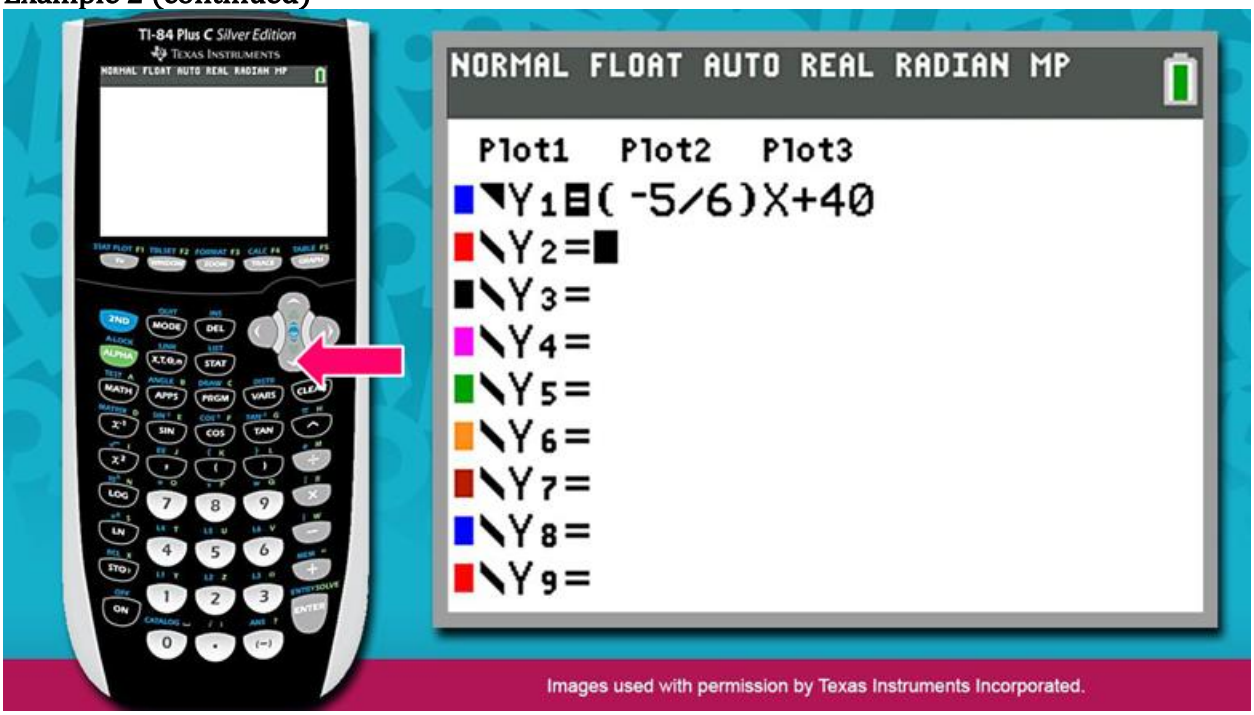
Example 2 (continued)



Now press GRAPH.

The graph of the first inequality will appear with the appropriate shading.

Example 2 (continued)

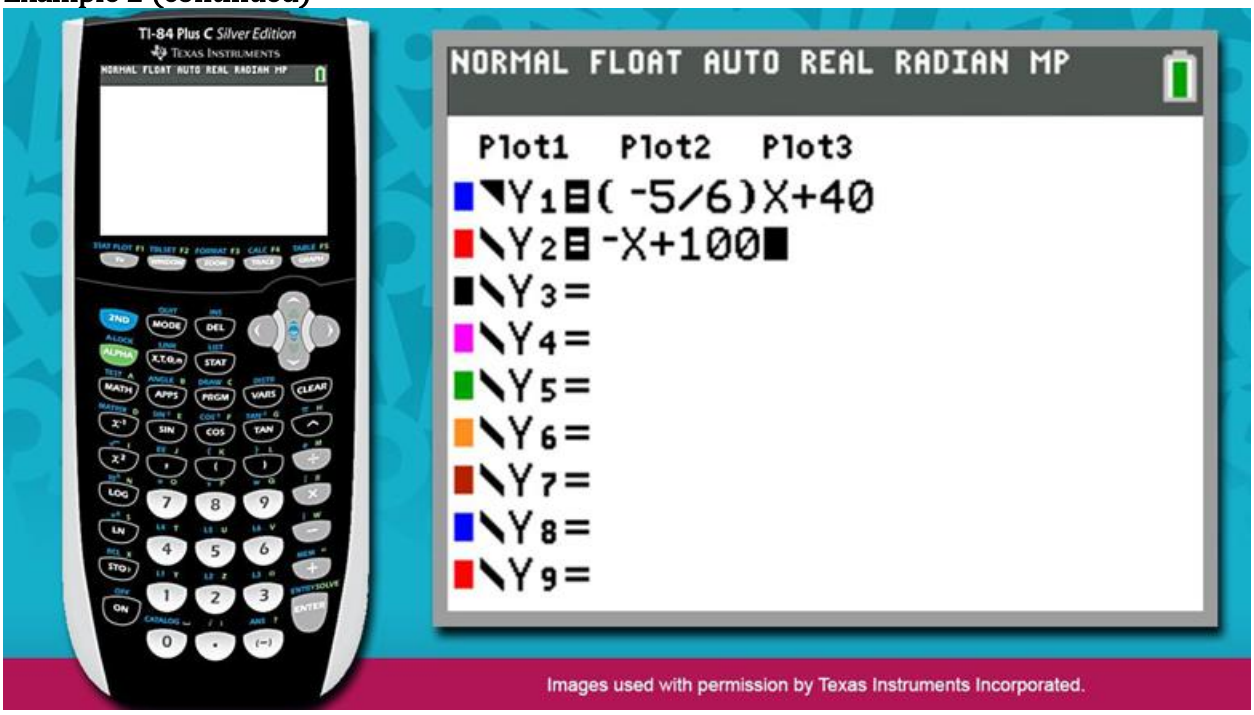


Now that you have graphed the first inequality, it is time to graph the second one, $y \geq -x + 100$. You'll begin by first viewing its boundary line.

