Module 1: Logic and Reasoning
Topic 3 Content: Law of Contrapositive Transcript

Hi, guys. Welcome to Geometry. This topic is going to focus on the law of contrapositive. You're going to continue to use your logic and reasoning skills in order to draw valid conclusions. You're ready to get started? Let's go.

Now, before we dive into the law of contrapositive and those deductive reasoning skills, I want to bounce back a bit to inductive reasoning. Again remember, inductive reasoning is the use of patterns and observations in order to draw a valid conclusion.

For example, here we have a set of shapes, and we're going to use inductive reasoning to determine the next shape in the pattern.

Notice here we have a triangle, a rectangle, a pentagon, and then we want to figure out what's next.

If you think about this here—because this one may take you a second—how many sides does a triangle have? Three sides. A rectangle? Four sides. A pentagon? Five sides. From the pattern I have here, you can anticipate that the next shape should be a six-sided figure. If we take a look, it actually is.

We followed—let's get the pen back—we followed with the hexagon. The pattern we had here from our shapes is each shape increased by one side. We started with the three-sided figure, moved up to four, five, and then six.

We used inductive reasoning to draw this conclusion here that the final shape should be a six-sided figure. Another study, or another aspect, of deductive reasoning—because we discussed that deductive reasoning is the use of laws and facts and definitions to draw our valid conclusion—is the law of contrapositive. I'm going to show you symbolically what that represents.

If p, then q. If the opposite of q is true; therefore the opposite of p is also true.

That's the law of contrapositive. If you're given a true conditional statement and if the opposite of your conclusion is true, you can conclude that the opposite of your hypothesis is also true.

Let's take a look at this example here.

> If today is Saturday, then yesterday was Friday.

So that's our conditional statement, our "If p, then q." And that is a true statement: "If today is Saturday, then yesterday was Friday."

> Yesterday was not Friday.

Notice that's the opposite of q. Let me make that stand out a little bit for you here.

"If today is Saturday;" that's our p. ". . .then yesterday was Friday," that's our q. "Yesterday was not Friday." So that's the opposite of q.

Based on the law of contrapositive, we can conclude—let's get this out of our way—"therefore, today is not Saturday." We can conclude that the opposite of p is also true.

That's the law of contrapositive in action. You're given a conditional statement, then you're given the opposite of your conclusion, or if you're given that, you can conclude that the opposite of your hypothesis is also true.

Let's take a look at this one, threw a little algebra in the mix.

> If a plus three equals nine, then a equals nine.

Well that's true, right? If we substituted six in there for a (we'll throw it back to Algebra I) six plus three does actually equal nine. So we know that is a true statement.

Now let's take the opposite of q, the opposite of our conclusion: a does not equal six.

Okay, so we'll take that to be true. If this is a true statement, if a does not equal six, then you can conclude—let's reveal it: therefore, a plus three does not equal nine; because if a is a value other than six, then there's no way that you can add three to it and have a sum of nine. Again here, you have the law of contrapositive in action, just with a little algebra example.

We've reached the point that it's your turn. I want you to use the law of contrapositive to write a valid conclusion. I'm going to click to the next slide. Be sure to press pause and take your time to write that conclusion out. Here you go.

Alright, let's see what conclusion you drew.

"If you live in Washington, D.C., then you live in the United States." Let me make sure I have my pen. Here's p: "You live in Washington, D.C." And here's q: "You live in the United States." Kind of got them out of the zoom for a second. "Mark does not live in the United States." Well, that is the opposite of q. If we're using the law of contrapositive, the conclusion should be the opposite of p: "Therefore, Mark does not live in Washington, D.C." Good job on that.

You've reached the conclusion of this topic on the law of contrapositive. I hope you saw the difference between inductive reasoning (the use of patterns to draw a conclusion) and deductive reasoning (the use of definitions and laws to draw a valid conclusion). I hope you continue to exercise those reasoning skills. Bye.