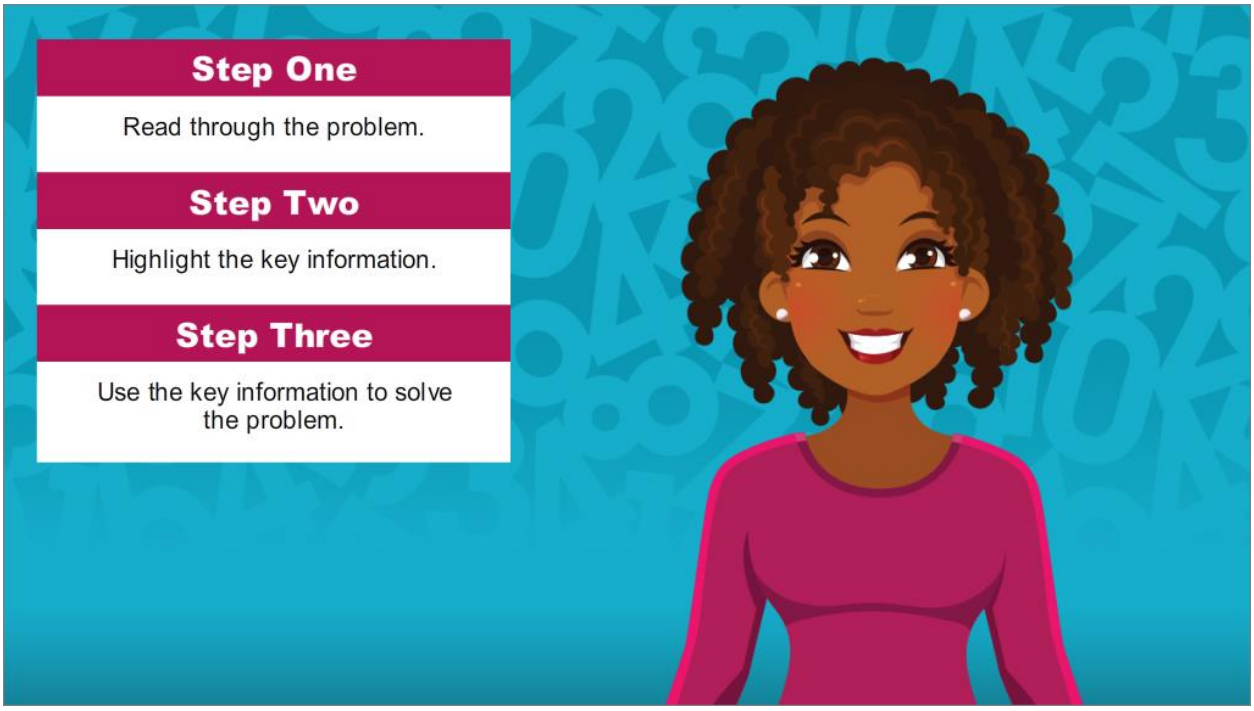
Module 7: Solving Linear Inequalities
Topic 4: Real-World Linear Inequalities in Two Variables

Introduction

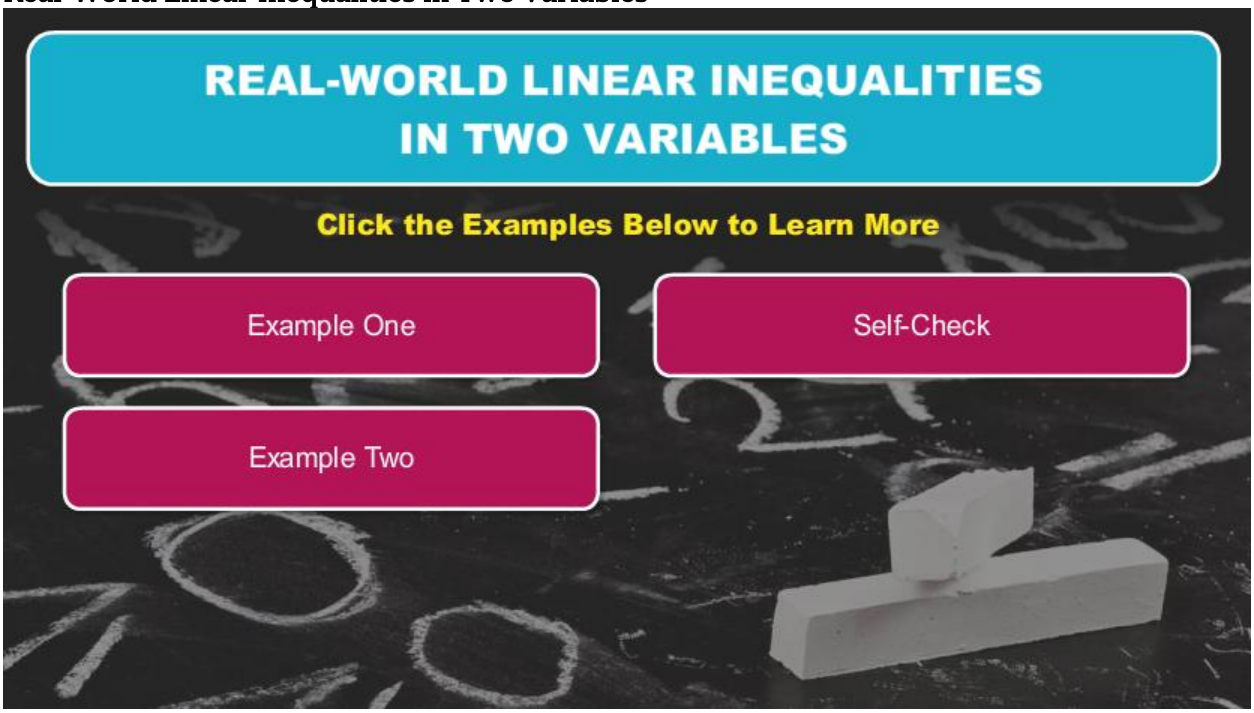


Hello there! I'm so happy you could join me for this lesson in Algebra I, where you will learn how to solve real world problems involving inequalities in two variables. These types of problems, also known as practical problems, can be intimidating, especially when there is a lot of information to analyze. But if you remember the three steps to solving word problems, I think you will find that they are not so bad after all:

Step 1: Read through the problem
Step 2: Highlight the key information
Step 3: Use the key information to solve the problem

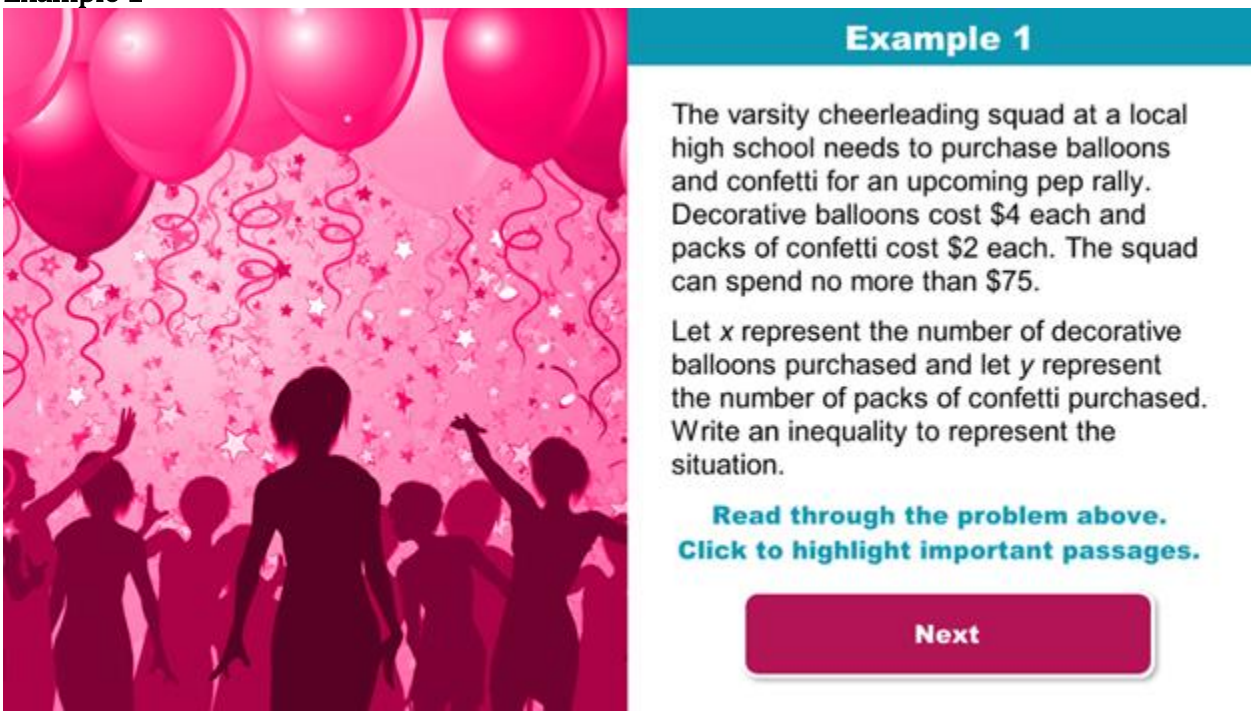
So without any further ado, let's start the first example.

Real-World Linear Inequalities in Two Variables



Click the examples below to learn more.

Example 1



Take a few moments to read Example 1. Highlight the information that you think will be helpful in solving the problem.

The varsity cheerleading squad at a local high school needs to purchase balloons and confetti for an upcoming pep rally. Decorative balloons cost $4 *each and packs of confetti cost* $2 *each. The squad can spend no more than* $75*.*

Let x represent the number of decorative balloons purchased and let y represent the number of packs of confetti purchased.

Write an inequality to represent the situation.

Read through the problem above. Click to highlight important passages.

Example 1 (continued)

Example 1

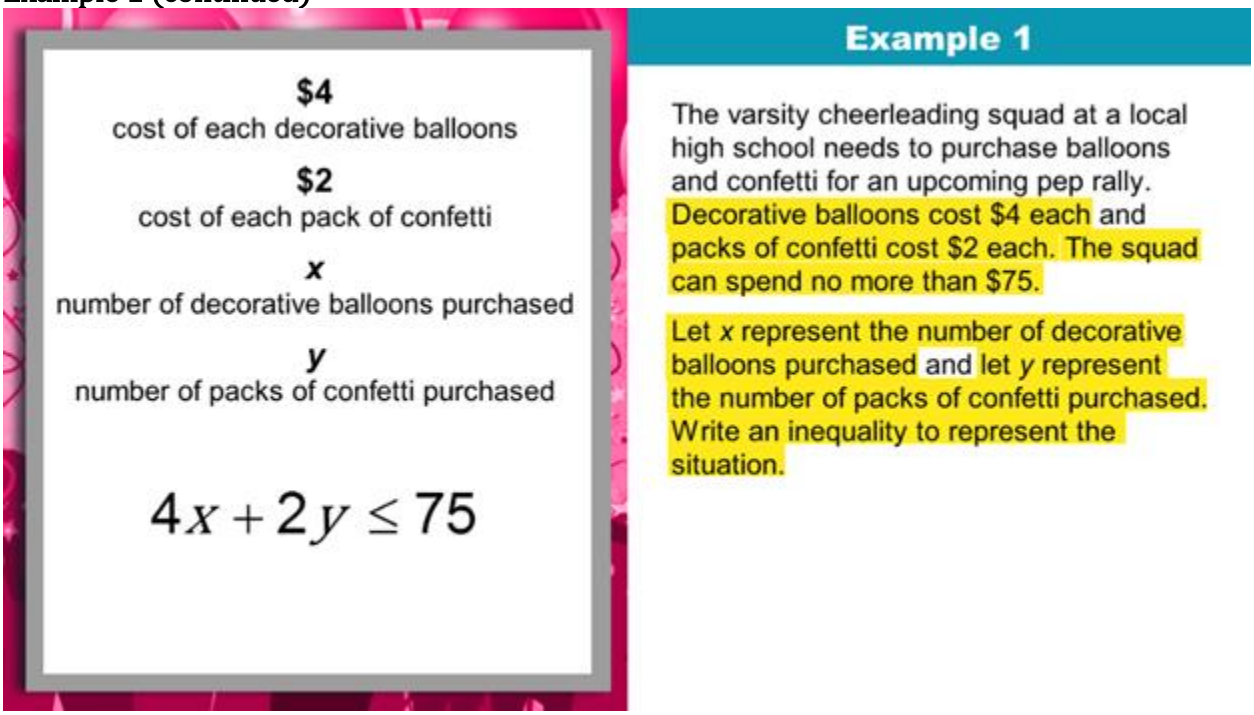
The varsity cheerleading squad at a local high school needs to purchase balloons and confetti for an upcoming pep rally. Decorative balloons cost $4 each and packs of confetti cost $2 each. The squad can spend no more than $75.

Let x represent the number of decorative balloons purchased and let y represent the number of packs of confetti purchased. Write an inequality to represent the situation.

Did you highlight these important facts?

- Decorative balloons cost $4 each
- Packs of confetti cost $2 each
- The squad can spend no more than $75
- x represents the number of decorative balloons
- y represents the number of packs of confetti
- Write an inequality to represent the situation

Example 1 (continued)



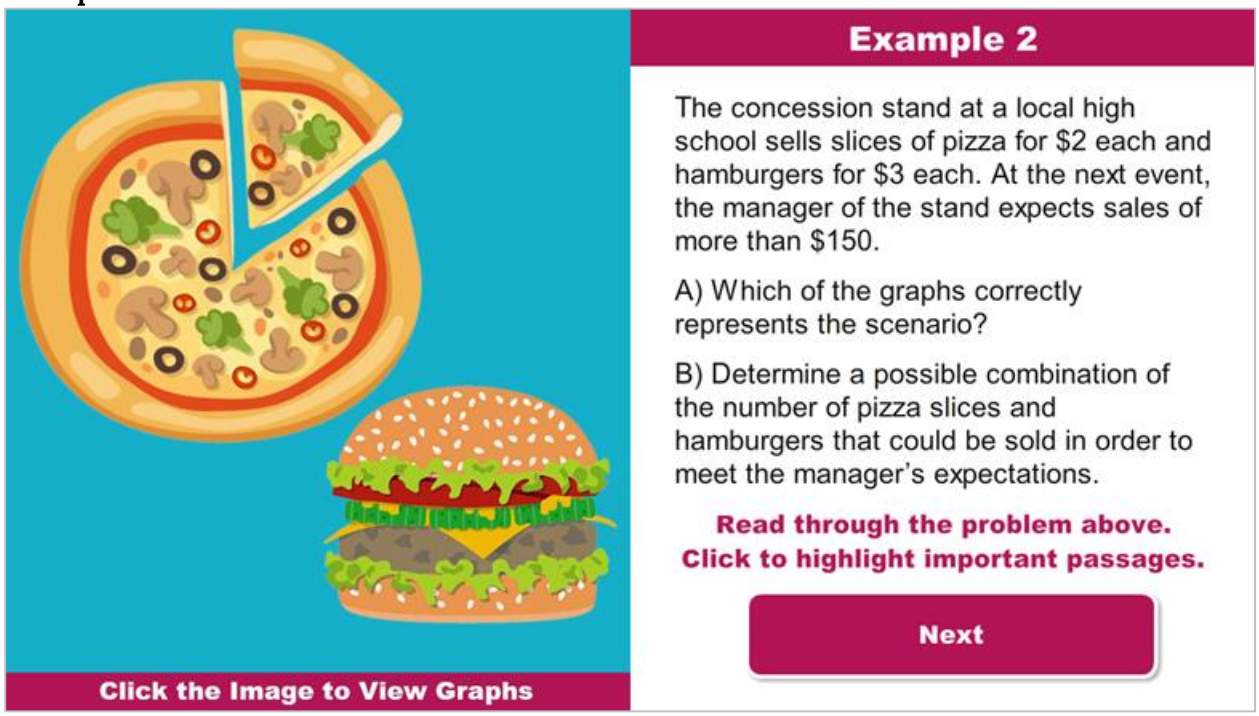
In the given scenario, decorative balloons cost \$4 each and packs of confetti cost \$2 each. Because x represents the number of decorative balloons purchased and y represents the number of packs of confetti purchased, you can model this part of the situation with the expression:

$$4x + 2y$$

Also, you know that the cheerleading squad can spend no more than \$75, or in other words \$75 or less. Therefore,

$$4x + 2y \leq 75$$

Example 2



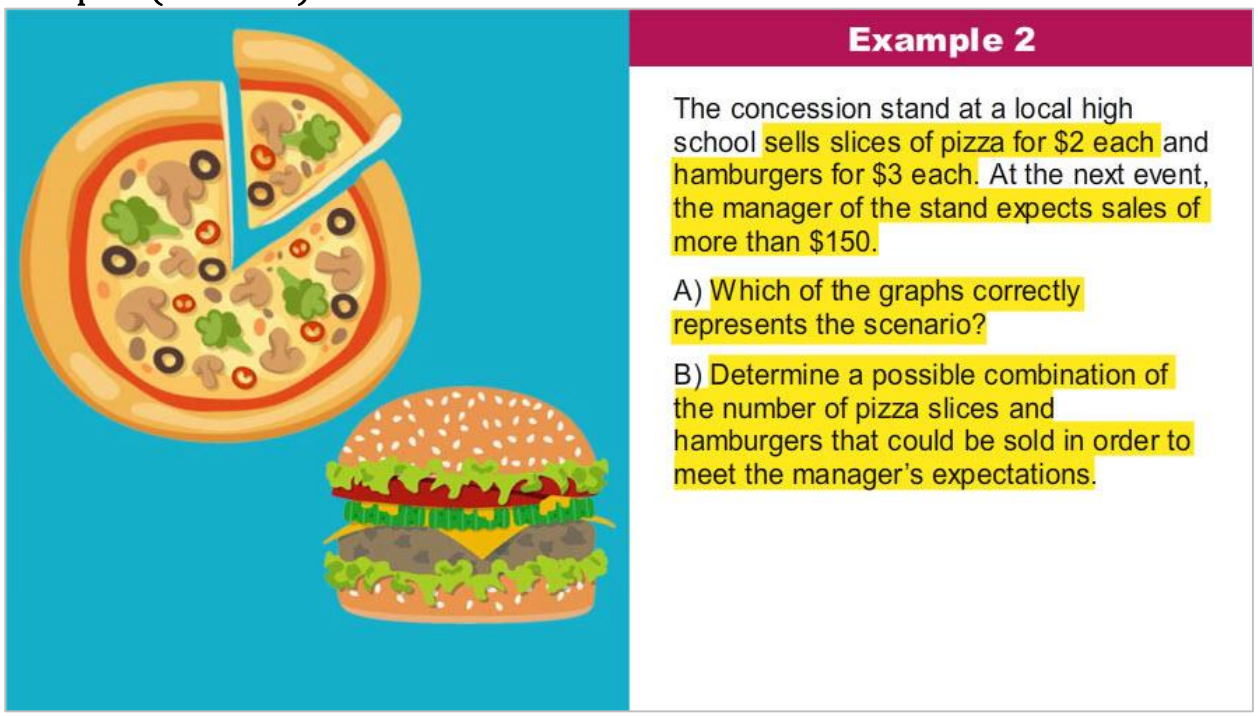
Now take a few moments to read Example 2. In order to determine which of the graphs correctly represents the scenario, you will need to write an inequality to model the situation. Then, you must use the graphing calculator to represent the inequality graphically. In order to write the inequality, remember the three-step process. Highlight the key information.