Press Y=

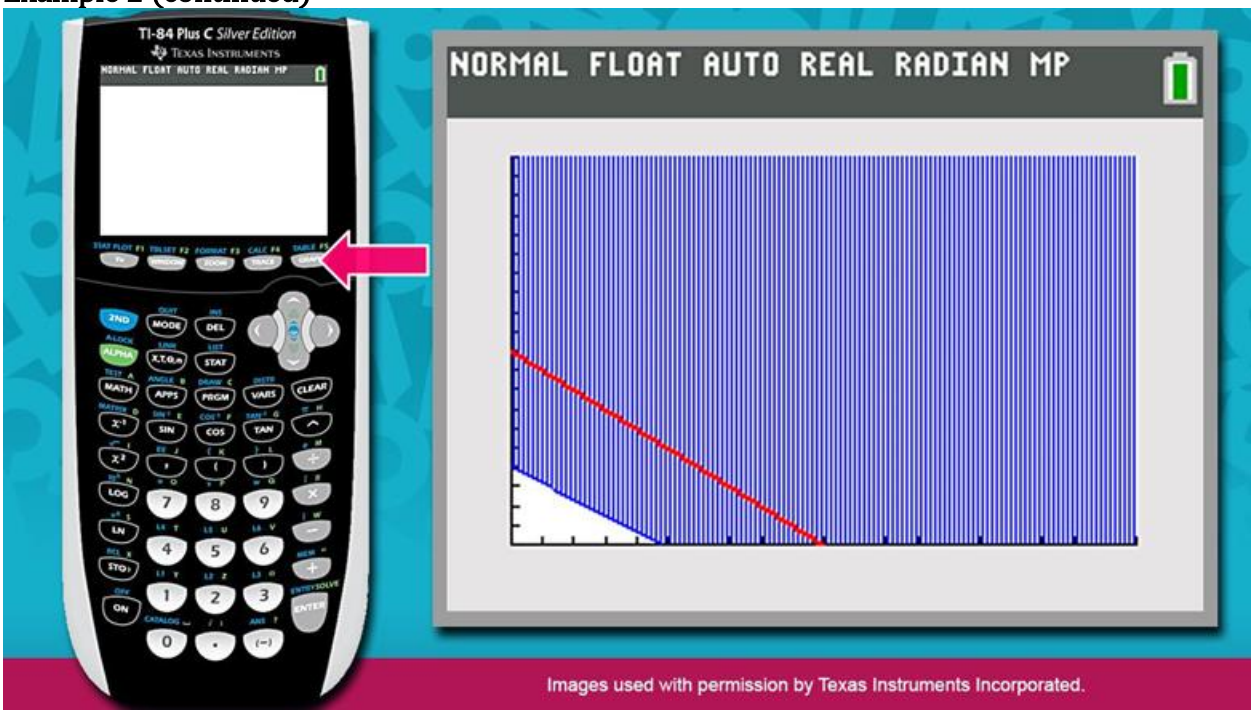
Then press the down arrow so that the cursor is blinking to the right of Y_2.

Example 2 (continued)



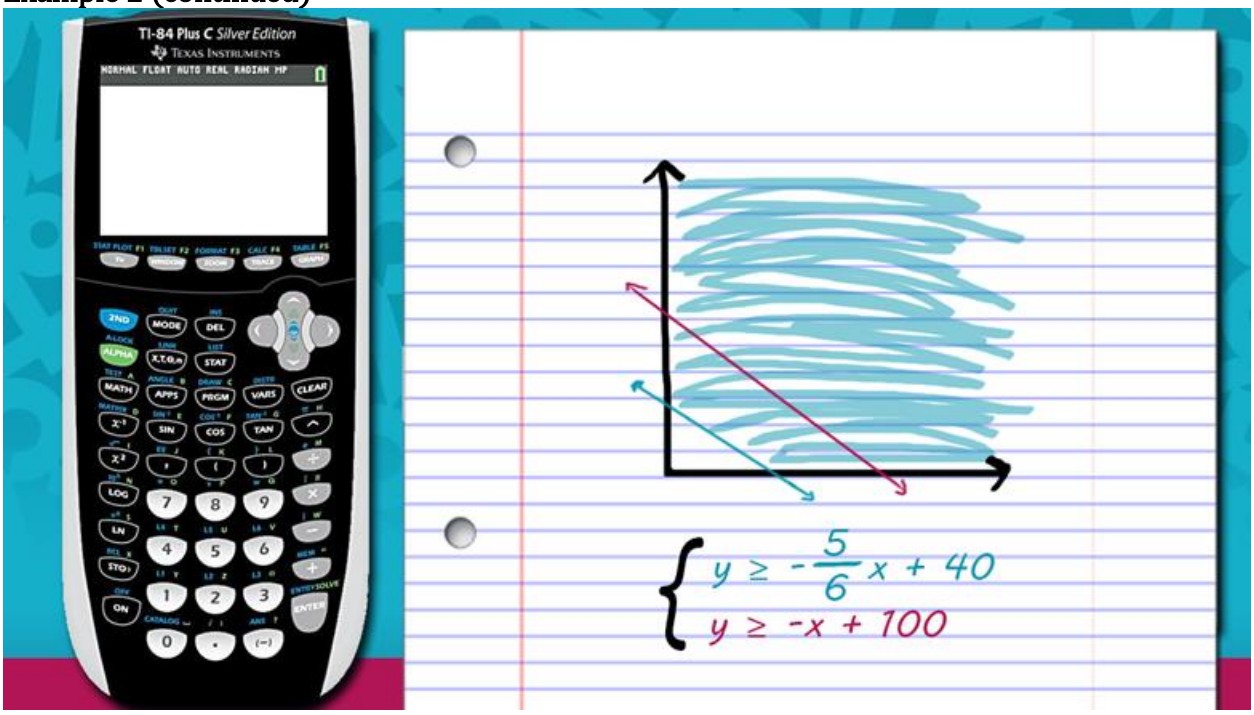
Enter the right side of second inequality to the right of Y_2.

Example 2 (continued)



