Module 10: Circles

Topic 1 Content: Angles Formed by Intersections in the Interior of a Circle Transcript

Hi guys. Welcome to Geometry. In this topic we're going to focus on angles formed by intersections and the interior of a circle. Your knowledge of angle relationships and your background knowledge on circles is going to come in handy for you during this topic. You ready to get started? Let's go.

Before we dive in to discussing those angles, I just want to review a little bit about the parts of a circle. You probably covered this in middle school, some of these parts, and even a little bit in elementary school. To just job your memory, a chord is a segment that has it's end points on the circle. This is one of the parts of the circle we're going to be working with, a diameter. It's just a special kind of chord. It's a chord that passes through the center of a circle. Its endpoints are on the circle but this chord passes through the center of a circle.

We have a radius. A radius is a segment that has one endpoint on the center and another endpoint on the circle.

Next we have a secant. A secant is a line that intersects a circle at two points. Here we have a secant.

A tangent. A tangent is a line that intersects a circle at exactly one point. You can think about it as it just touches the circle. It intersects the circle at exactly one point.

Let's keep going. Arcs and semicircles. You've probably learned, like I said, in elementary or middle school, that a circle contains 360 degrees. We call the circumference of the circle, the edge of it, these are our arcs. The sum of the measures of these arcs is 360 degrees. A semicircle, let's reveal this. A semicircle is exactly 180 degrees. It's the measure of the arc that's a 180 degrees. You name a circle by three points. I'm sorry, you name a semicircle by three points. Arc A, B, C, that's a semicircle, and arc A, D, C, that's another semicircle.

Notice when I name these arcs you name it by its endpoints, so A and C. That tells me that's where that arc starts and stops. B is a point contained in the interior of the arc. Again, it's semicircles A, B, C and A, D, C. Those are two right there.

What about minor arc? Let's reveal that. A minor arc is an arc that measures less than 180 degrees. Notice here that we only use it's endpoints to name it. Two minor arcs here, arc BC, that's a minor arc. It's less than 180 degrees or you think about it as it's less than a semicircle, and arc CD. That's another minor arc. Let's keep going.

Major arcs. Major arcs measure more than 180 degrees. They are more than a semicircle. Again, they're named with three letters. They're named by their endpoints, and then also one point contained in its interior. For example, arc A, B, D, that'd be starting at A. Arc A, B and taking it all the way around to D. Even though I passed through B and C to get to D you'll only name a major arc by its endpoints and just one point in its interior, one point that you passed through.

Arc A, B, D that was a major arc. Arc C, D, B, that's another major arc. There's arc C, D, B. There are several more we could focus on here, but that's just a couple of them just to get you warmed up about major arcs, minor arcs and semicircles. Let's keep going.

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Angles formed by intersections in the interior of a circle. In this example we're going to focus on this angle right here noted by X. It was formed by these two secants intersecting in the interior of the circle. The measure of this angle, how we'd calculate that, X will equal the half the sum of the measures of its intercepted arcs. Let me show you what I mean by that. Let's reveal this right here.

X equals half the sum of the measures of its intercepted arcs. What I mean by intercepted arc, in this case AB, this arc right here, arc AB, that's an intercepted arc. CD, it's another intercepted arc. X will equal half the sum of the intercepted arcs. You can think about that as the arc that was cut off by the sides of that angle. It's endpoints lye on the sides of the angle and the interior parts of the arc and interior points of the arc lye in the interior of the angle.

We focus on these two arcs because if you think back, recall that these are vertical angles so they'll have the same measure. When we're trying to find the measure of one of those vertical angles, you could say it's measure, one of those angles, will equal half the sum of the measures of the intercepted arcs. We're going to use this fact to work through a few examples.

Take a look at this one. Given circle A find X and Y. Notice it says circle A. We name a circle by its center. Just from looking at this statement I'm told that the center of this circle is A. I just know that for information. I'm asked to find X and Y here. Look what we have here. We have an angle formed by an intersection in the interior. We're given the measure of a couple of arcs. In fact, it's intercepted arcs. The first thing I'm going to do is start out by finding the measure of X and then I'm going to use that to figure out what Y is.