The concession stand at a local high school sells slices of pizza for $2 *each and hamburgers for* $3 *each. At the next event, the manager of the stand expects sales of more than* $150*.*

A) Which of the following graphs correctly represents the scenario?
B) Determine a possible combination of the number of pizza slices and hamburgers that could be sold in order to meet the manager's expectations?

Read through the problem above. Click to highlight important passages.

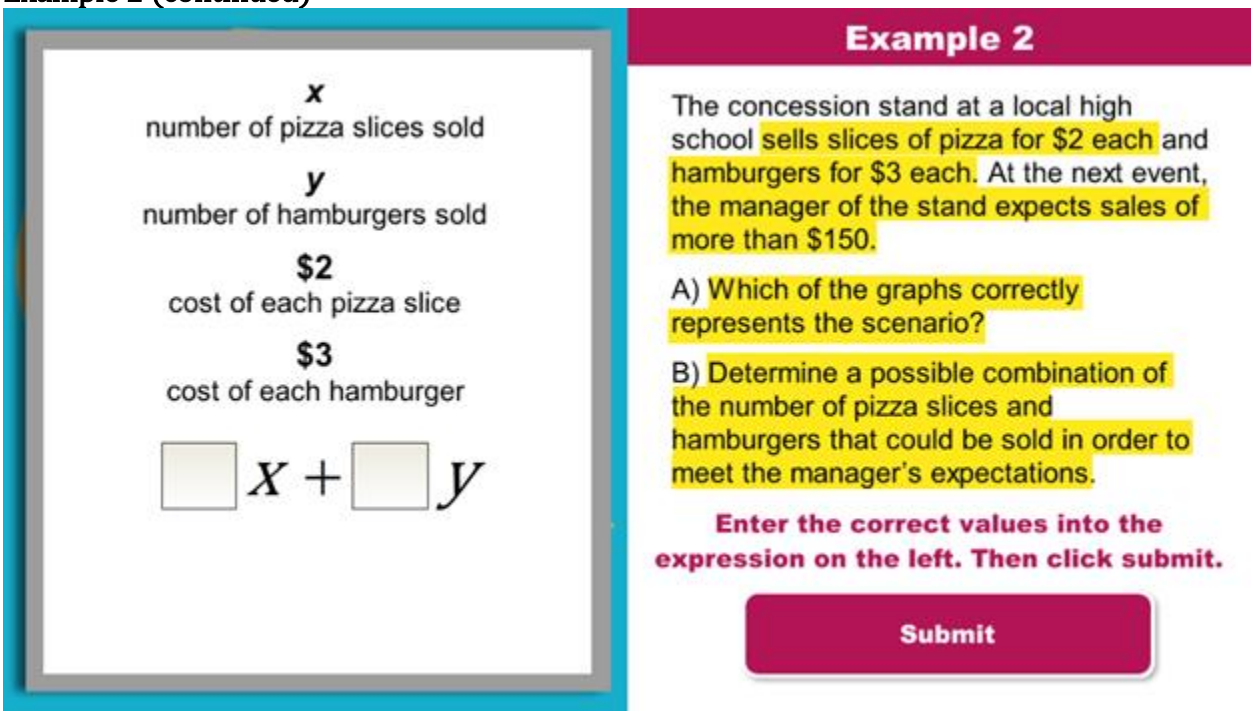
Example 2 (continued)



Did you highlight these important facts?

- Pizza slices are $2 each
- Hamburgers are $3 each
- The manager expects sales of more than $150
- Which of the following graphs correctly represents the scenario?
- Determine a possible combination of pizza slices and hamburgers that could be sold in order to meet the manager's expectations.

Example 2 (continued)

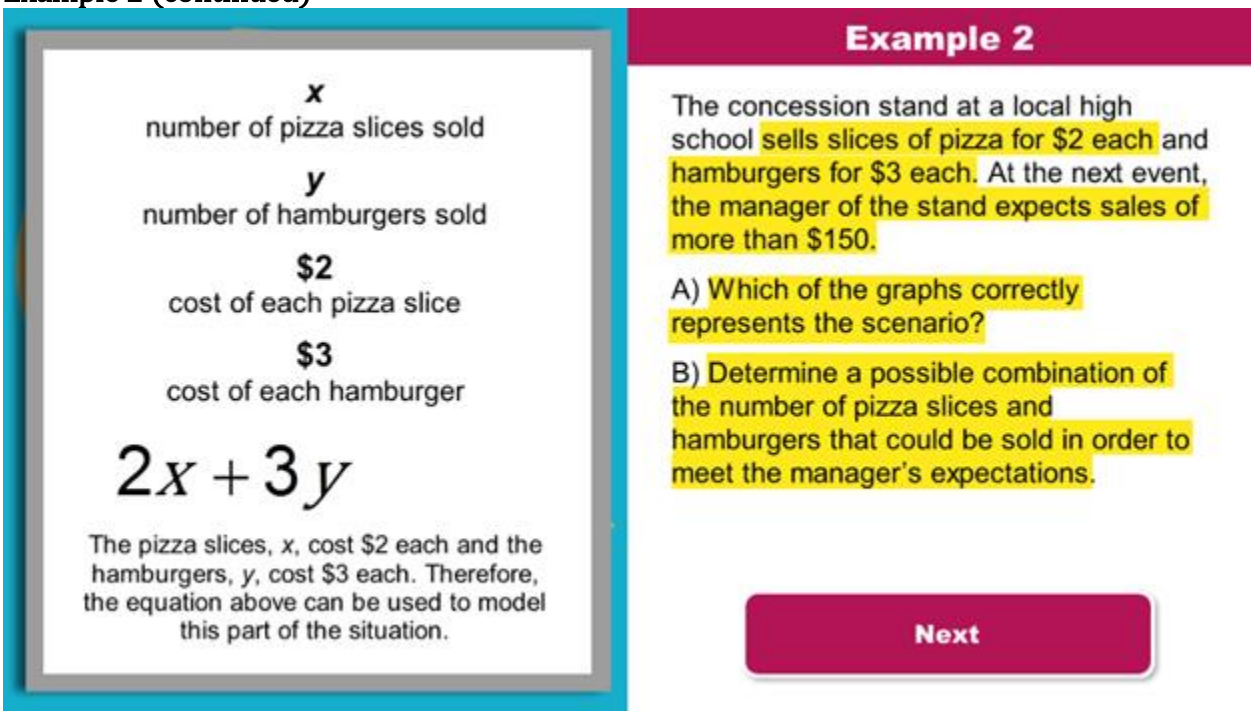


By analyzing the graphs given as answer choices, you are able to determine that x represents the number of pizza slices sold and y represents the number of hamburgers sold. You know that pizza slices are $2 each and hamburgers are $3 each. So you can model this part of the situation by using the expression:

$$?x + ?y$$

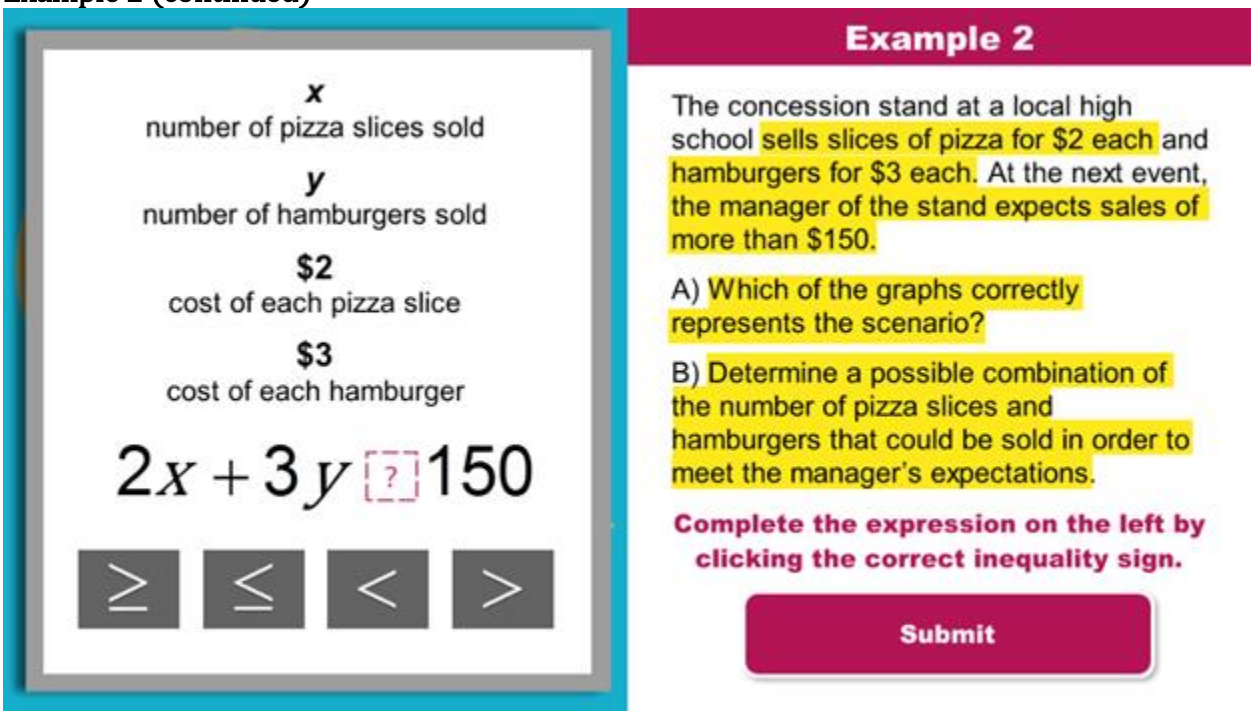
Enter the correct values into the expression on the left. Then click submit.

Example 2 (continued)



The pizza slices, x, cost \$2 each and the hamburgers, y, cost \$3 each. Therefore, $2x + 3y$ can be used to model this part of the situation.

Example 2 (continued)

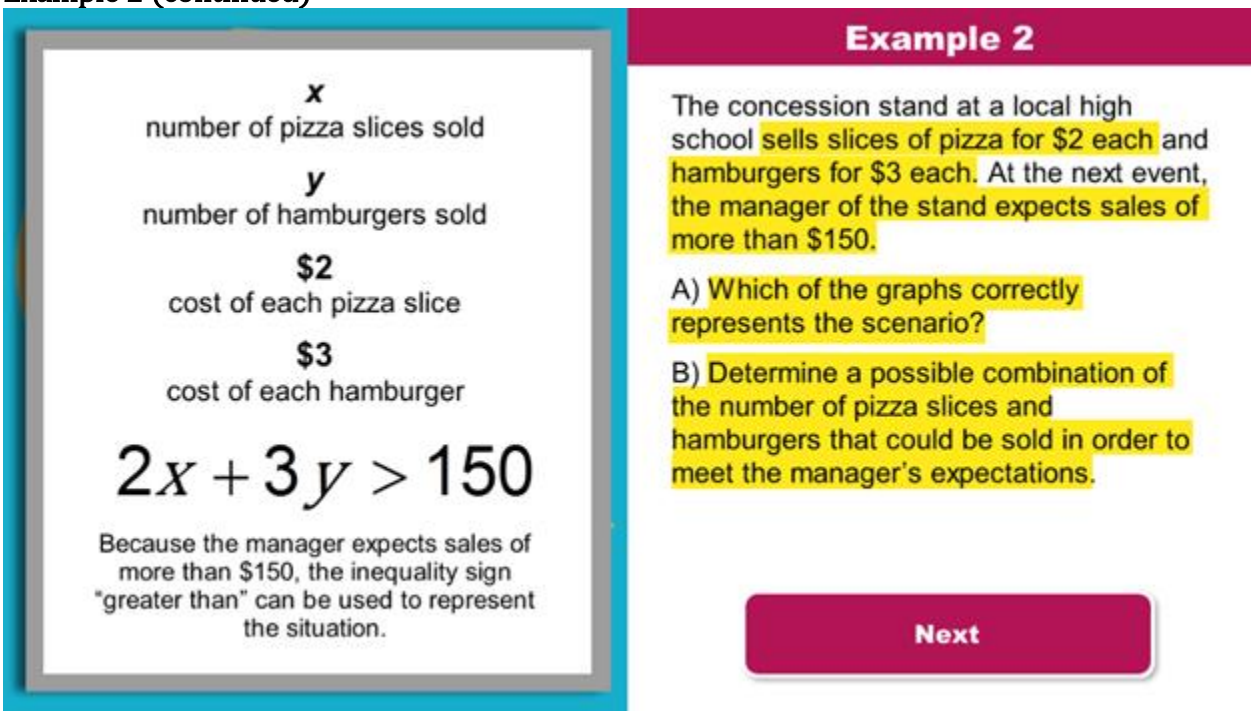


You also know that the manager expects sales of more than \$150. Therefore?

$$2x + 3y \quad ? \quad 150$$

Complete the expression on the left by clicking the correct inequality sign.

Example 2 (continued)



Because the manager expects sales of more than \$150, the inequality $2x + 3y > 150$ can be used to represent the situation.

Example 2 (continued)

Now that you have an inequality to model the situation algebraically, you can use the graphing calculator to model it graphically. In order to do this, you must solve the inequality for y.

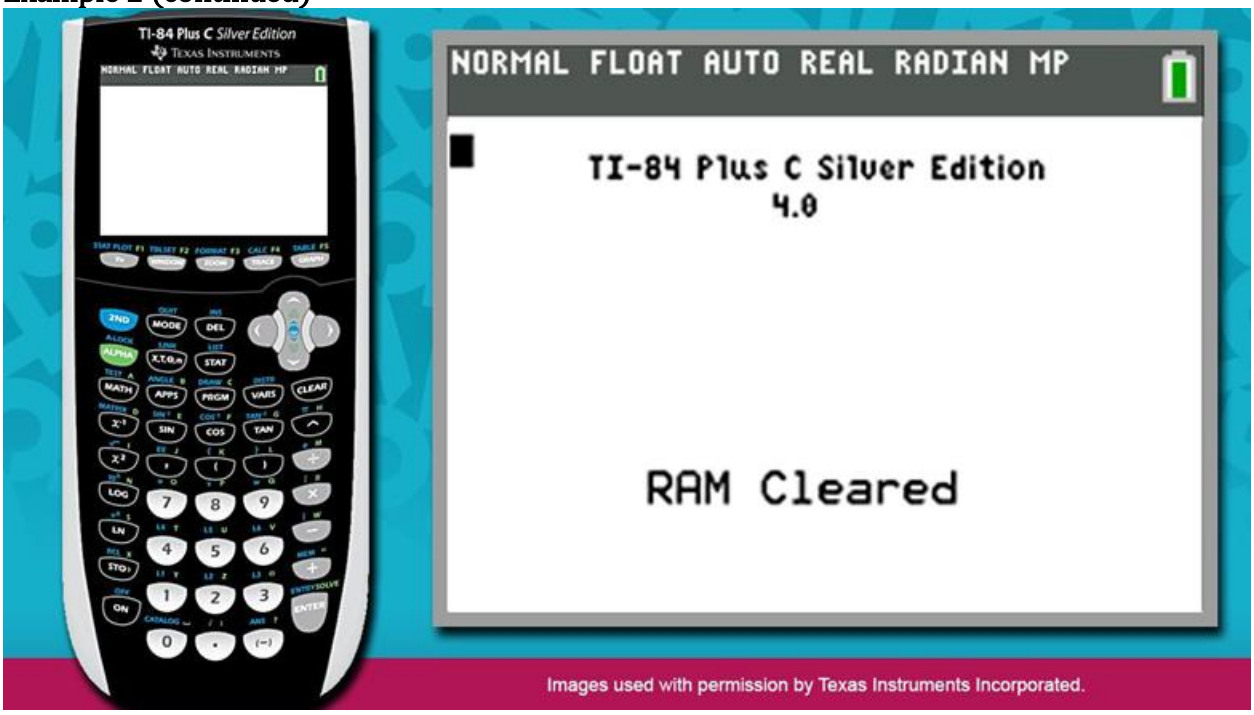
$2x + 3y > 150$

$-2x \qquad -2x$ Subtract $2x$ from both sides.

$\frac{3y}{3} > \frac{-2x}{3} + \frac{150}{3}$ Now divide each term by 3.