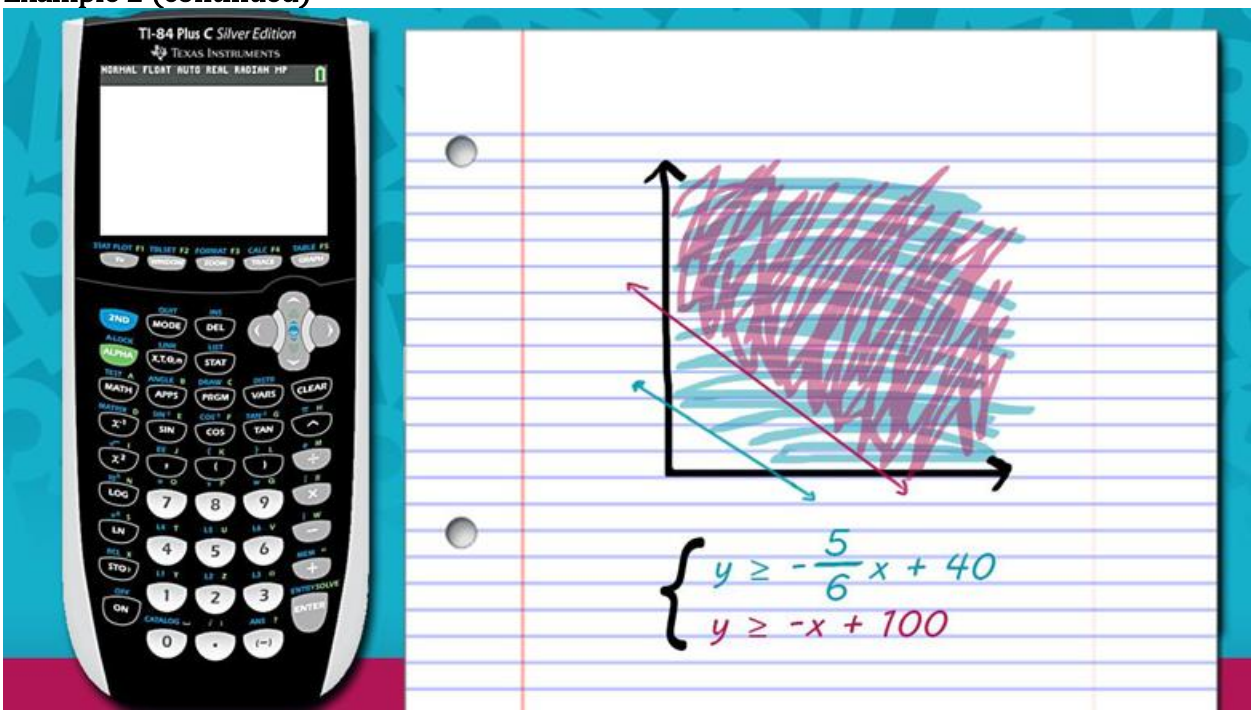
Now press GRAPH.

Example 2 (continued)



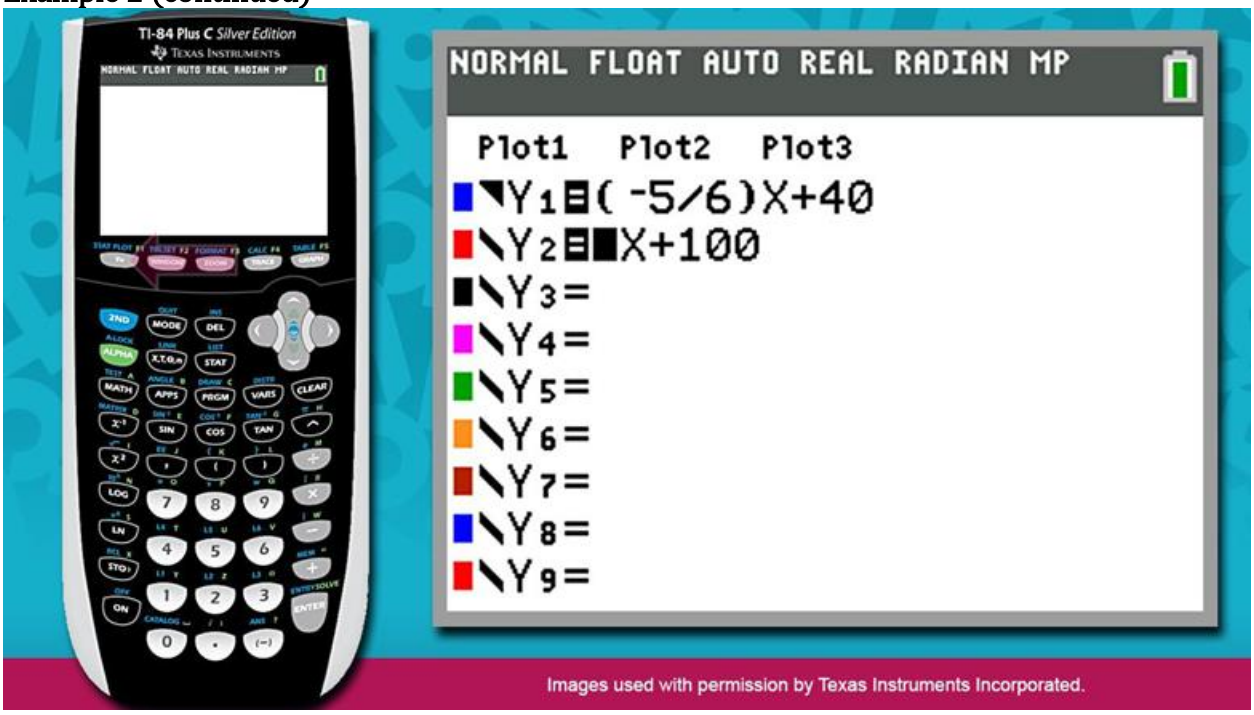
On your rough sketch, include the boundary line of the second inequality. The inequality, $y \geq -x + 100$, is a non-strict inequality, so you should include a solid boundary line. Also, make sure to use a different colored pencil when sketching this time, so that you will be able to differentiate between the two inequalities.

Example 2 (continued)



It's time to move on to the shading. The inequality, $y \geq -x + 100$, includes $\geq$. So, you will need to shade above the boundary line. On your rough sketch, shade above the boundary line of the second inequality. You'll notice that the shaded regions overlap.

Example 2 (continued)

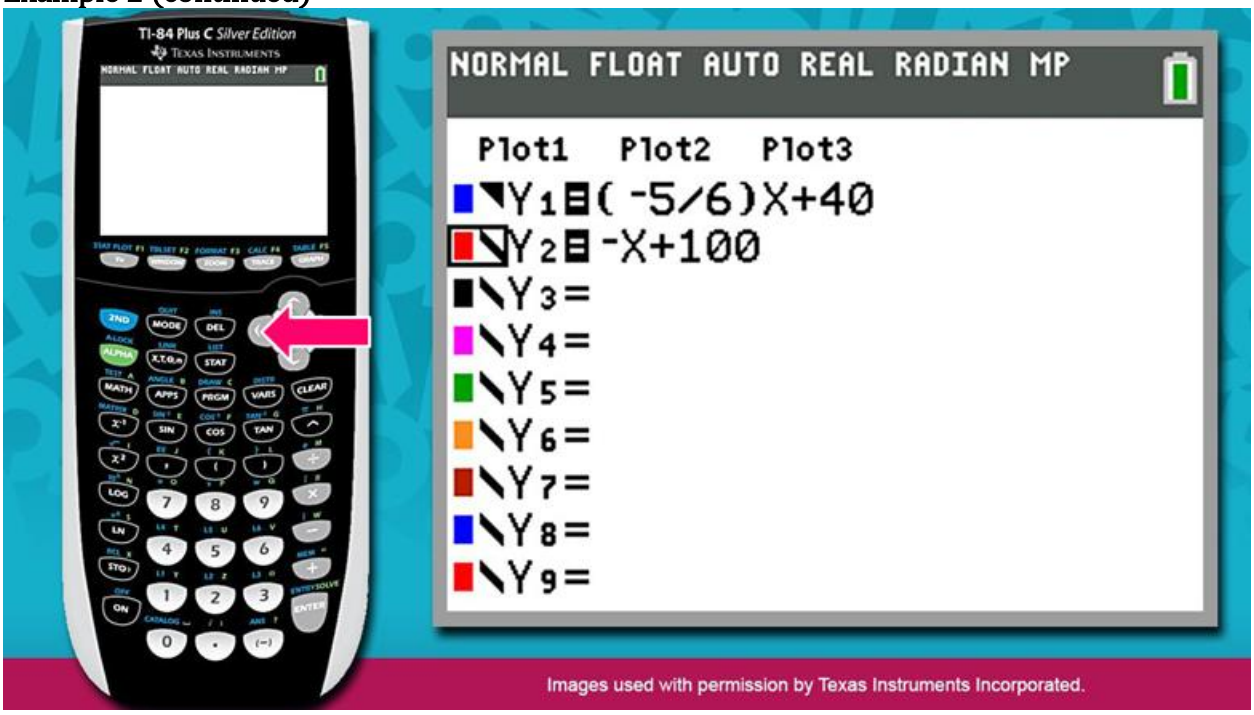


To view the shading on the graphing calculator:

Press Y=

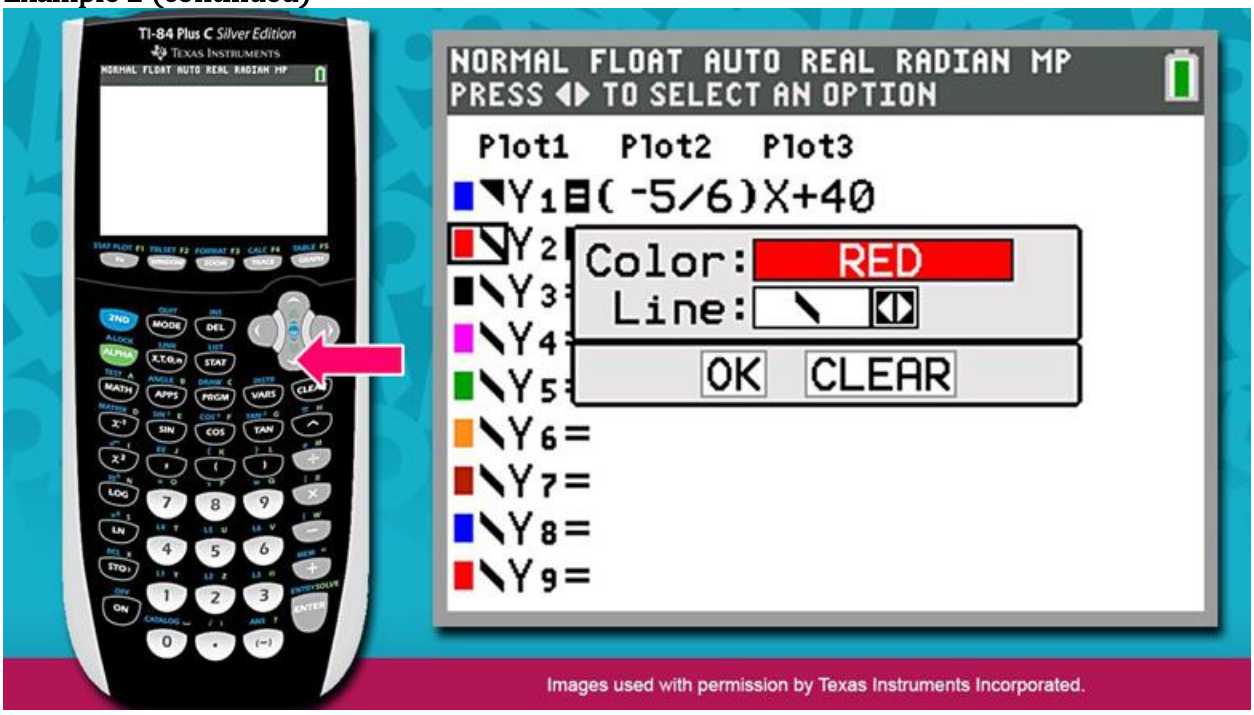
Then press the down arrow until the cursor is blinking to the right of Y_2.

Example 2 (continued)



Now press the left arrow until the cursor is blinking to the left of Y_2.

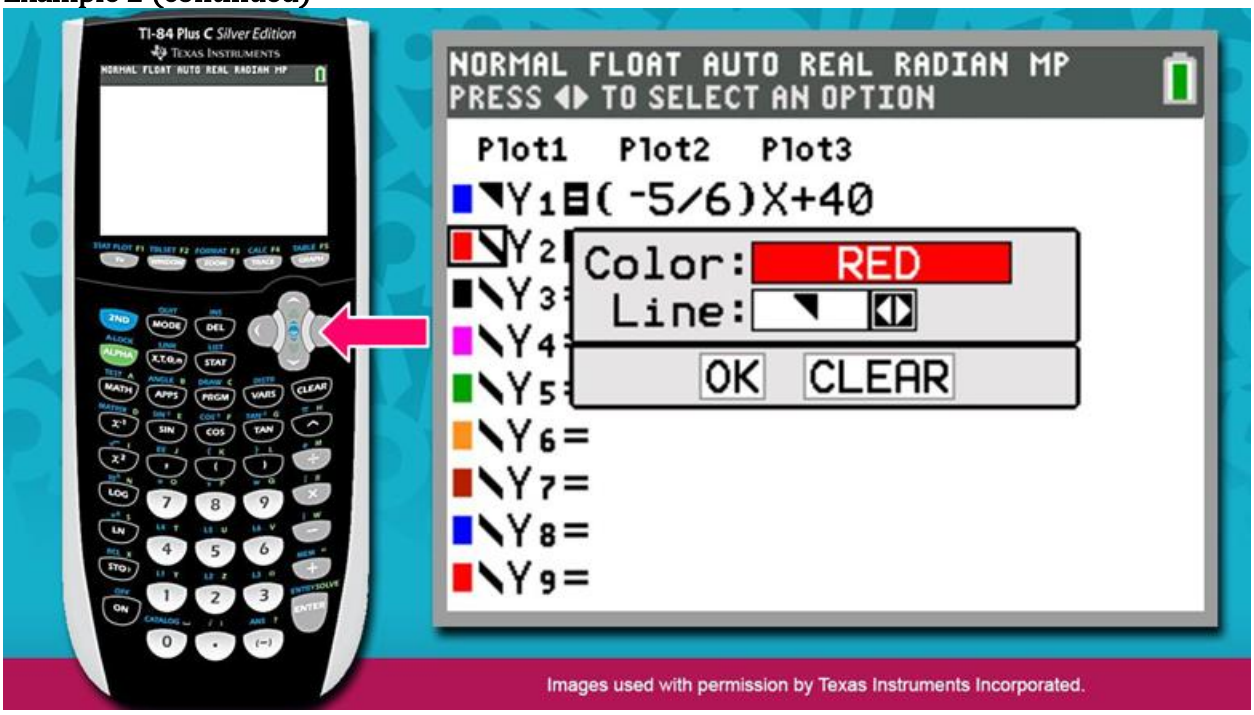
Example 2 (continued)



Press ENTER

Then press the down arrow so that the cursor is blinking beside the "Line" prompt.