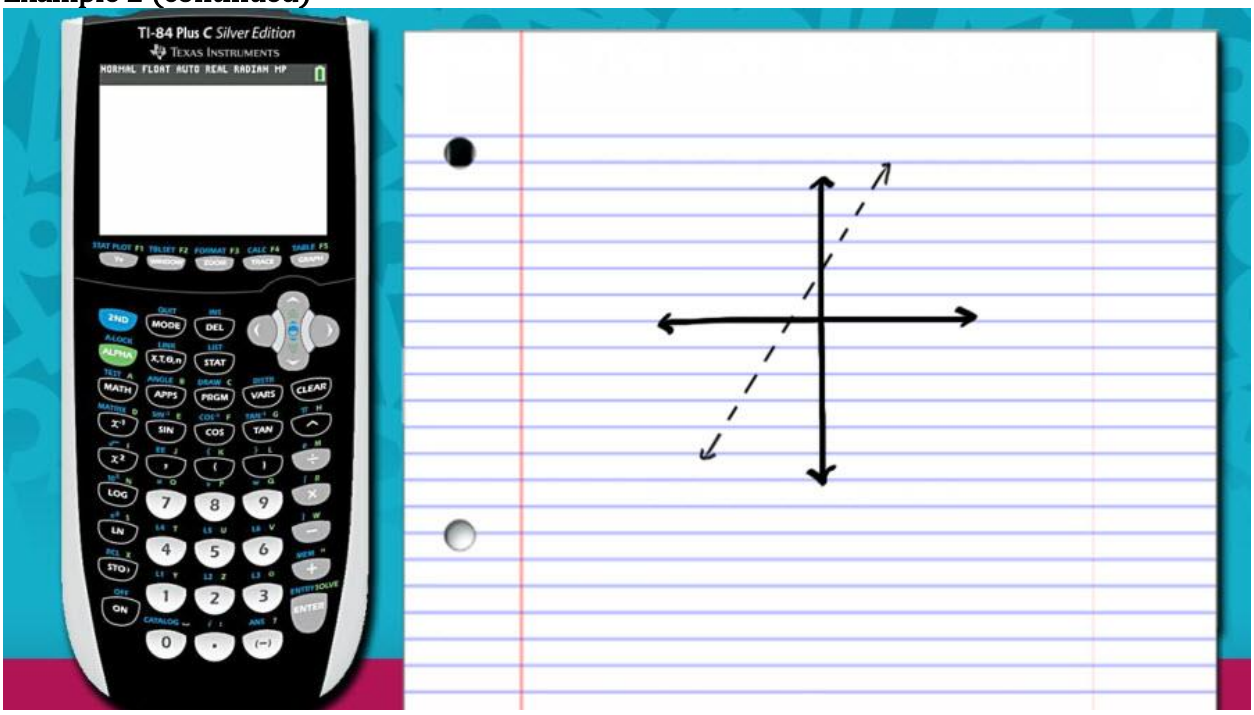
$y > -\frac{2}{3}x + 50$

Example 2 (continued)



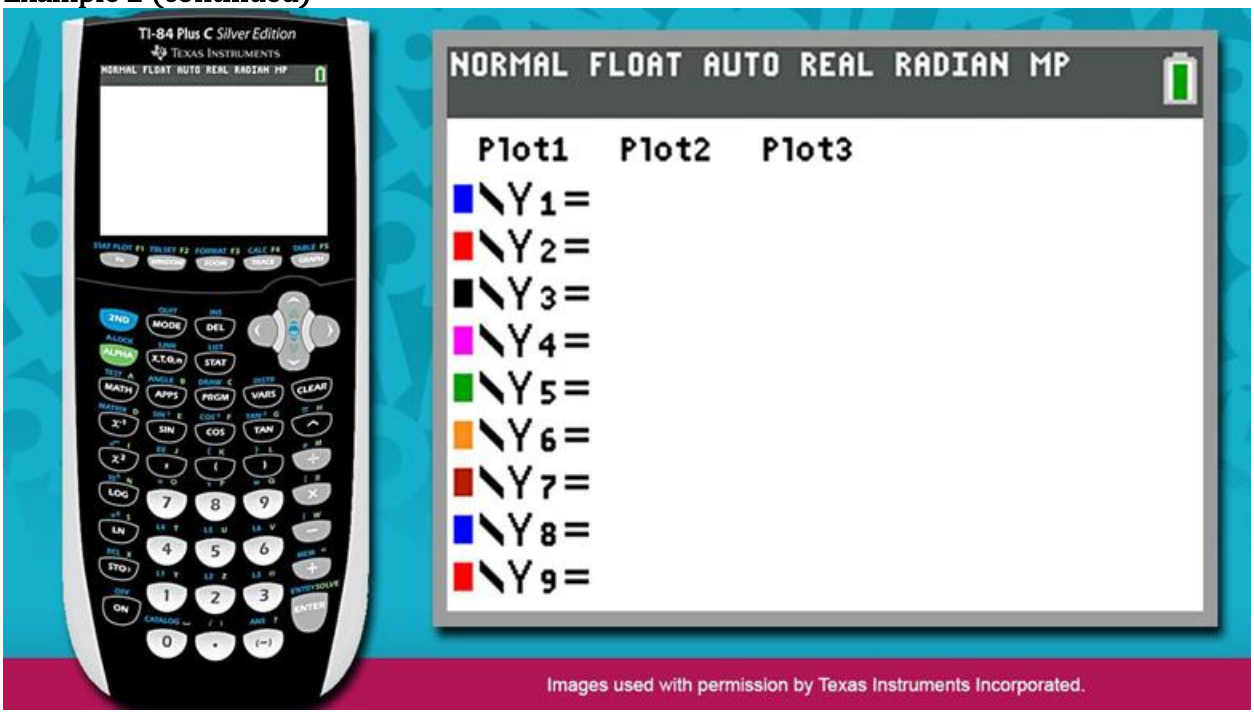
Now that the inequality is solved for y, you can move on to use the graphing calculator to represent it graphically. This will help you determine which of the graphs correctly represents the scenario. Begin by clearing the calculator's memory.

Example 2 (continued)



Remember that when using the graphing the calculator to graph a linear inequality, the calculator will only graph a solid boundary line, it will not graph dashed boundary lines. For this reason, it is often helpful to prepare a rough sketch of the graph, so that you will remain mindful of whether your inequality actually includes a solid line or a dashed line. Begin to prepare your rough sketch by having the calculator graph only the boundary line at this time. Then, you will go back and include the necessary shading.

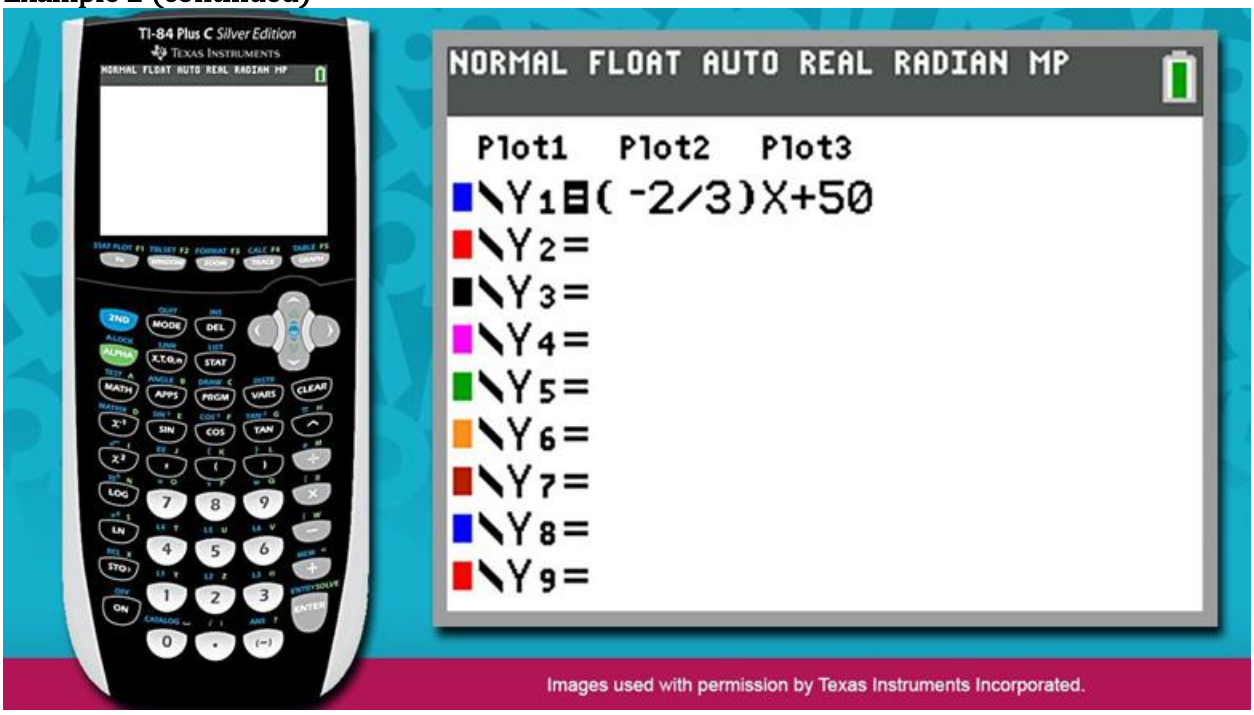
Example 2 (continued)



Press Y=

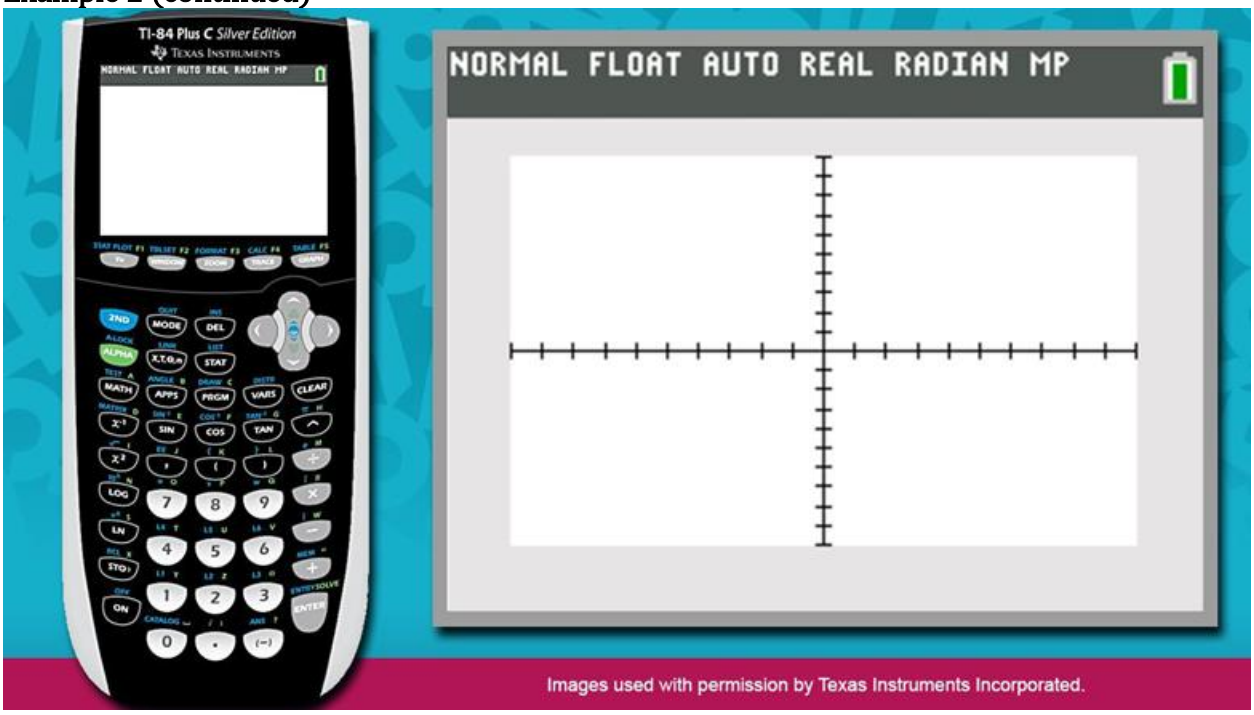
Now enter in the right side of the inequality to the right of Y_1. Remember when entering a fraction, you should use parentheses.

Example 2 (continued)



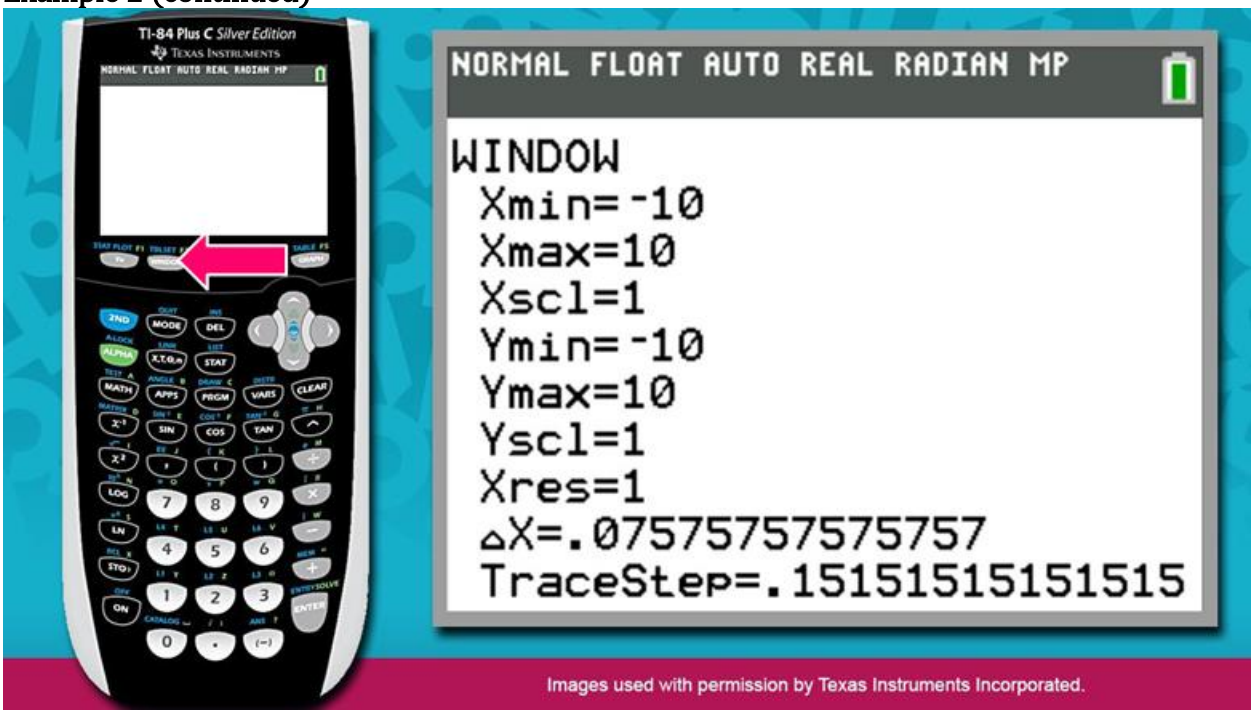
Press GRAPH

Example 2 (continued)



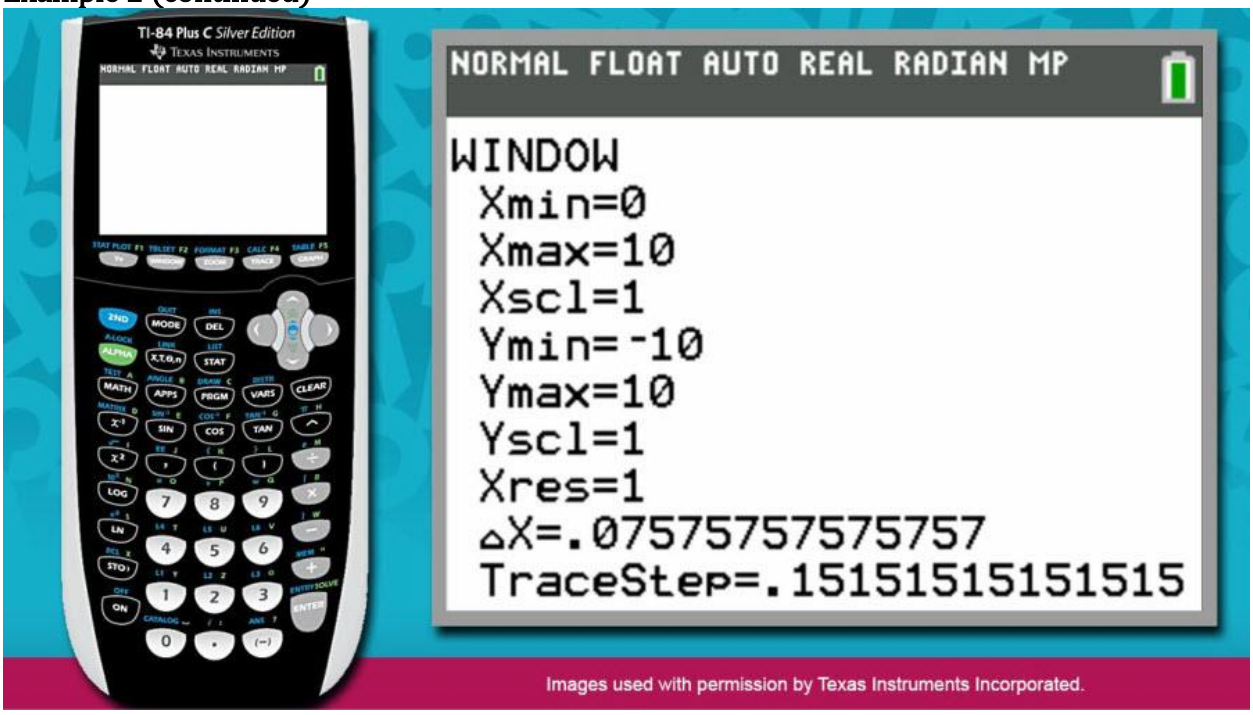
You will immediately notice that you don't see a line appear on the calculator's screen. This is a signal that the values along the x- and y-axes are not large enough for the line to appear. You will need to adjust the scale on the x- and y-axes. This is the time for you to consider what values of x and y make sense for this real-world scenario.

Example 2 (continued)



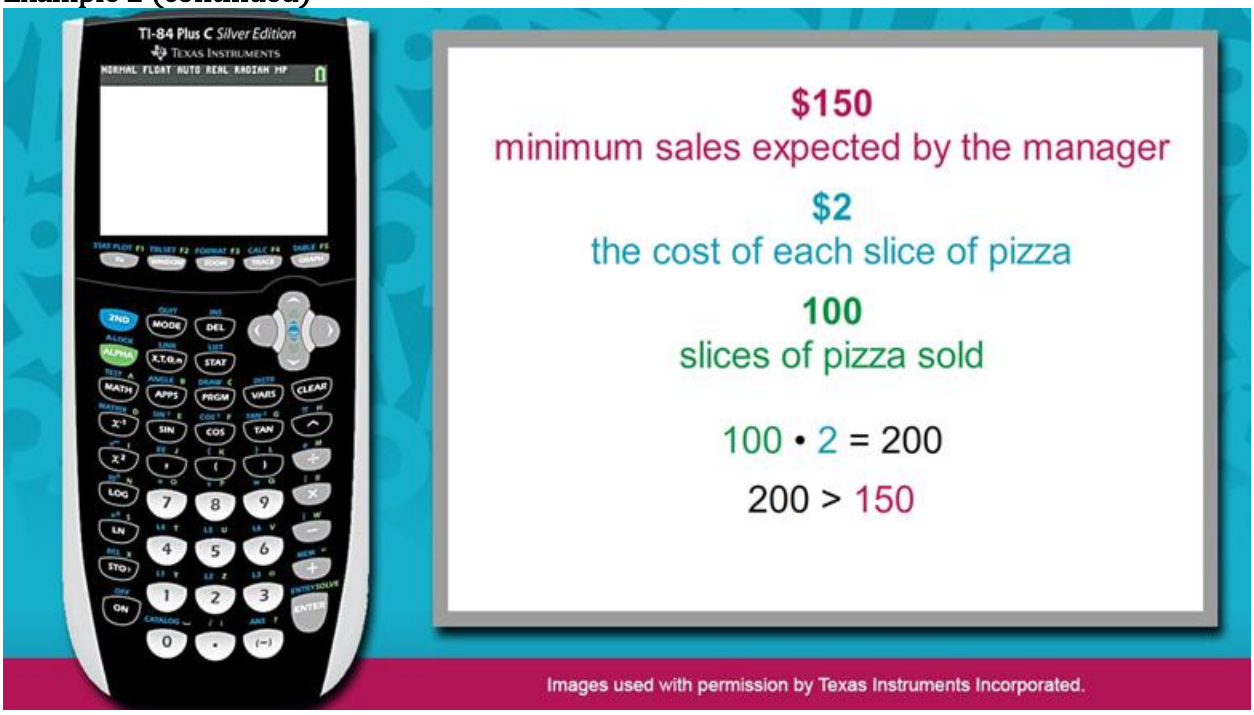
Press WINDOW

Example 2 (continued)