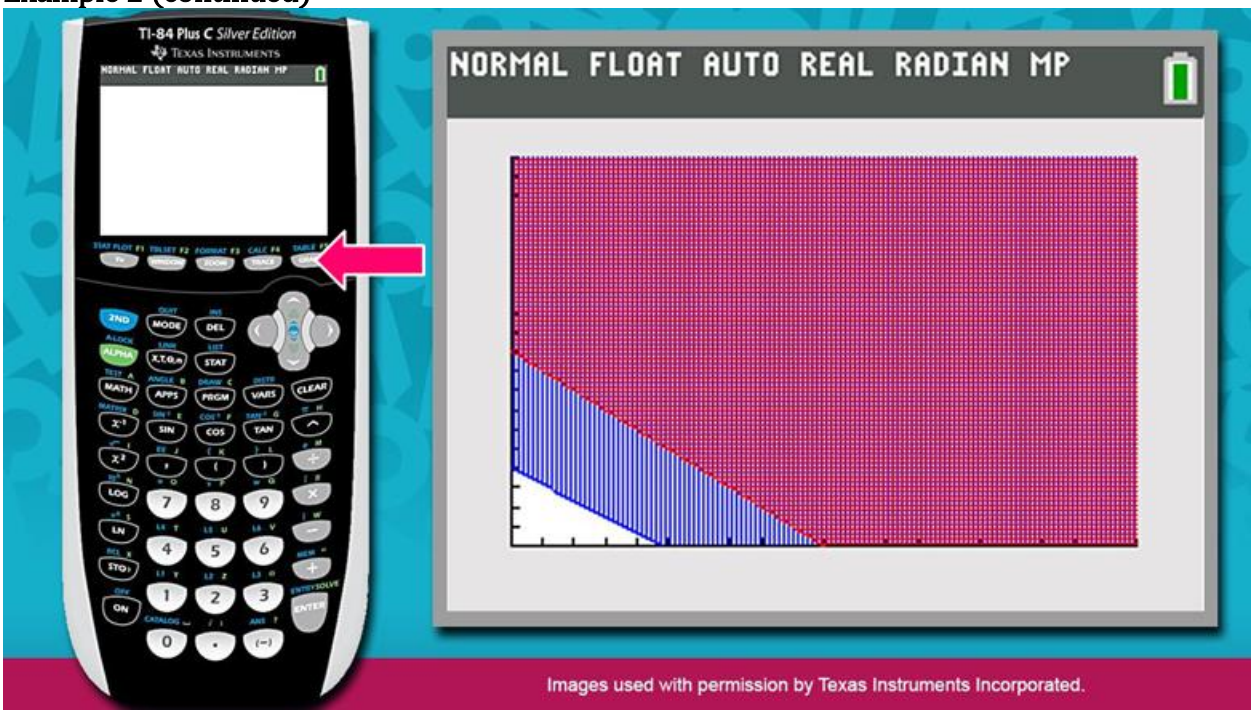
Example 2 (continued)



Press the right arrow until the option for shading above the line appears.

Press enter to select the shading option.

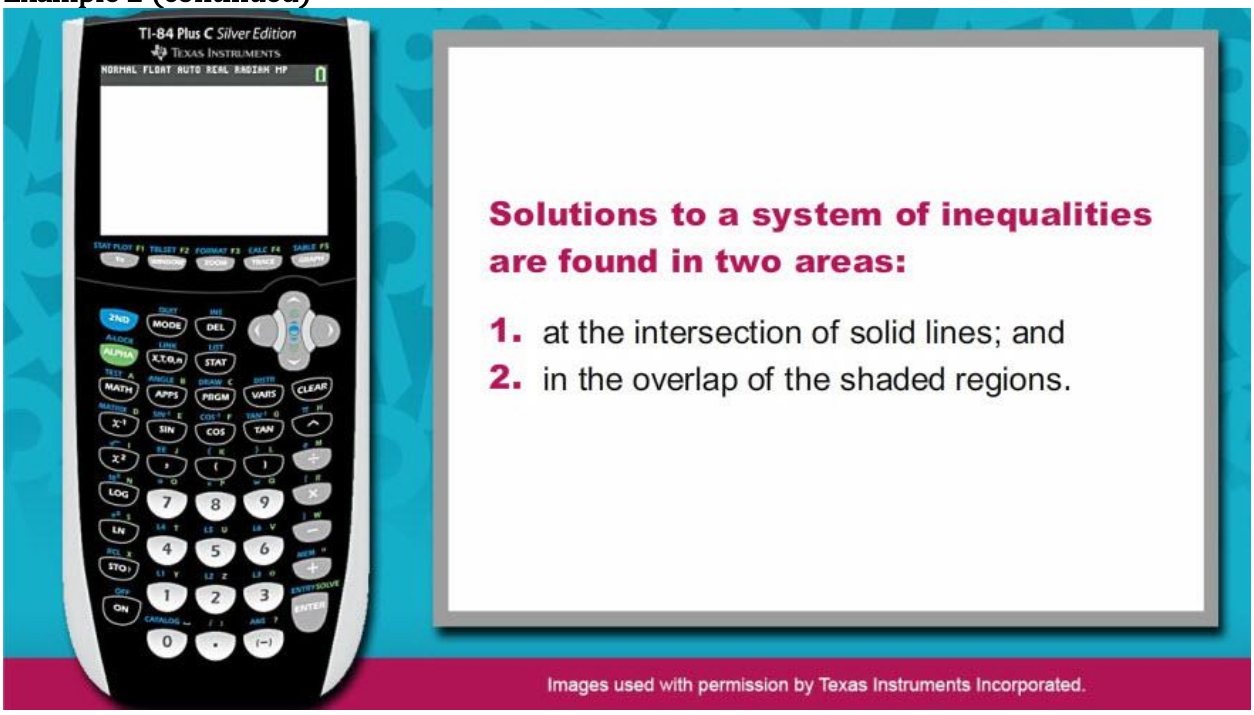
Example 2 (continued)



Press enter again once the cursor moves to OK.

Now press GRAPH.

Example 2 (continued)

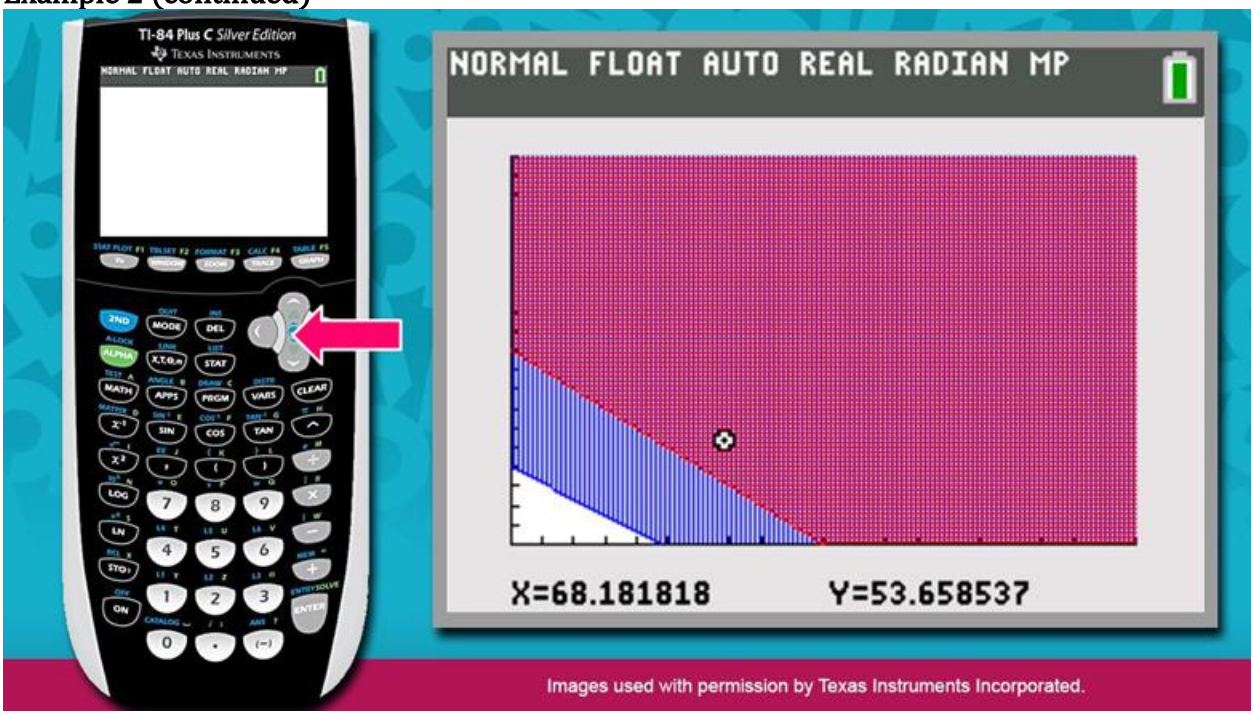


Now that you have graphed the system of inequalities in the calculator, with the appropriate shading, you are ready to identify a solution. Recall that solutions to a system of inequalities are found in two areas:

1. at the intersection of solid lines; and
2. in the overlap of the shaded regions.

The inequalities in this system each include a solid boundary line; however, the lines do not intersect. Solutions for this system will only be found in the overlap of the shaded regions.

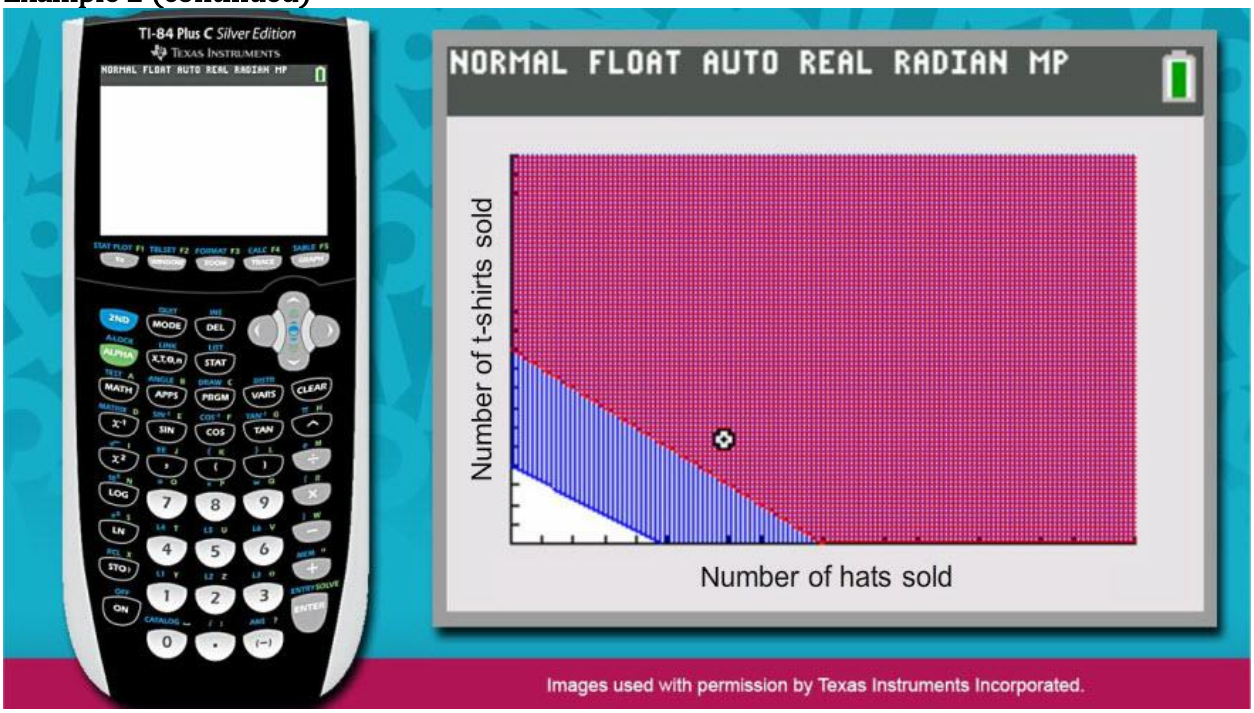
Example 2 (continued)



Press the right, left, up, and down arrow keys in order to locate points included in the overlap of the shaded regions