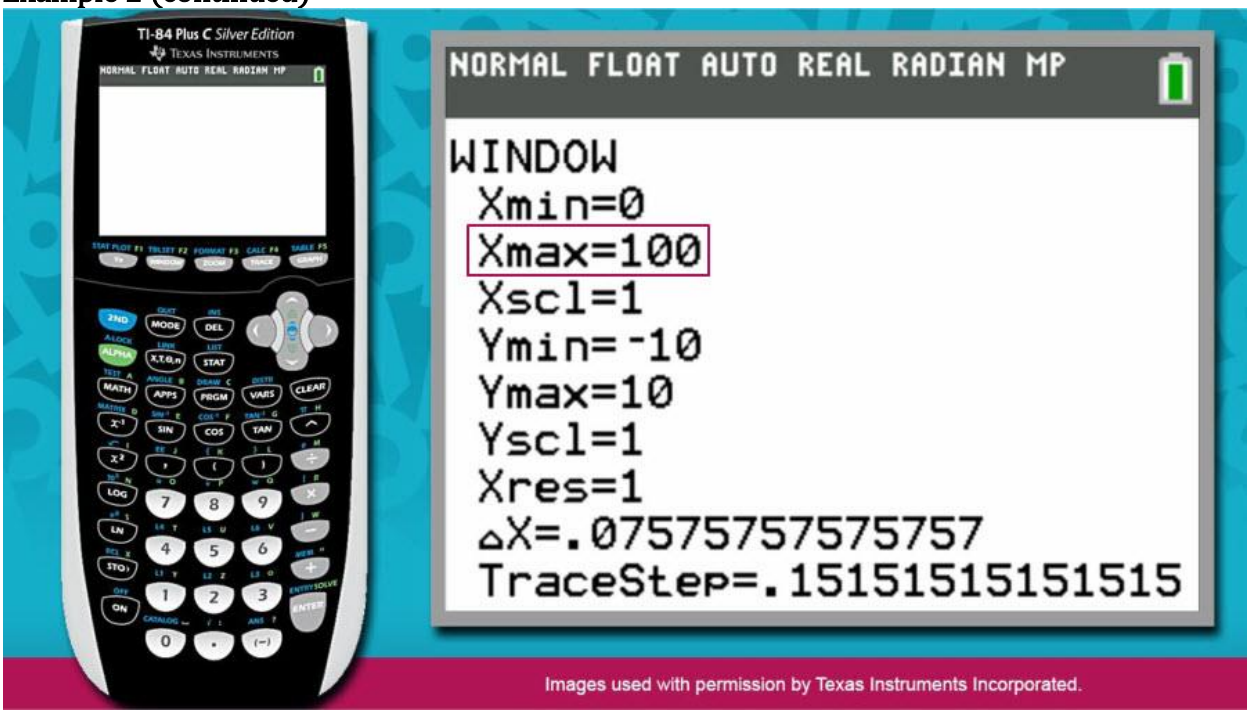
Because x represents the number of pizza slices sold, negative x-values will not make sense. It is impossible to sell a negative number of pizza slices. The smallest number that can be sold is 0. So set the minimum x-value to 0.

Example 2 (continued)



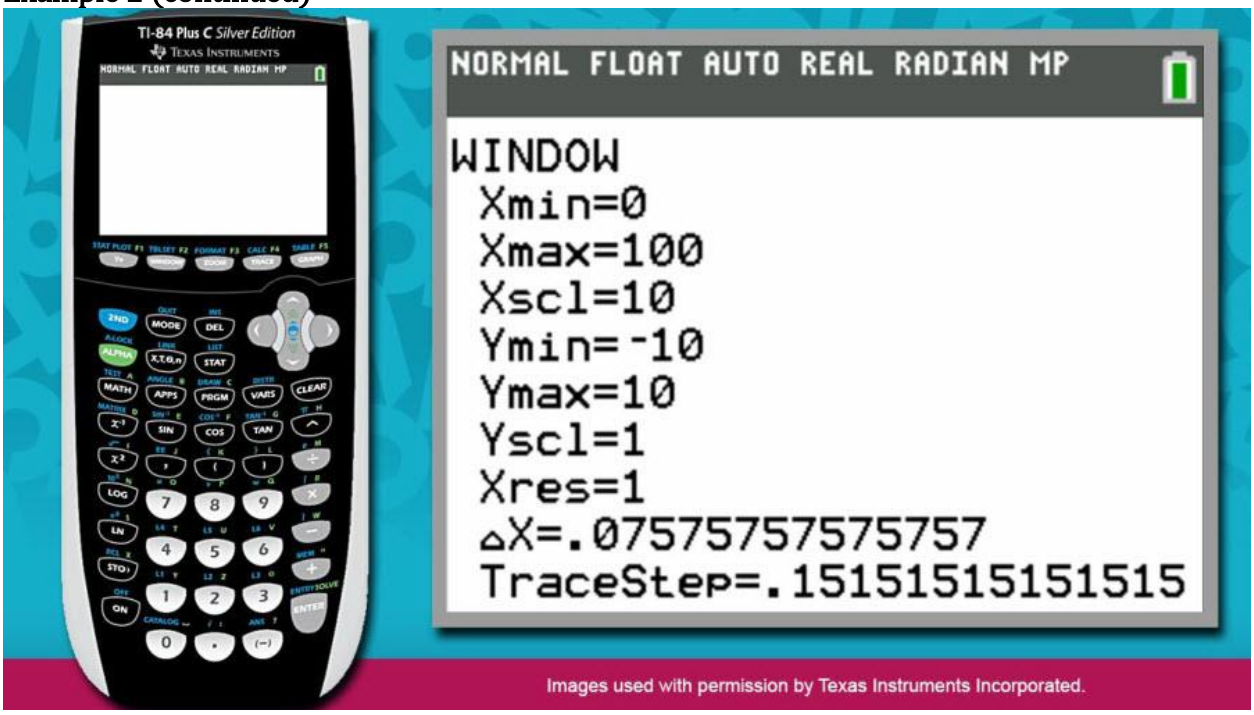
You aren't told how many slices of pizza, in total, are available for sale. So you can't be certain exactly what your maximum x-value should be. You will have to reason your way through this part, a little. You know that the manager expects sales of more than \$150 and the pizza slices are sold for \$2 each. If the concession stand sold 100 pizza slices, sales would be \$200…which is more than \$150.

Example 2 (continued)



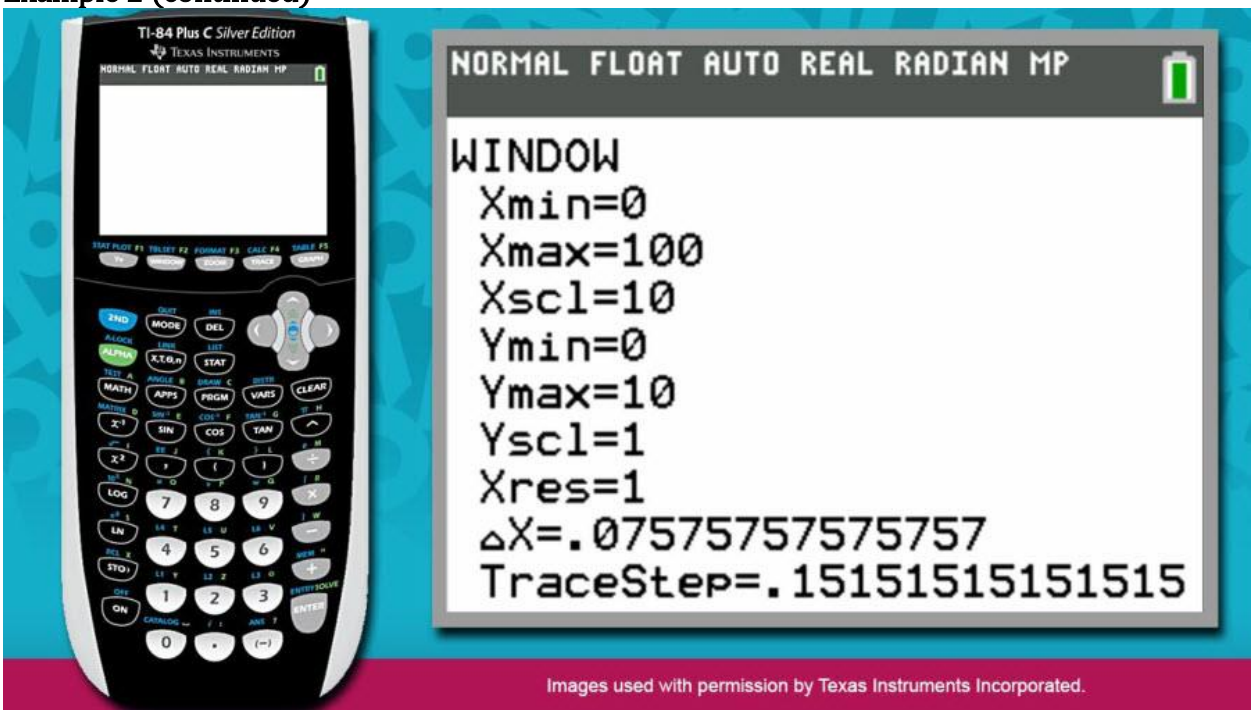
So, set the maximum x-value to 100. If you notice that this value is too small or too large, you can change it later.

Example 2 (continued)



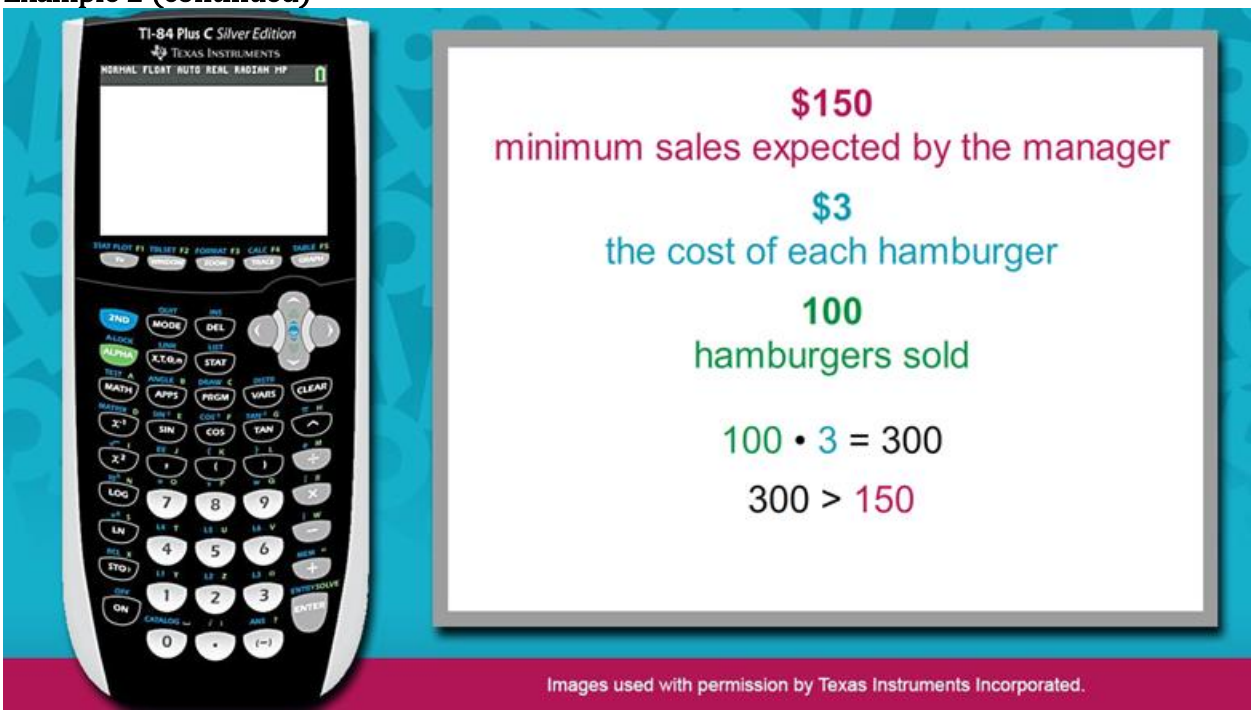
The x-scale lets the calculator know how much distance to put between the tick marks on the x-axis. Because the x-min is set to 0 and the x-max is set to 100, it is appropriate to increase the x-scale, so that you will have a good view of the line once it is graphed. Change the x-scale from 1 to 10. Now, the x-axis will count off by tens, instead of counting off by ones.

Example 2 (continued)



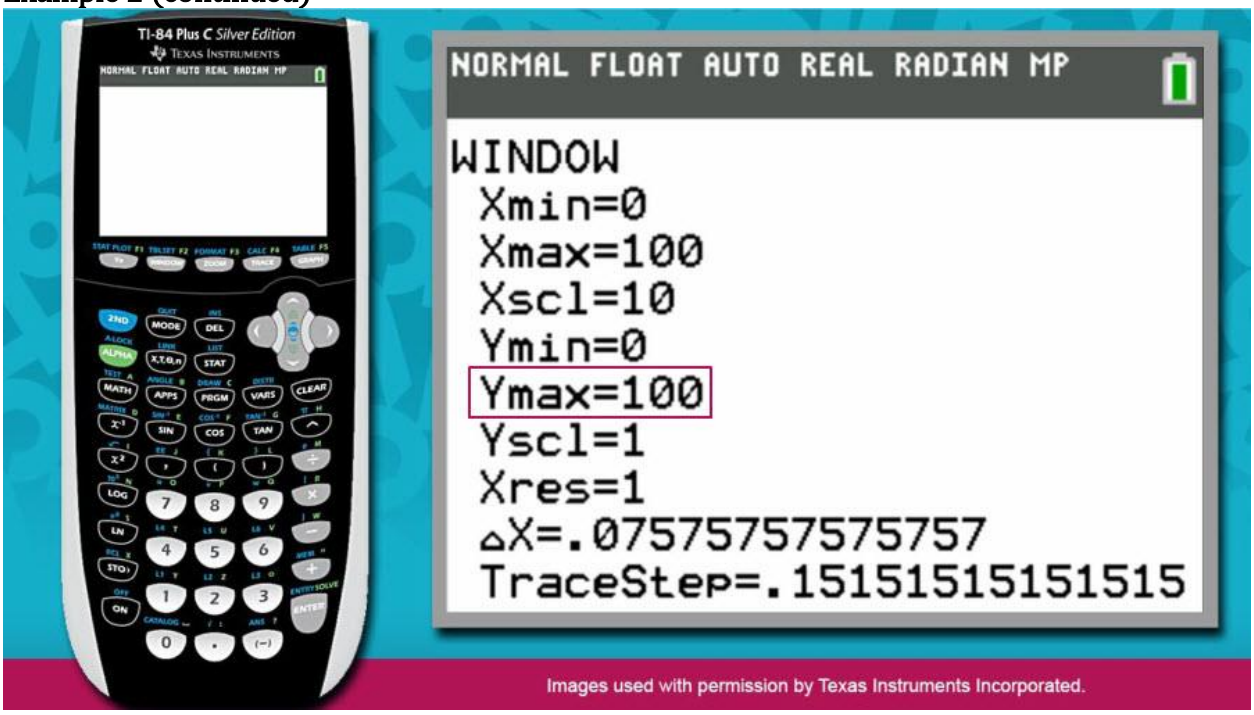
Now move on to adjust the values on the y-axis. Because y represents the number of hamburgers sold, negative y-values will not make sense. It is impossible to sell a negative number of hamburgers. The smallest number that can be sold is 0. So set the minimum y-value to 0.

Example 2 (continued)



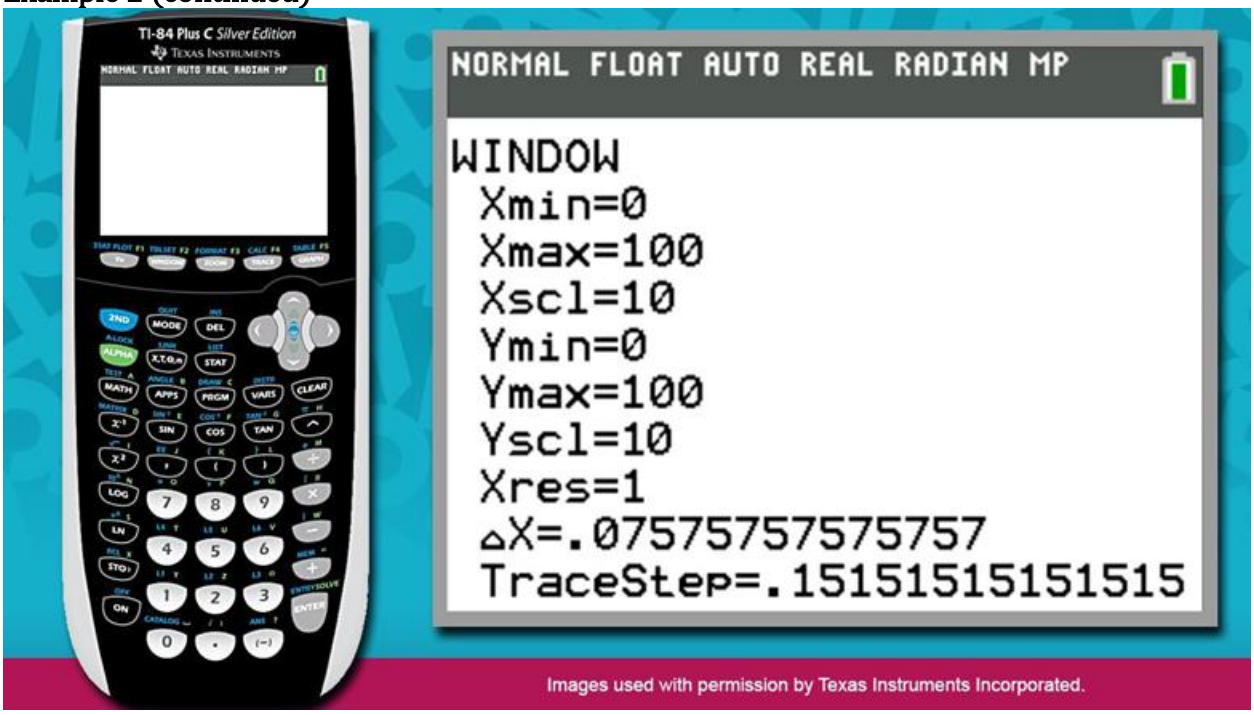
You aren't told how many hamburgers, in total, are available for sale. So you can't be certain exactly what your maximum y-value should be. You will have to reason your way through this part, as well. You know that the manager expects sales of more than \$150 and the hamburgers are sold for \$3 each. If the concession stand sold 100 hamburgers, sales would be \$300…which is more than \$150.

Example 2 (continued)



So, set the maximum y-value to 100. If you notice that this value is too small or too large, you can adjust it later.