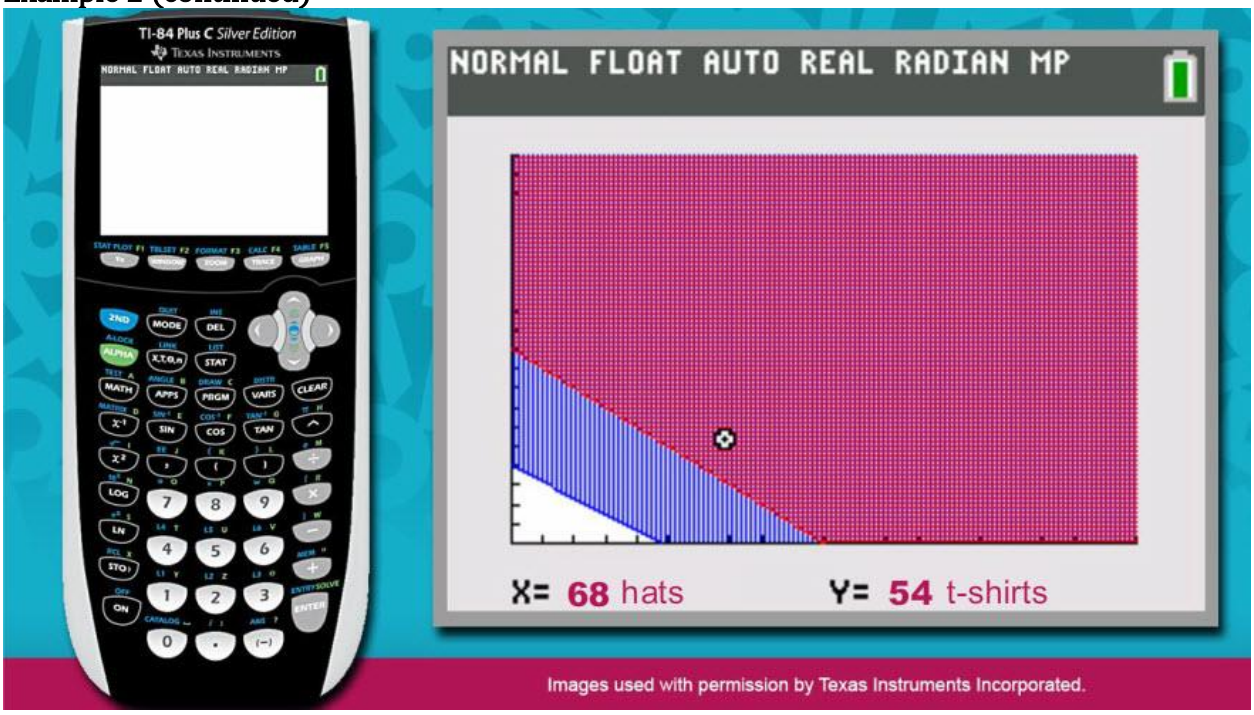
As you press the arrow keys to move around the shaded area, you will notice that the cursor will most likely move to points that include decimal values. Because you are searching for points that are appropriate for this real-world situation, you must determine if decimal values are appropriate for x and y.

Example 2 (continued)



Remember that in the given scenario, x represents the number of hats sold and y represents the number of t-shirts sold. It isn't possible to sell a portion of a hat or a portion of a t-shirt. So decimal values of x and y-values are not appropriate. You must round any decimal value to the nearest whole number that is included in the solution set.

Example 2 (continued)



Notice this point included in the solution set of the system. If you round the x- and y-values to the nearest whole numbers included in the solution set, the result is $(68, 54)$. This means that the sophomores could sell 68 hats and 54 t-shirts and meet the class goal.

Example 2 (continued)

You can verify this algebraically, by substituting the values into the original inequalities that make up the system:

$$\begin{cases} 10x + 12 \geq 480 \\ x + y \geq 100 \end{cases}$$

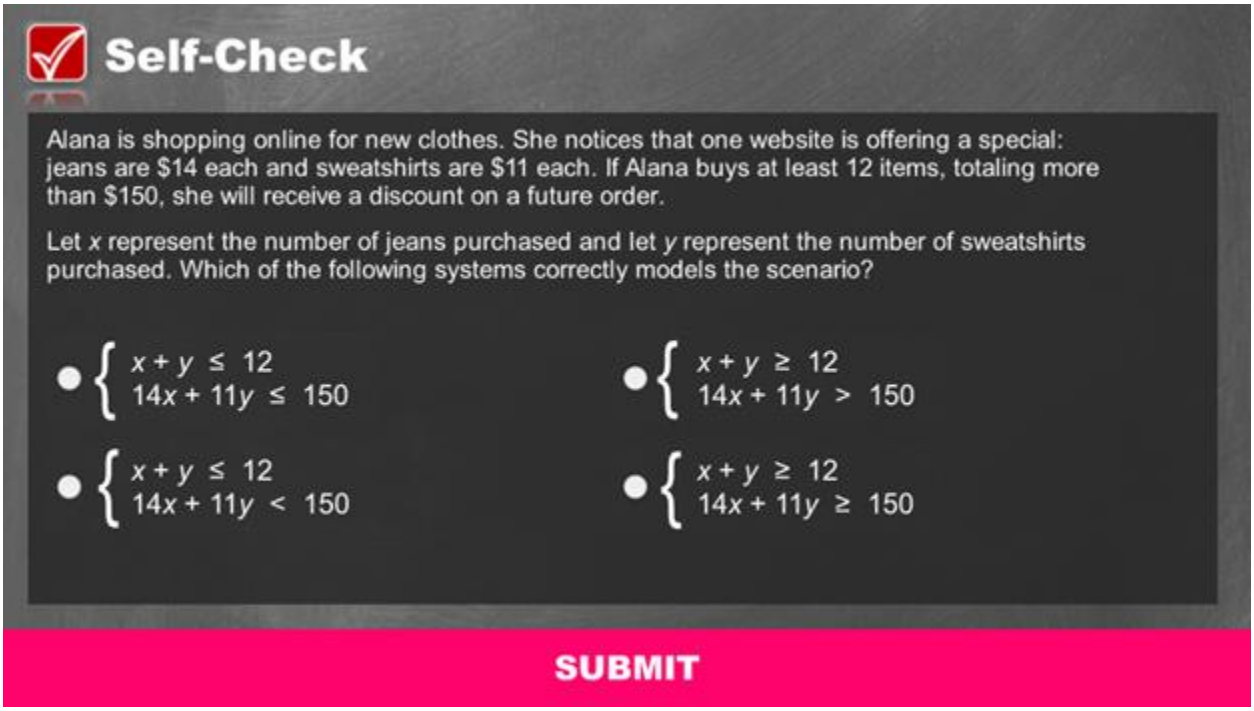
$(68, 54)$

$10x + 12y \geq 480$
$10(68) + 12(54) \quad 480$
$680 + 648 \quad 480$
$1328 \geq 480$

$x + y \geq 100$
$68 + 54 \quad 100$
$122 \quad 100$
$122 \geq 100$

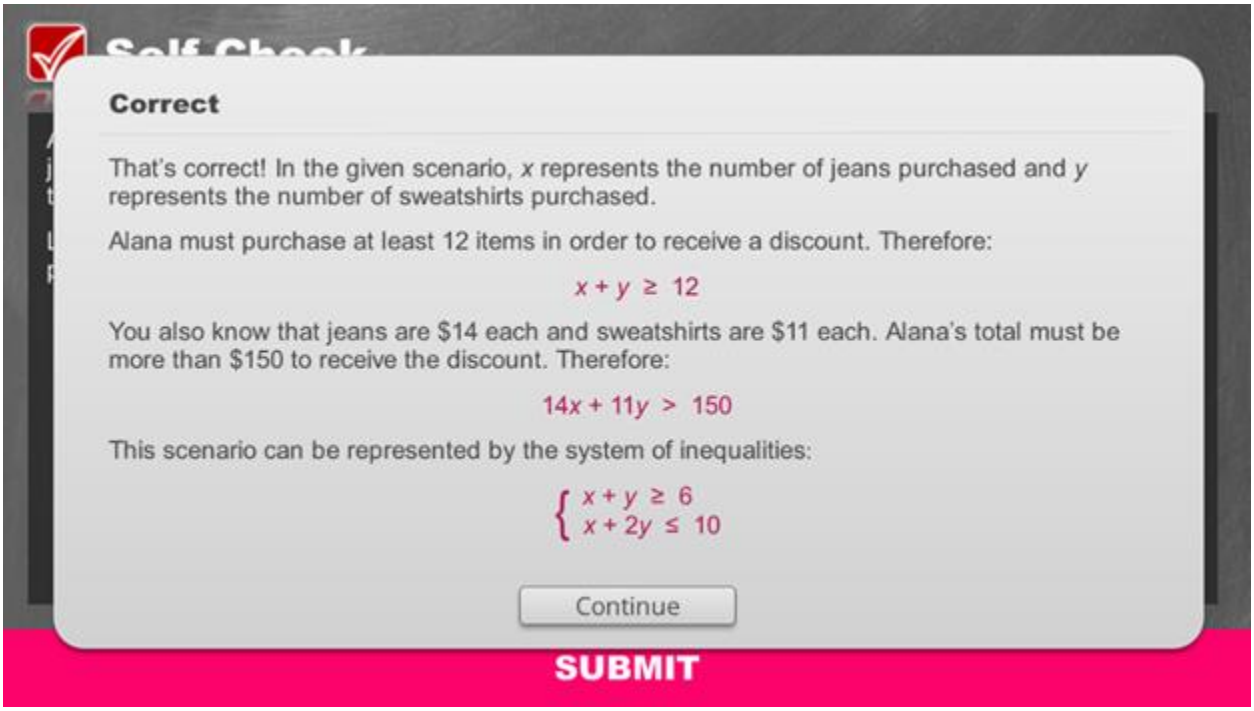
You have now verified, algebraically, that $(68, 54)$ is a solution to the system of inequalities. Any other points, included in the overlap of the shaded regions, that include whole-number values for x and y are also solutions to this real-world scenario.

Self-Check



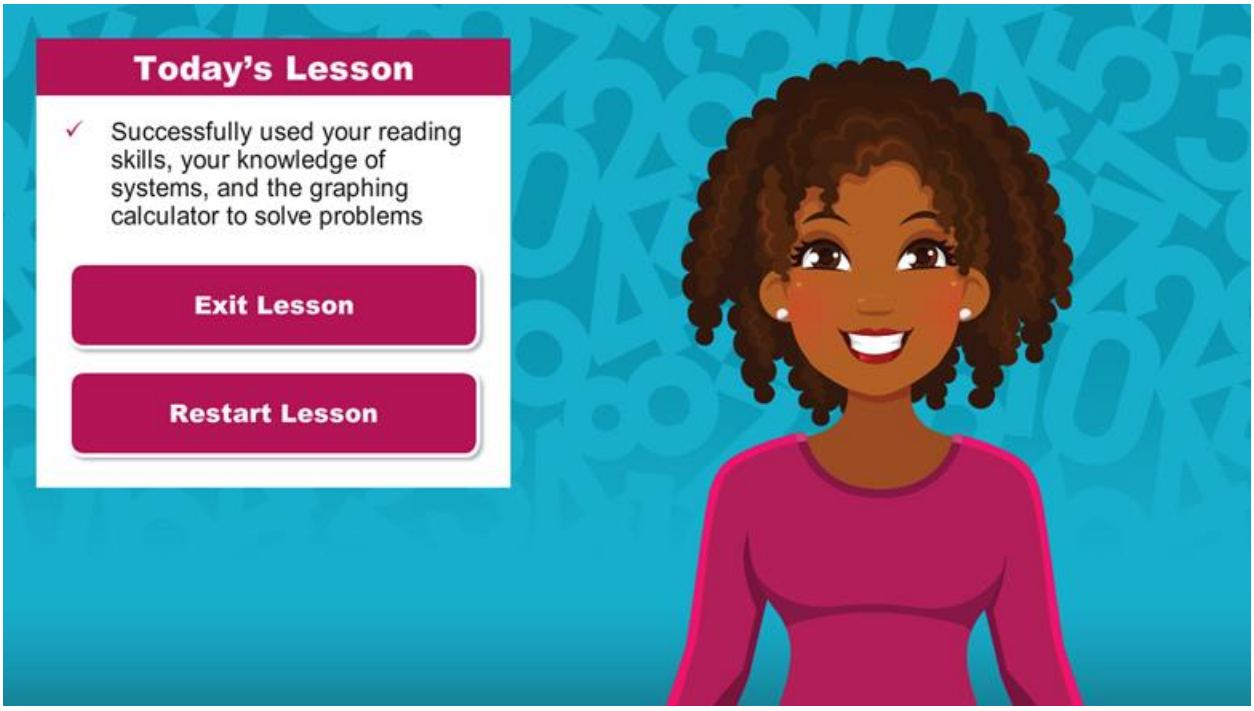
Solve the problem in the image above to check your understanding of the content.

Self-Check: Answer



For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



Congratulations! You have reached the conclusion of this lesson on solving practical problems involving systems of inequalities. Your reading comprehension skills, along with your knowledge of systems, and the use of the graphing calculator, each played a role in helping you to successfully complete this lesson.