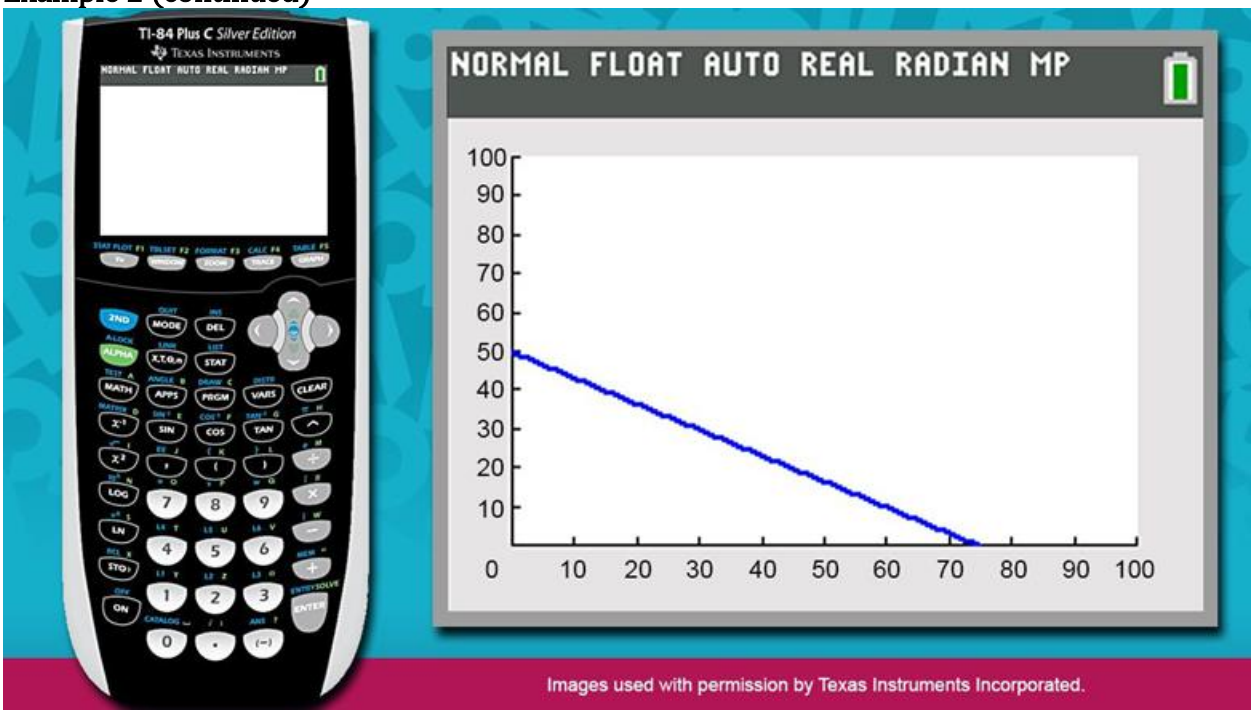
Example 2 (continued)



Just as the x-scale lets the calculator know how much distance to put between the tick the marks on the x-axis, the y-scale lets the calculator know how much distance to put between the tick marks on the y-axis. The x-min and y-min are each set to 0 and the x-max and y-max are each set to 100. It is appropriate then for the x-scale and y-scale to be set to the same values as well. This way, both the x-axis and the y-axis will have the same distance between their respective tick marks. So, change the y-scale to 10.

Leave the remaining values as they are. It is now time to look at the graph.

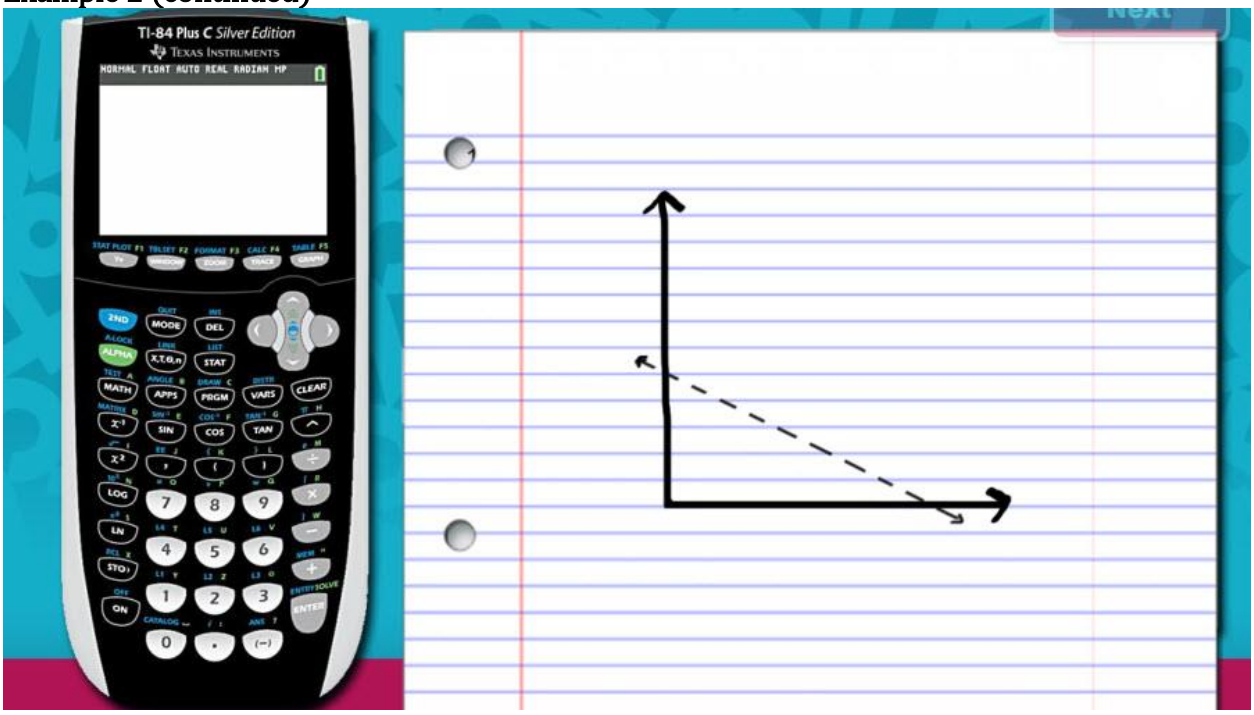
Example 2 (continued)



Press GRAPH

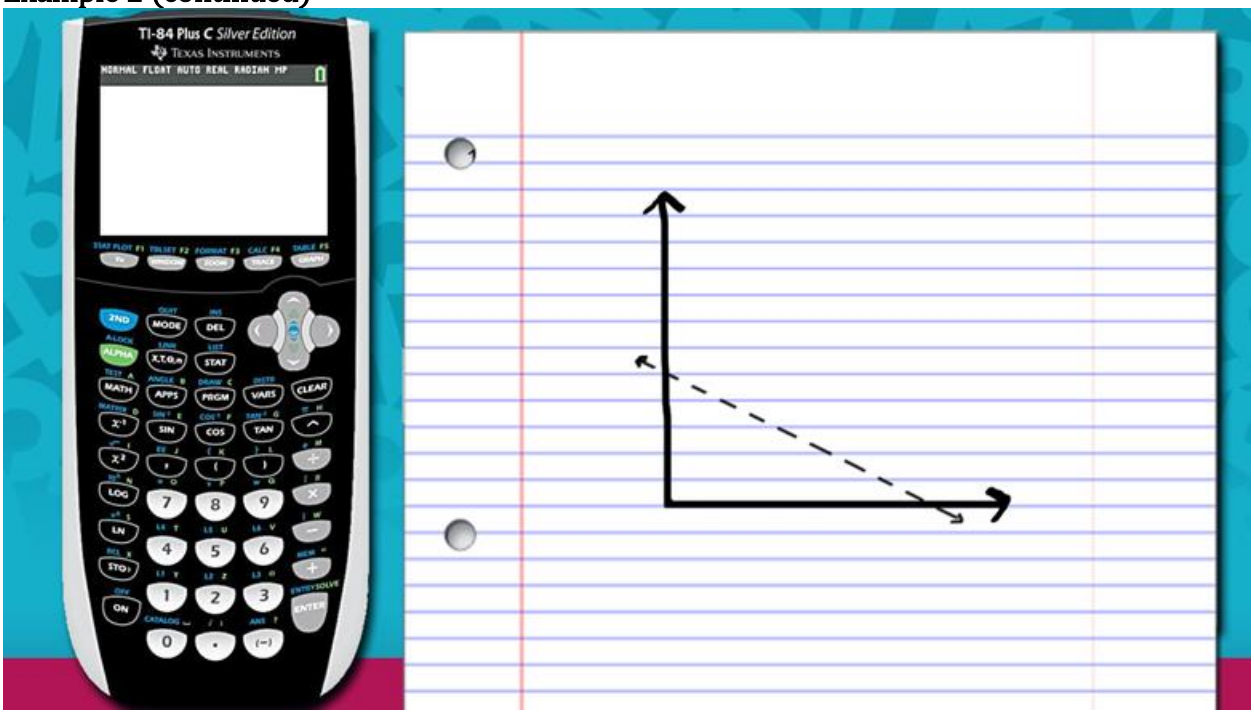
Notice that the calculator only shows the portion of the line included in the first quadrant. The minimum x-value is 0, the maximum x-value is 100, and there are 10 units between consecutive tick marks along the x-axis. Or in other words, the x-axis counts off by 10, beginning at 0 and ending at 100. The same pattern is true of the values along the y-axis.

Example 2 (continued)



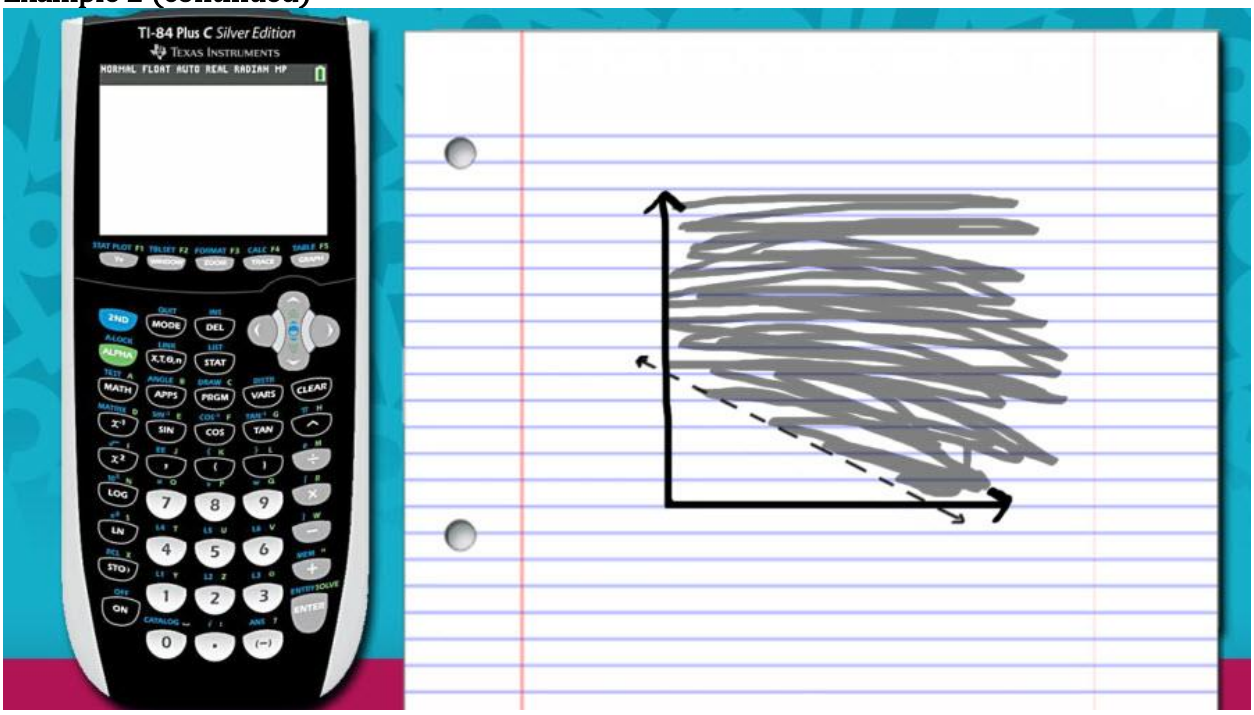
On a piece of paper, roughly sketch the graph that you see on the calculator. Remember, however, that because the inequality $y > -\frac{2}{3}x + 50$ is a strict inequality, your rough sketch should include a dashed line, not a solid line.

Example 2 (continued)



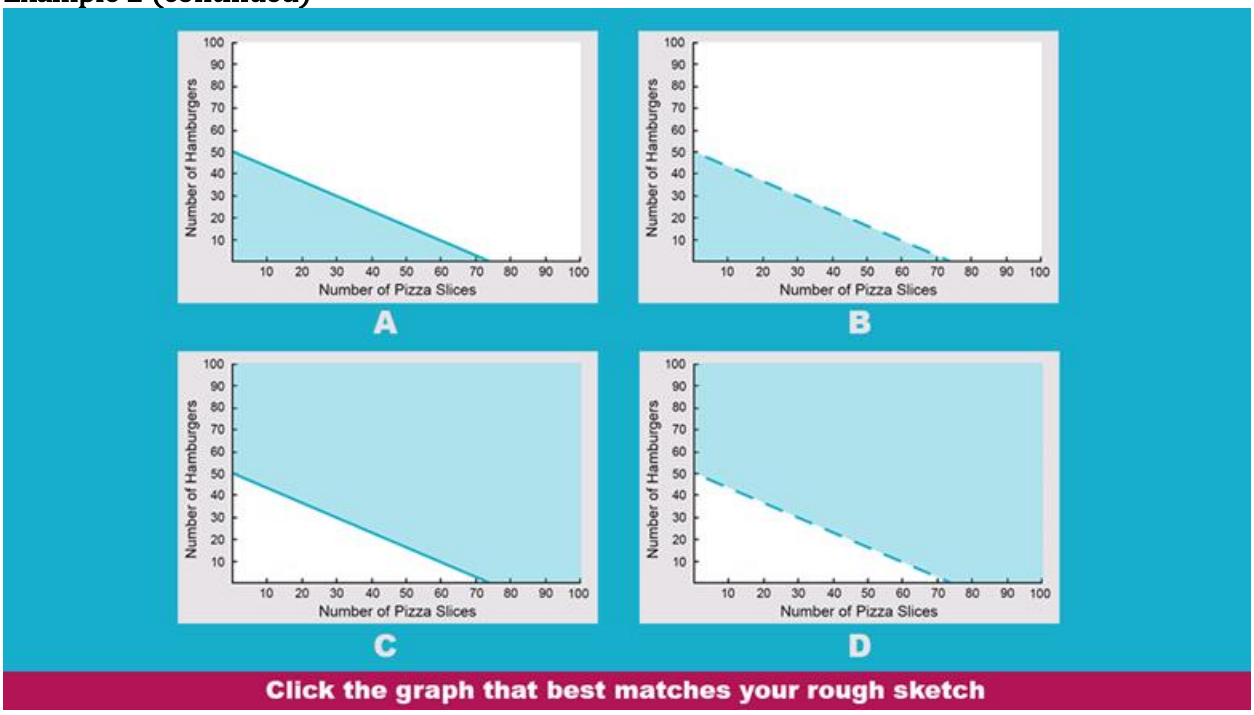
Now that you have a rough sketch of the boundary line of the inequality, it is time to determine the appropriate shading.

Example 2 (continued)



Because the inequality includes "greater than," the graph will include shading above the boundary line. Add the shading to your rough sketch.

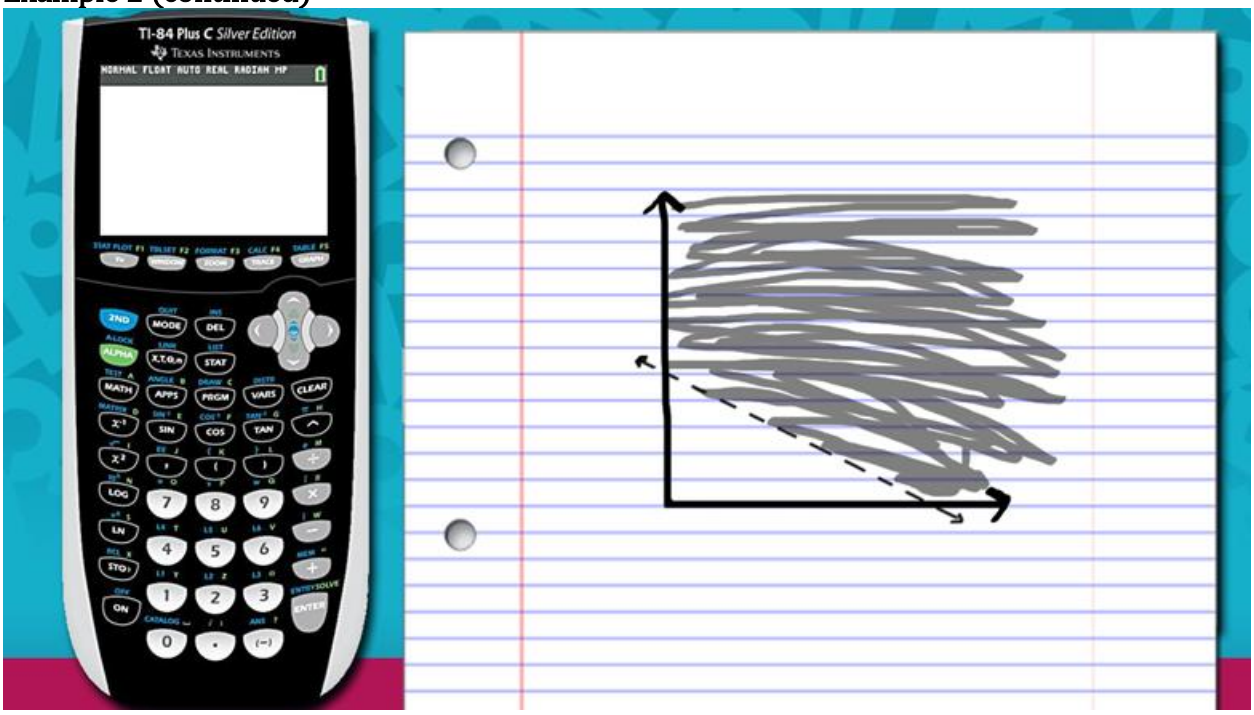
Example 2 (continued)



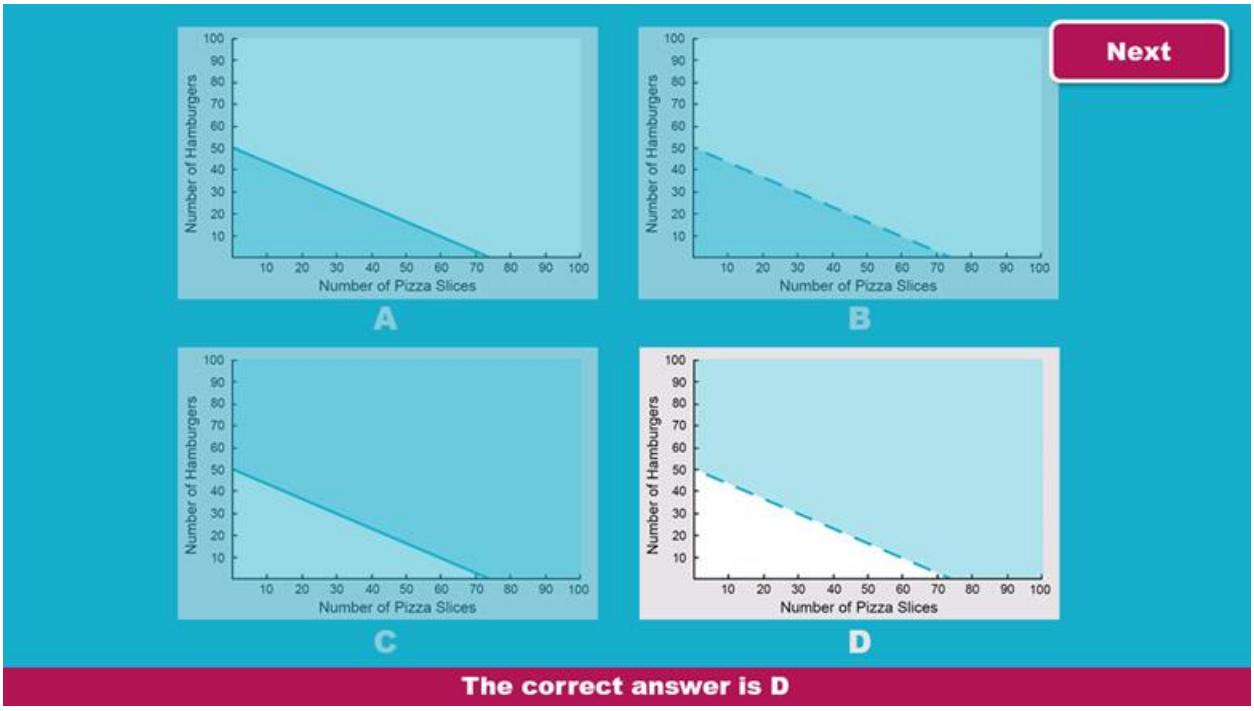
Which of the answer choices best matches your rough sketch?

Click the graph the best matches your rough sketch.

Example 2 (continued)

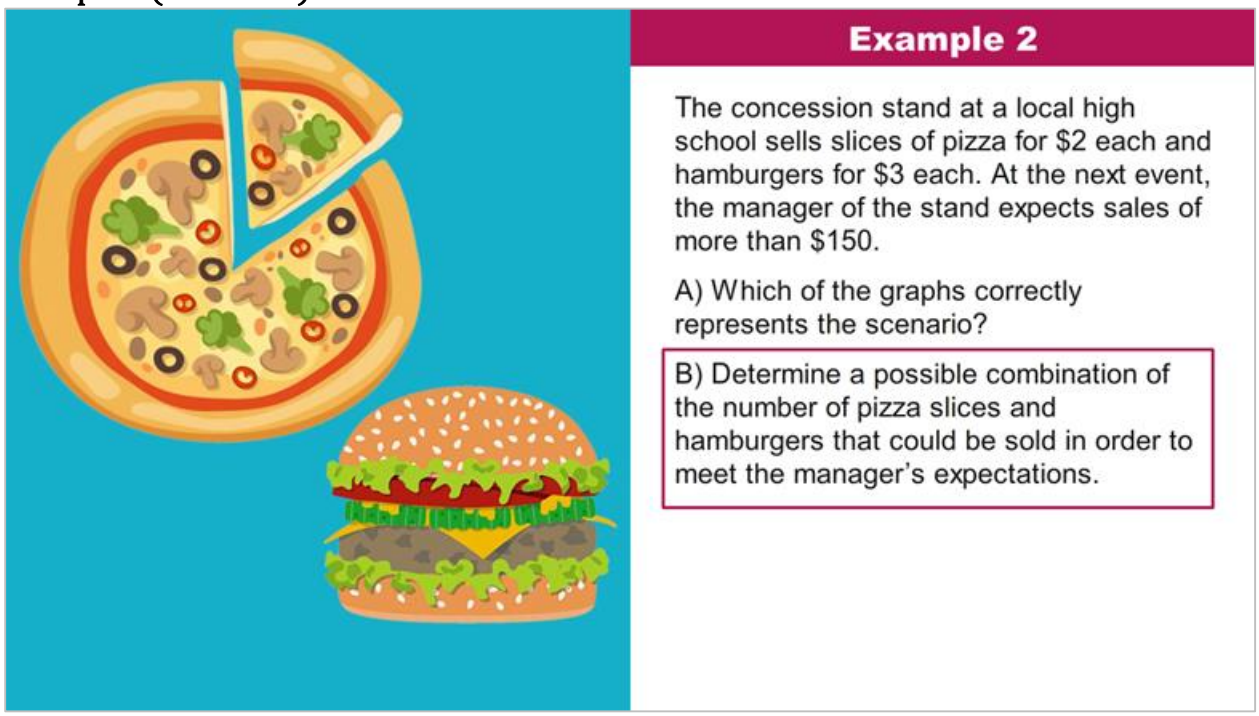


Your sketch should look similar to this.



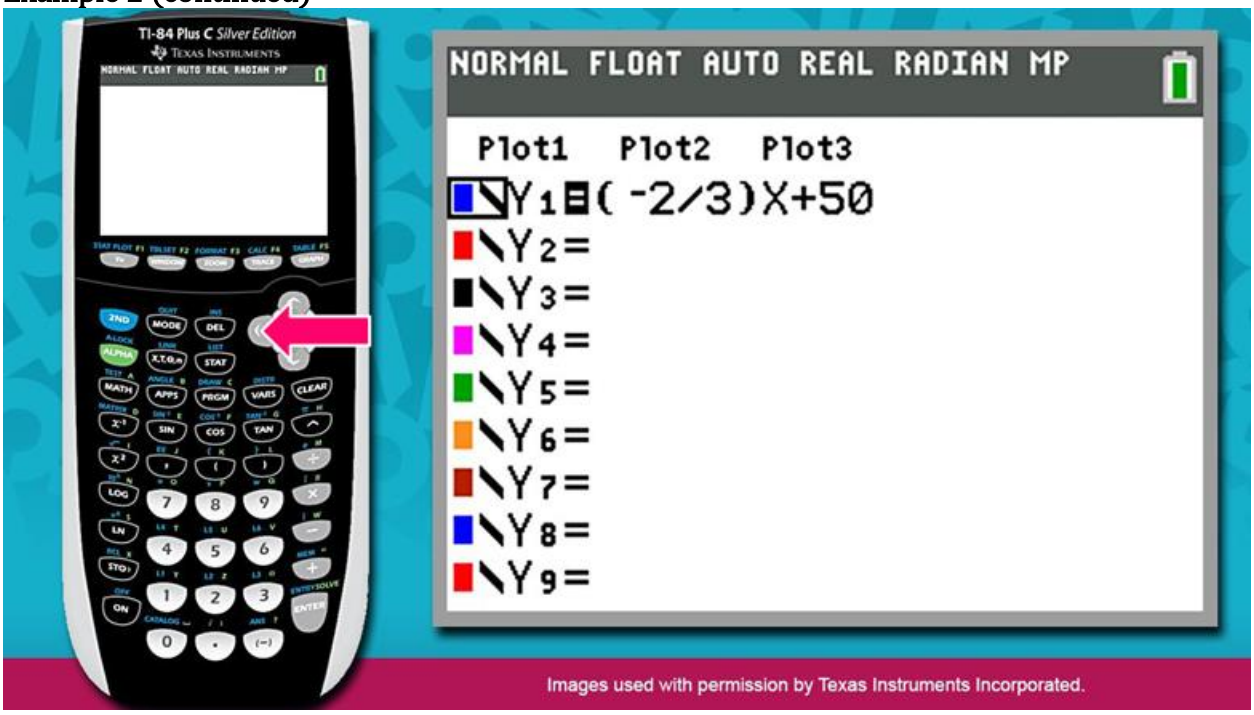
The correct answer is D.

Example 2 (continued)



Now that you know which graph correctly represents the scenario and have answered Part A, it's time to move on to Part B. In order to determine a possible combination of pizza slices and hamburgers that could be sold to achieve sales of more than $150, you must analyze the graph and determine the points that are included in the solution set. You will need to view the graph, including its shading, on the calculator.

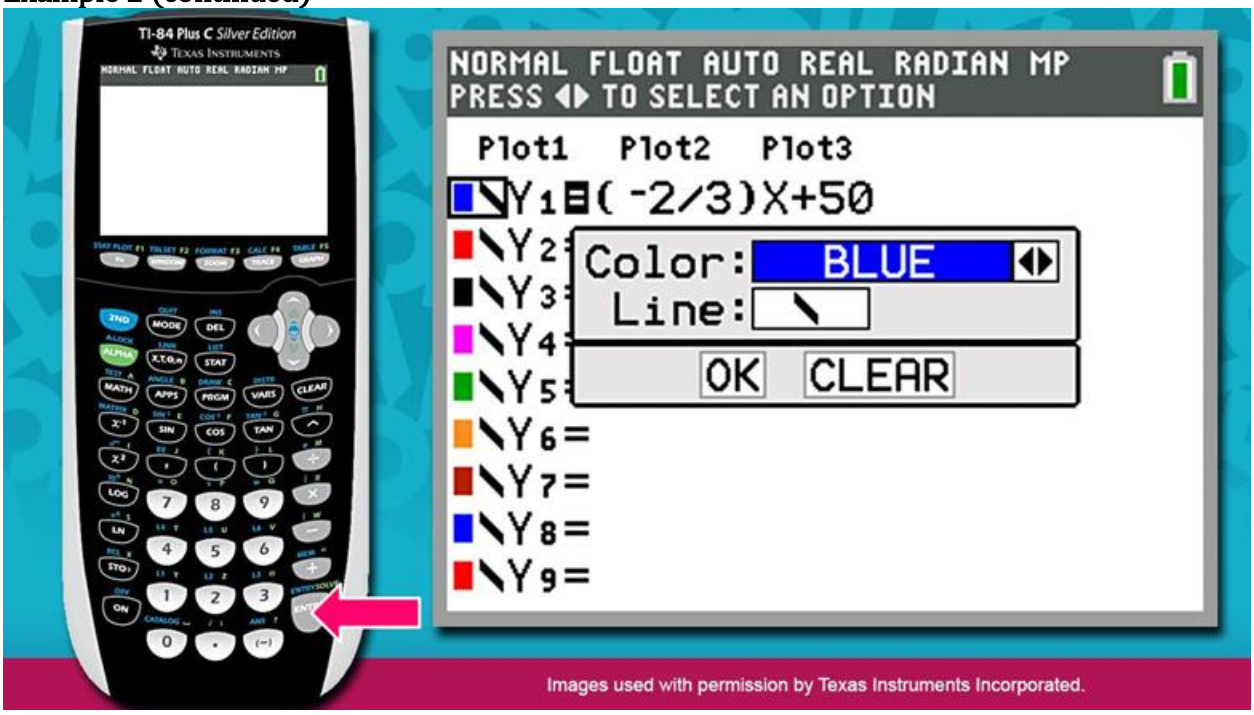
Example 2 (continued)



Press Y=

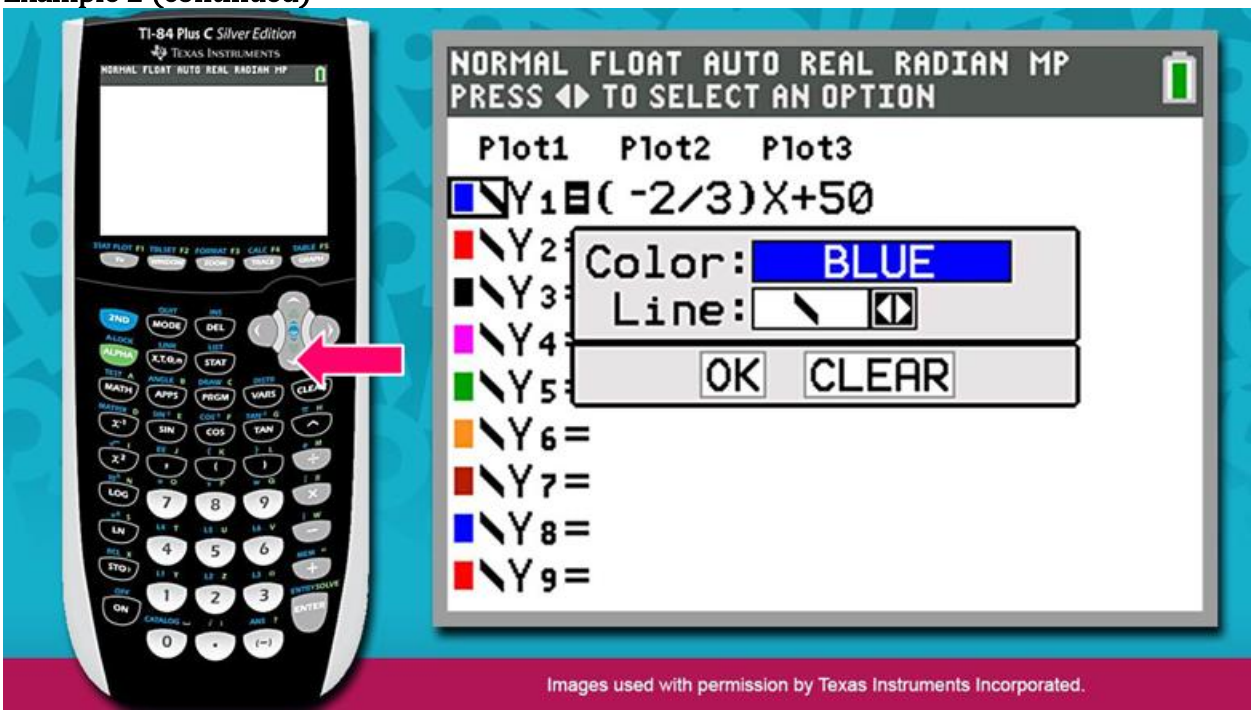
Press the left arrow until the cursor is blinking on the line beside Y_1.

Example 2 (continued)



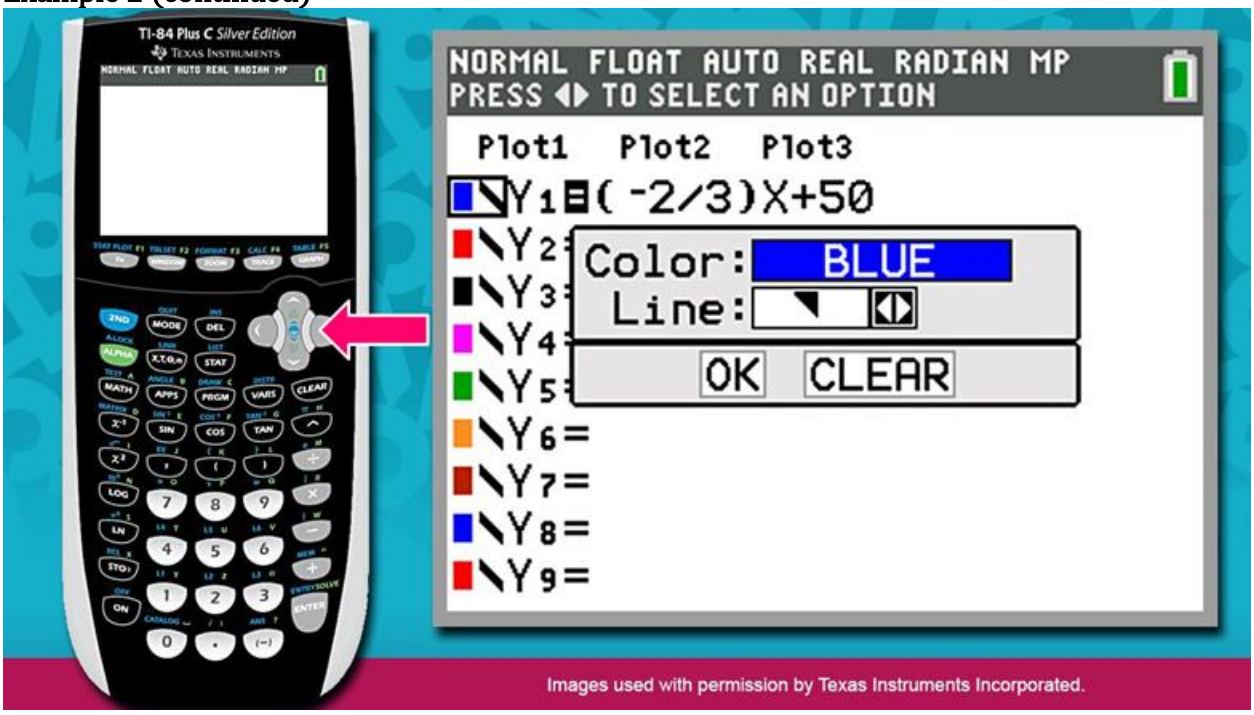
Press ENTER.

Example 2 (continued)



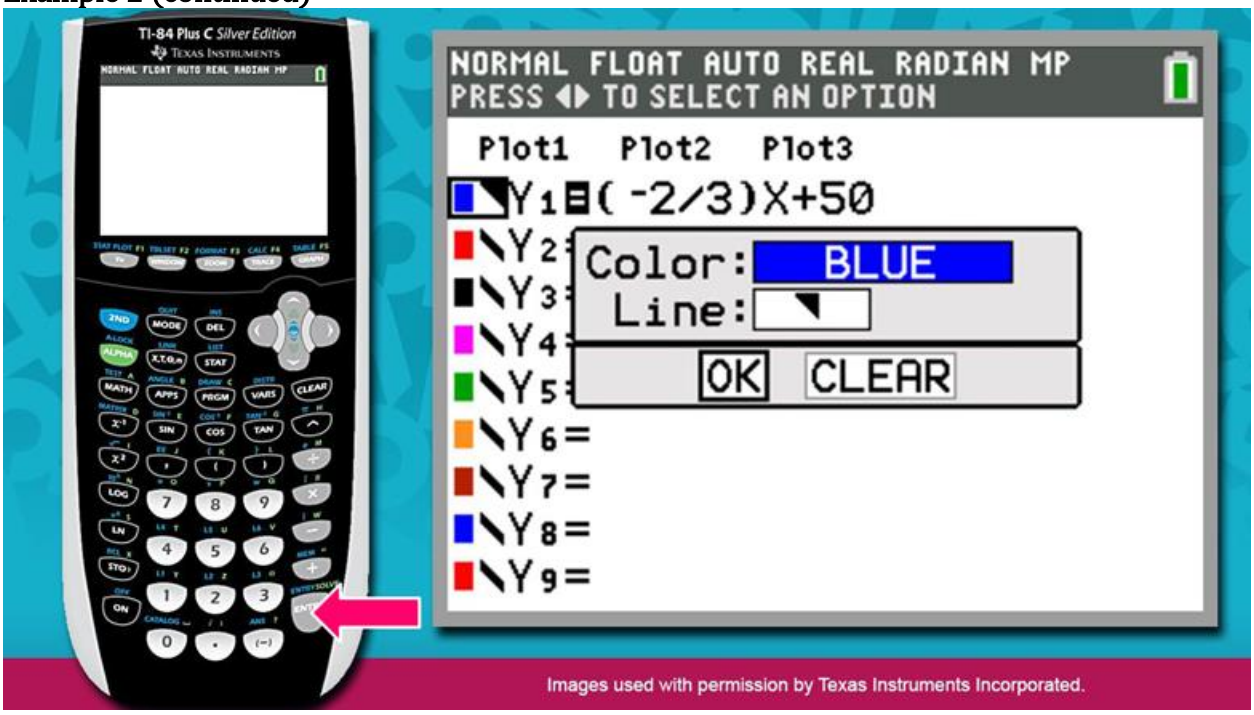
Press the down arrow until the cursor is blinking beside the "Line" prompt.

Example 2 (continued)



Press the right arrow until the option for shading above the line appears.

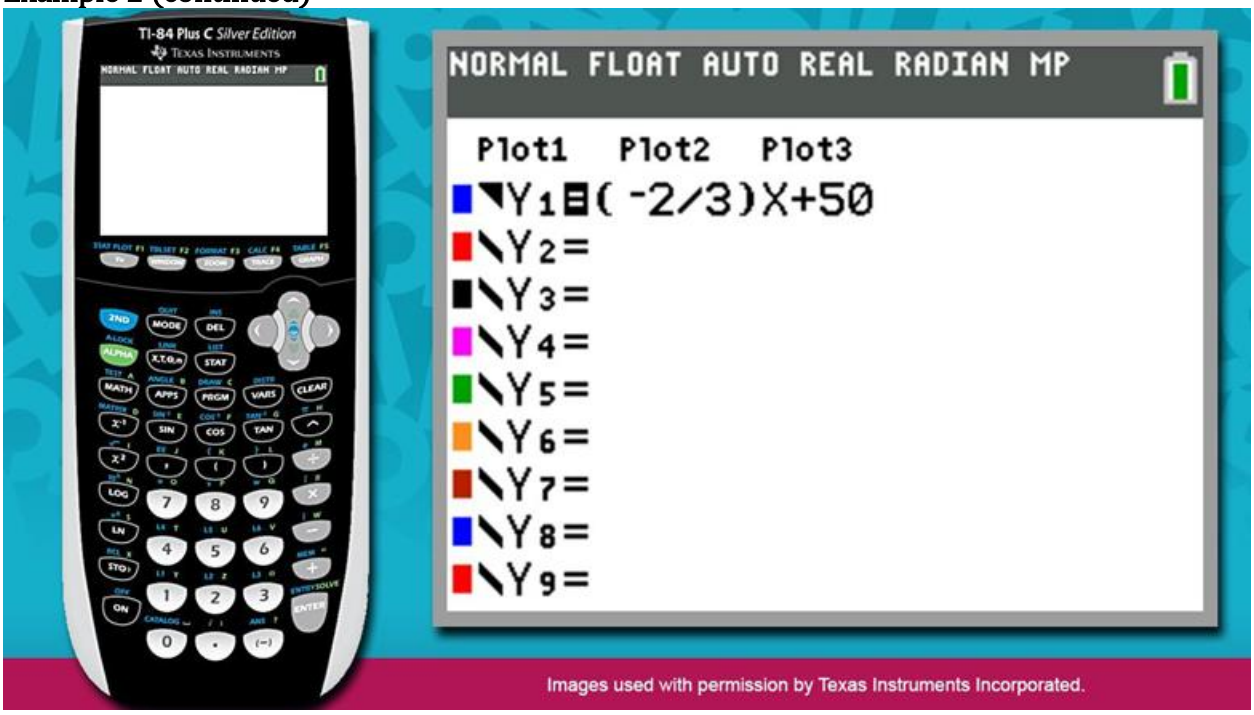
Example 2 (continued)



Press ENTER to confirm your selection.

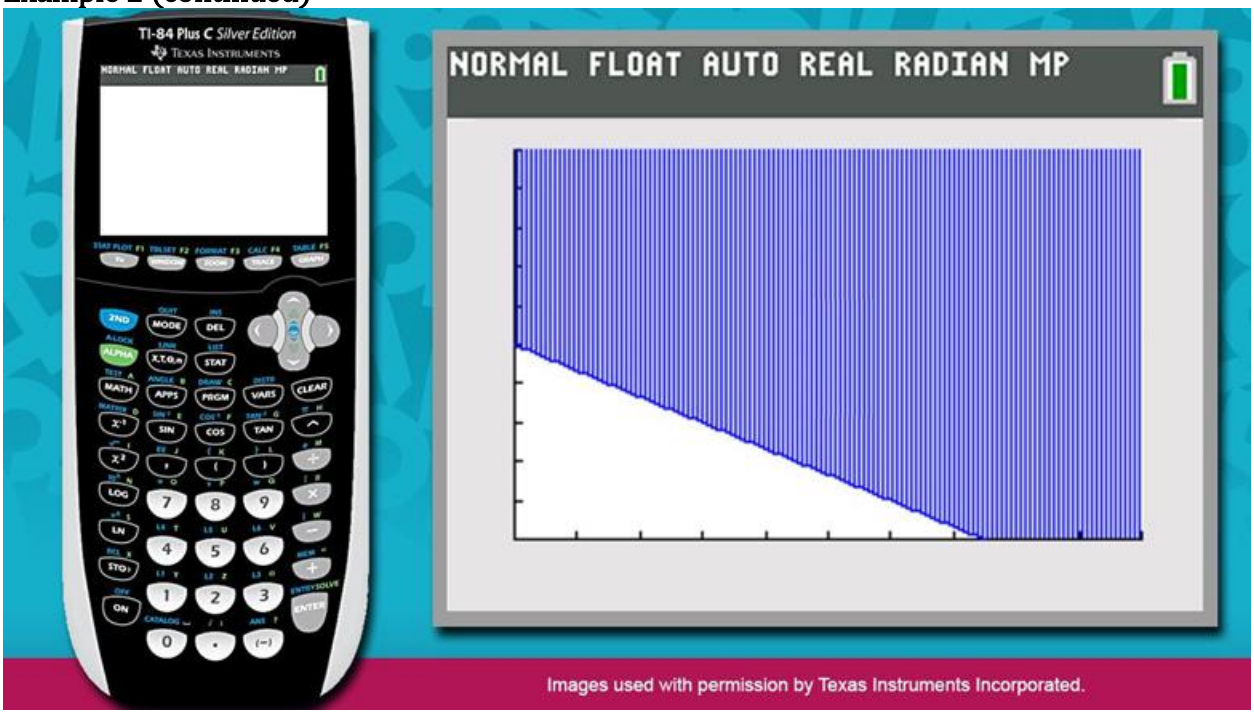
Then press ENTER again once the cursor moves to OK.

Example 2 (continued)



Now the calculator is set to shade the appropriate region.

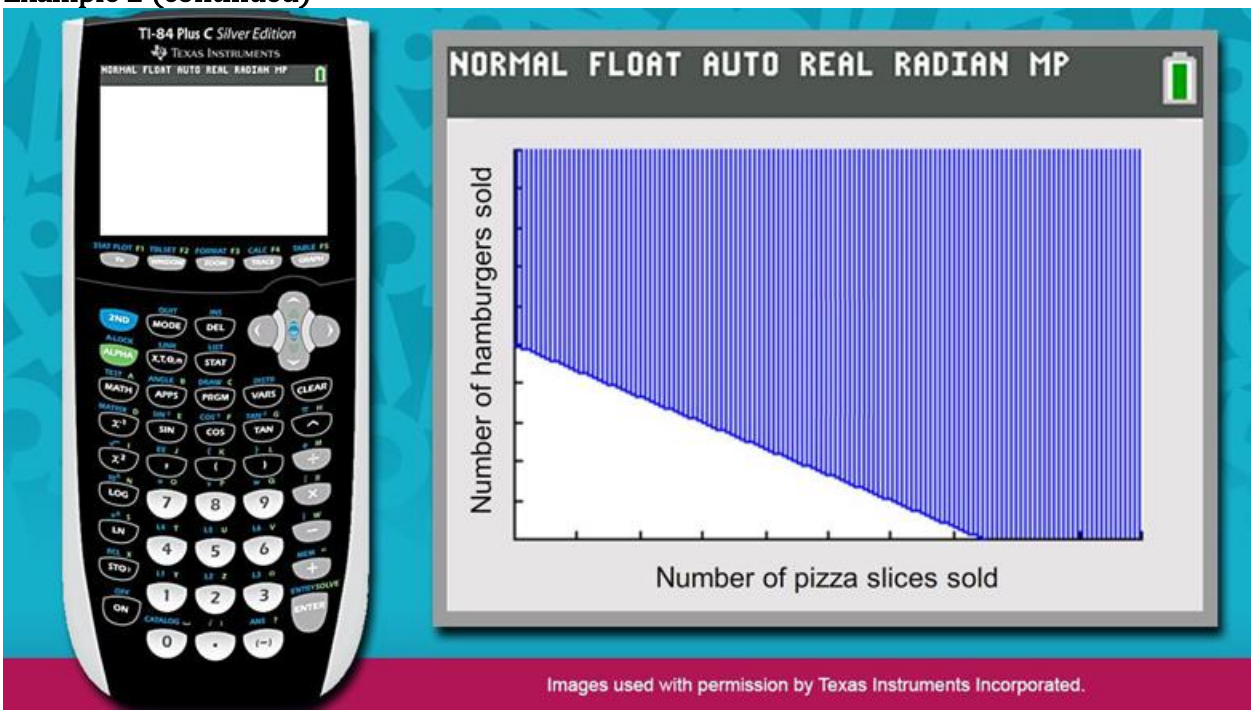
Example 2 (continued)



Press GRAPH

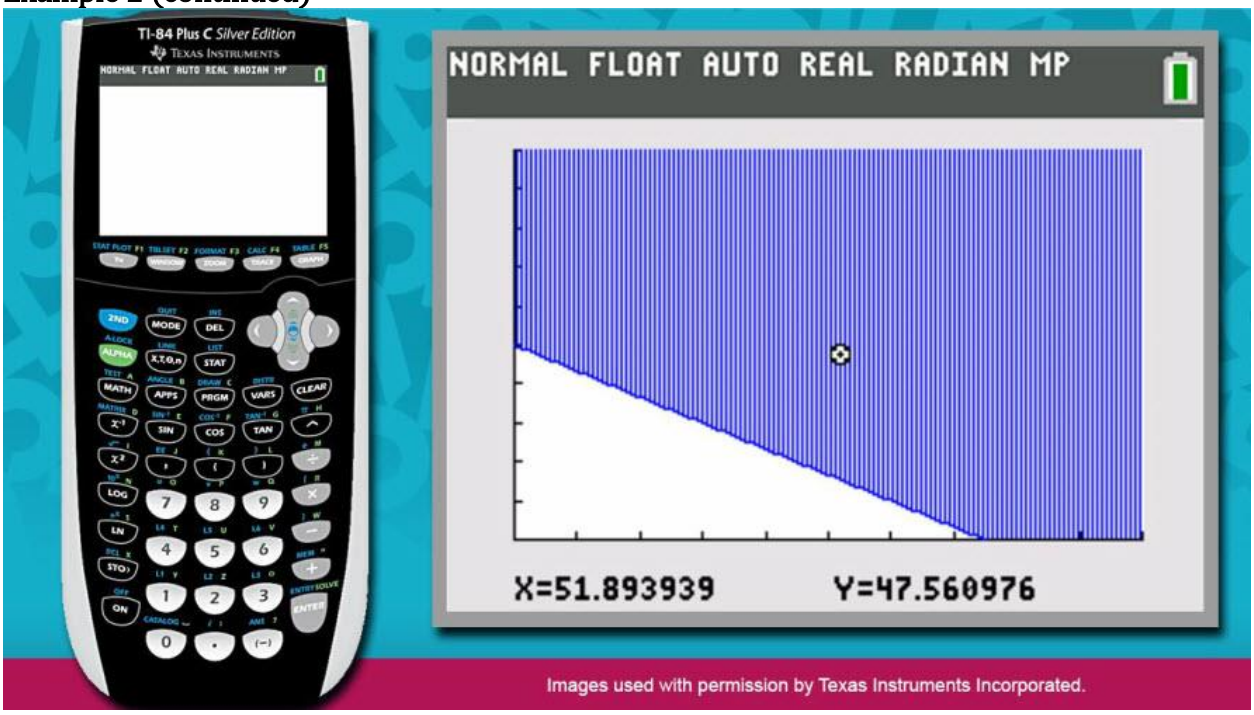
The graph now includes the appropriate shading. Now you can begin analyzing the graph to determine solutions. Although the calculator shows a solid boundary line, you know that the graph of the inequality includes a dashed boundary line. Solutions will not be found along this line. You will only find solutions in the shaded area.

Example 2 (continued)



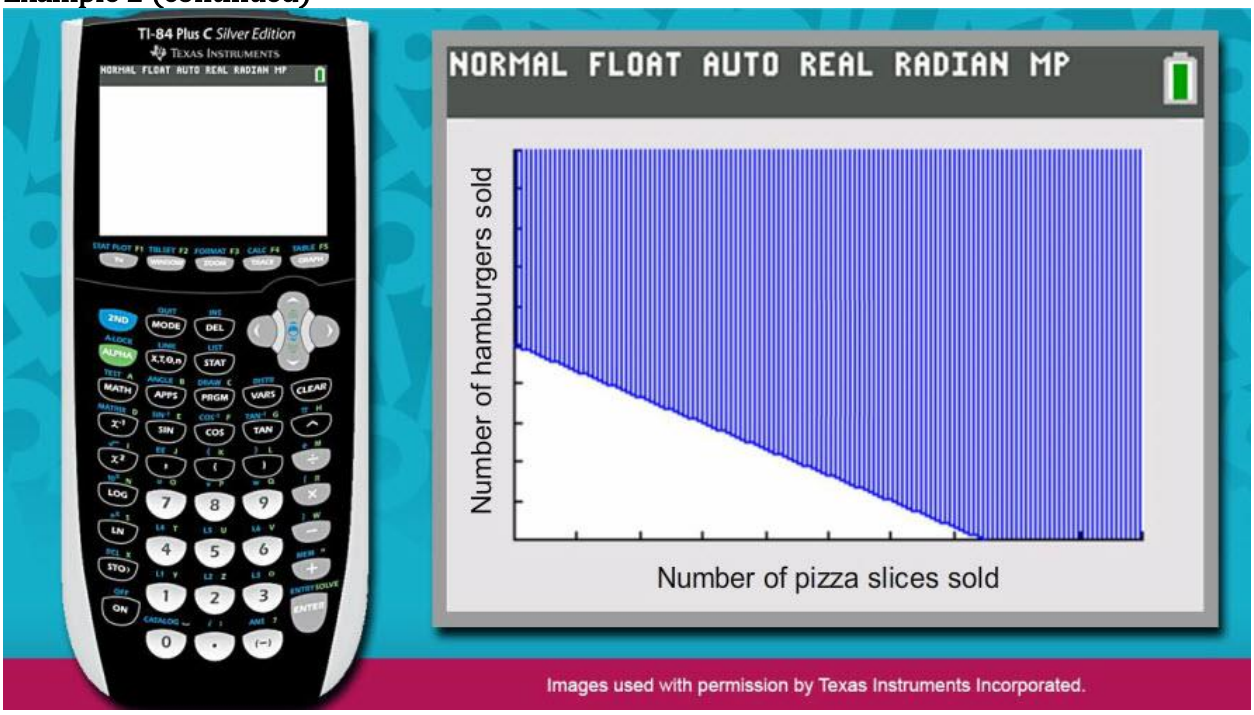
Press the right, left, up, and down arrow keys in order to locate points included in the shaded area

Example 2 (continued)



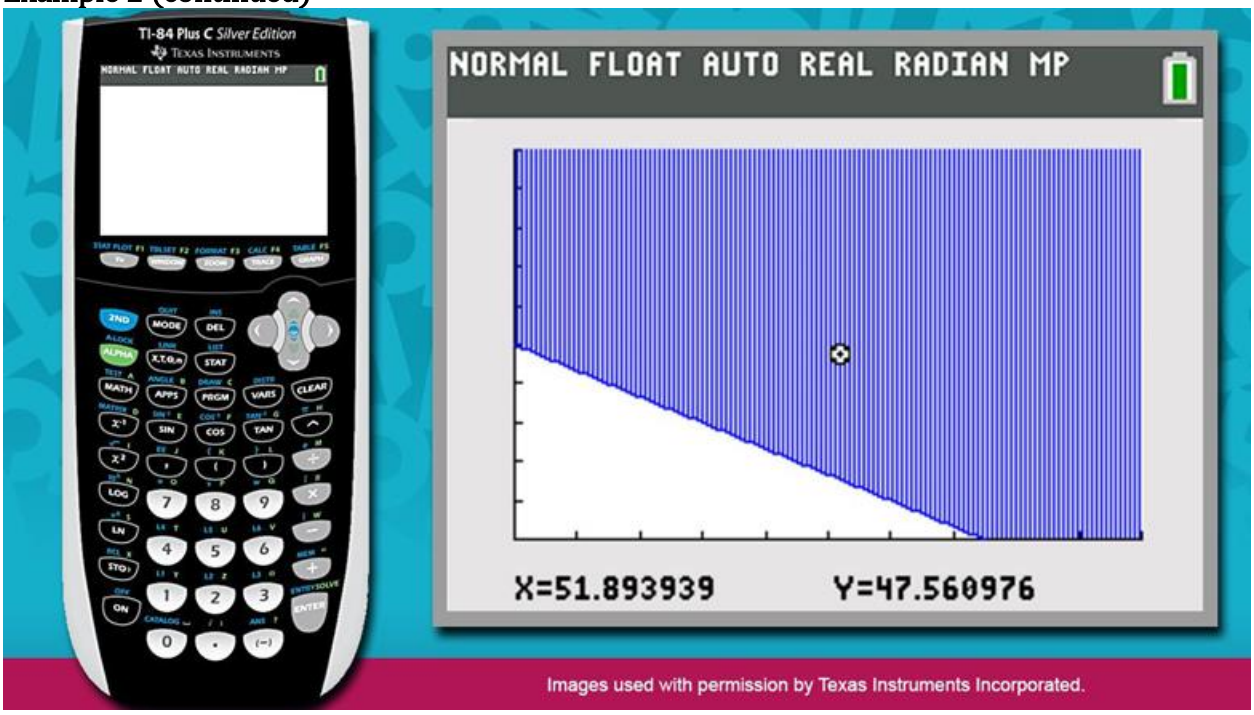
As you press the arrow keys to move around the shaded area, you will notice that the cursor will most likely move to points that include decimal values. Because you are searching for solutions that are appropriate for this real-world situation, you must determine if decimal values of x and y are appropriate.

Example 2 (continued)



Remember that in the given scenario, x represents the number of pizza slices sold and y represents the number of hamburgers sold. It isn't possible to sell a portion of a pizza slice or a portion of a hamburger. So decimal values of x and y are not appropriate. You must round any decimal value to the nearest whole number that is included in the solution set.

Example 2 (continued)



Notice this point included in the shaded area. If you round the x- and y-values to the nearest whole numbers included in the solution set, the result is (52, 48). This means that the concession stand could sell 52 slices of pizza and 48 hamburgers and achieve sales of more than \$150.

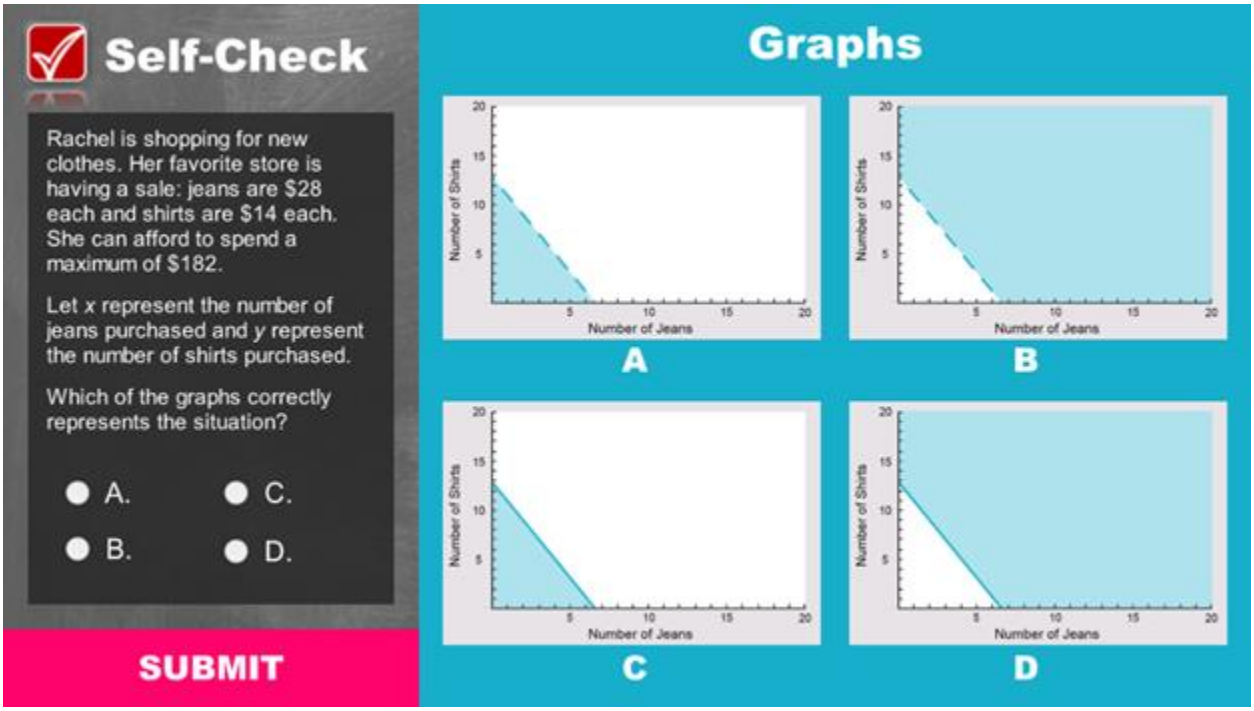
Example 2 (continued)

You can verify this algebraically, by substituting the values into your original inequality. Substitute 52 for x and 48 for y.

$$
\begin{array}{lllll}
(52, 48) & & & & \\
2x & + & 3y & > & 150 \\
2(52) & + & 3(48) & & 150 \\
104 & + & 144 & & \\
 & & 248 & > & 150
\end{array}
$$

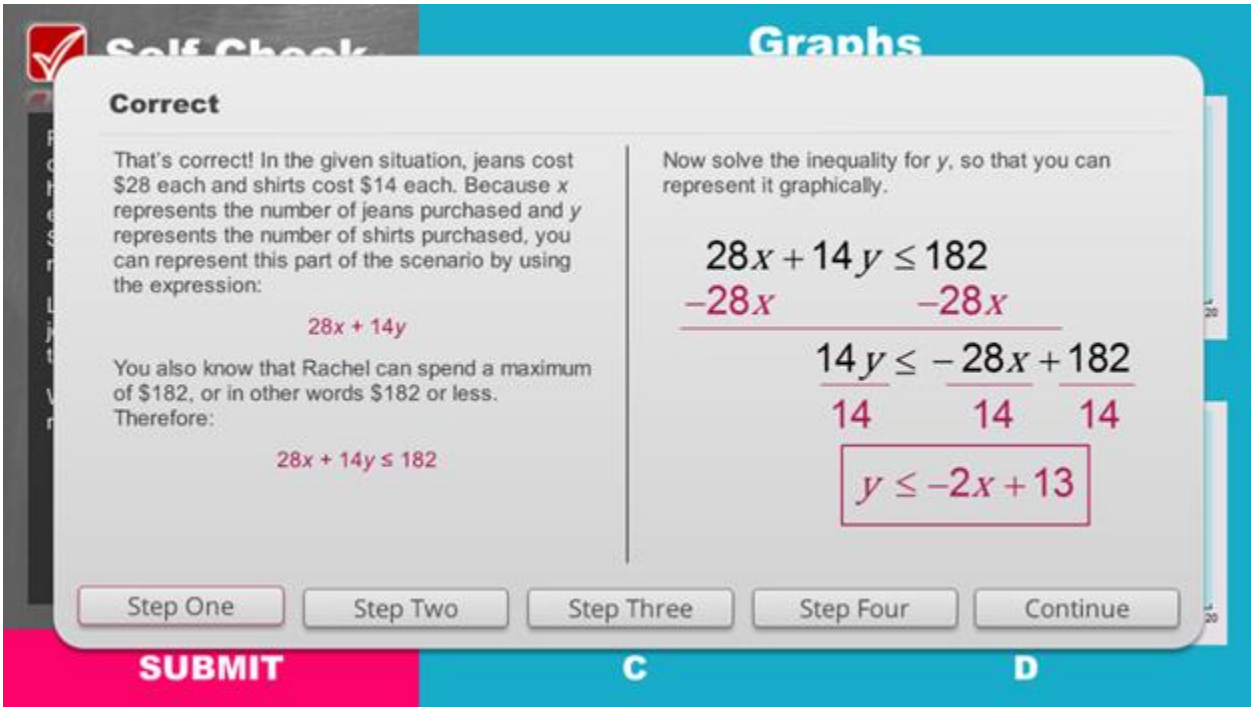
So you can also see, algebraically, that $(52, 48)$ is a solution to the inequality. Any other points, in the shaded area, that include whole-number values for x and y are also solutions to this practical problem.

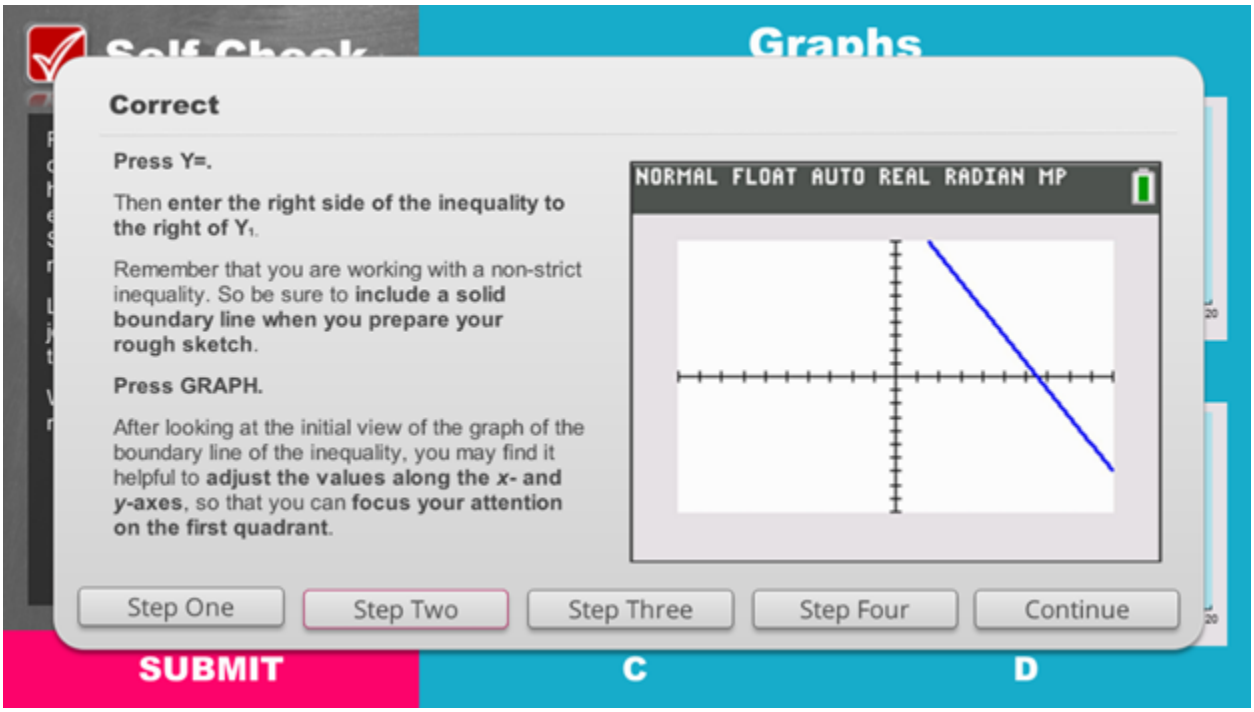
Self-Check



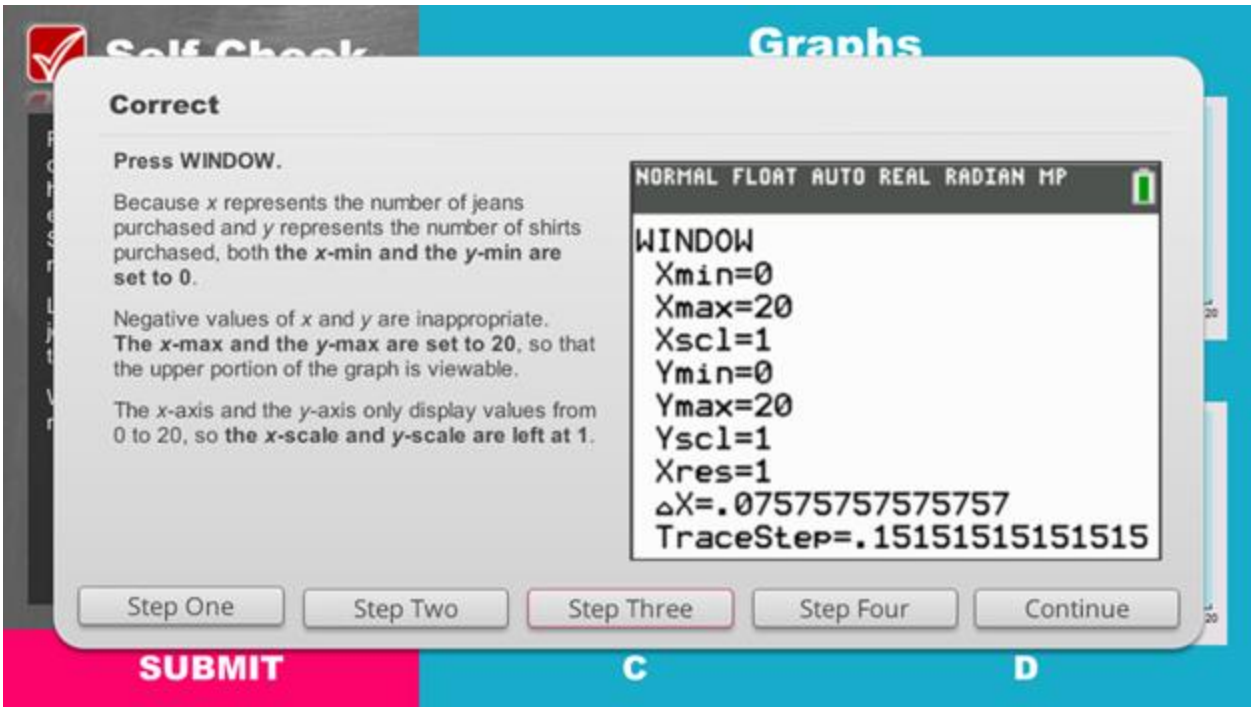
Solve the problem in the image above to check your understanding of the content.

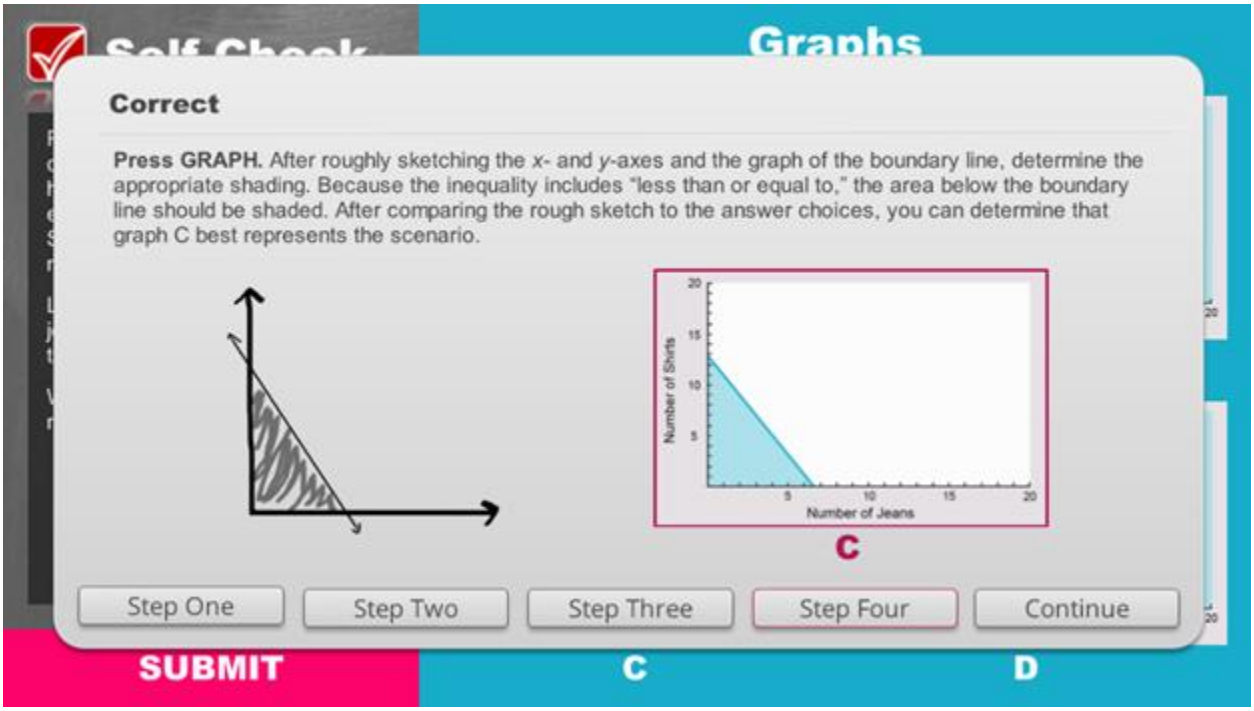
Self-Check: Answer





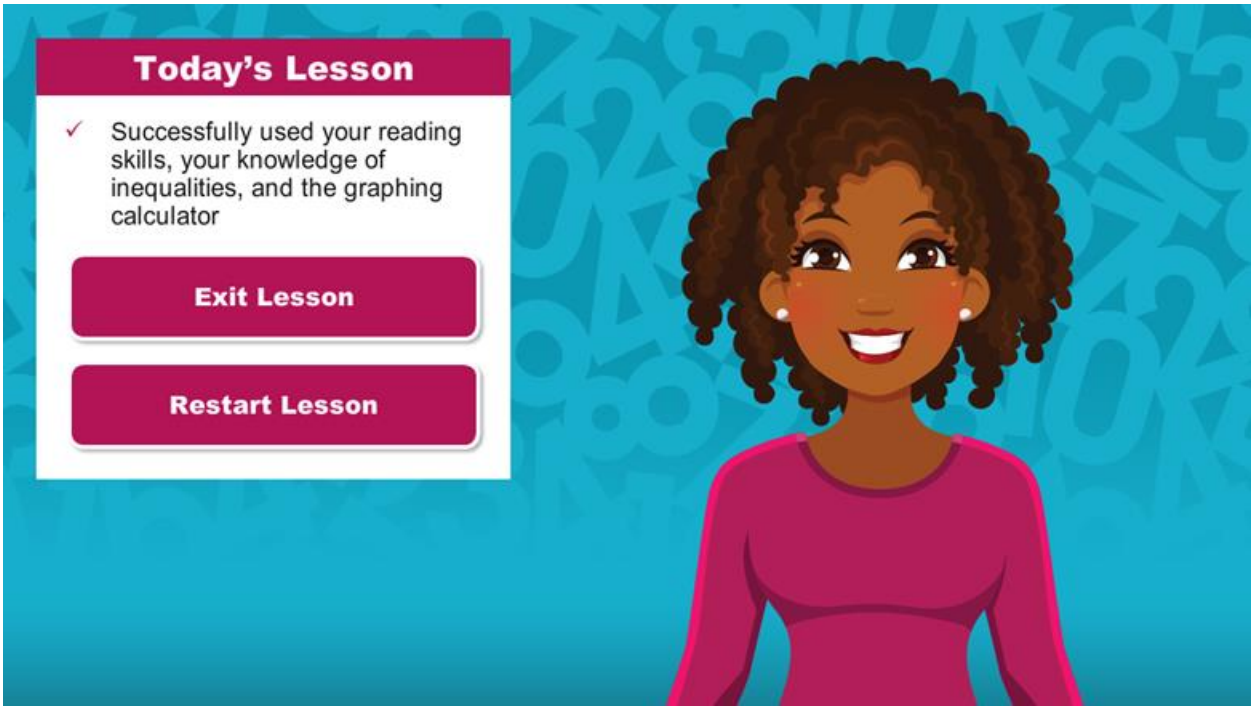
For your reference, the image above shows the correct solution to the self-check problem.





For your reference, the image above shows the correct solution to the self-check problem.

Conclusion



Congratulations! You have reached the conclusion of your lesson on solving practical problems involving inequalities in two variables. You were able to combine your reading skills, along with your knowledge of inequalities, and the use of the graphing calculator in order to successfully complete this lesson.