I know that X will equal half the sum of 70 plus 50. That means X is half of 120, get a little more space here, and half of 120 is 60. I know that X equals 60. I could say that the measure of that angle is 60 degrees. Now look what we have here. Recall an older angle relationship that you've learned. If you just focused your attention right here, notice that you have a linear pair. Recall that a linear pair is always supplementary. Now that I know that X is 60 I can use that fact to figure out what Y equals, because I know the sum of these angles has to equal 180 degrees.

Let me get my pen back, get a little more work space here. Y will equal, I'll go ahead and switch colors, Y equals 180 minus 60, which is 120. Y equals 120 or scroll back to the circle. I could say the measure of this angle 120 degrees, and you're all done with that one. I'll box my answers in here. X equaled 60. Y equals 120. Let's recall how we got that. We started out by getting X. X equaled half the sum of these intercepted arcs, so half of 120 degrees, which was 60 degrees. We used the fact that these angles formed a linear pair. 180 minus 60 gave us that 120. Getting warmed up a bit with circles? Let's keep going.

Check this example out. Sample two. Given circle A find X. I'm given different information here. A lot of times with circles they can be a little intimidating because you're getting a lot of different pieces sometimes. Sometimes you have the measures of angles, sometimes you have the measures of arcs, and you're not always quite sure where to begin. Just begin with what you know. A lot of times those pieces will just come right together.

In this case I see I know the measure of one of the angles in the interior. I know the measure of one of my intercepted arcs, but not the other. I know that formula that I can use that will help me figure it out. I know that the measure of this angle is equal to half the sum of the measures of these arcs. I know that 70 equals half the sum of 60 plus X. I'm going to use this equation to solve for X. Let's get solvent on this one. I'm going to need a little more room to work it out. I'm just going to move it right down here just so I can do my algebra right there.

The first thing you're probably noticing here is that fraction. A lot of times with equations, when we see a fraction, we just want to get rid of it. You want to clear that fraction. Let's go ahead and just start there. Because my fraction's one half like you learned in algebra one, you can clear it by multiplying both sides of your equation by two. I multiply my right side by two and the left side so this fraction clears out. I'm left with one times this quantity, which really doesn't change anything. I'm just going to write that quantity down, that 60 plus X, and two times 70, that's 140. I'm going to go ahead and solve for X. Subtract 60 from each side. That cancels out 140 minus 60, that's 80. In this case X equals 80, and you're done with that one.

I'm going to take it up to the circle just so we can really look at what that means. If X is 80 degrees, let's just check that work. I know that this angle is equal to half the sum of these intercepted arcs. Let's check that. Half the sum of 60 and 80, so that's half of 140 which is 70, which is what we started with. You can go ahead, you can get your answer, you can check your work sometimes just so you can feel confident that you did end up with the right answer. Good job on that one.

It's your turn. Go ahead and press pause, take a few minutes, redo this example, go ahead and work it out, press play when you're ready to check your work. Let's see how you did here. The circle below represents a decorative pattern to be embroidered on a sweater. Find X, Y and Z. We were given a lot of different things to solve for here. We were given the measures of a couple of arcs. I'm just going to start with my given information and see what I can come up with here.

I noticed these are the intercepted arcs. I'll focus on angle X at first, this measure. Let's go ahead and get a little room to do some work. I know that X will equal half the sum of 120 and 110. X equals one half of 120 plus 110. Let's get a little space here. X equals one half, let's do a little mental math here, that's 230. One half of 230, that's 115. I know that X equals 115. Got one of the answers there.

I'm going to go back up to the circle, I'm going to write that answer in there and see if it can help me figure out some more answers. I know that X, 115 degrees. If I look at X and Y I've got another linear pair right there. I'm going to focus my attention there now. I'm going to go ahead and highlight it so I'm focusing right here on this linear pair. I know that these angles have to be supplementary. I need to figure out what 180 minus 115 is. I'm going to scroll down here, get a little space. Y equals, let's get rid of that highlighter, get the pen back, so Y equals 180 minus 115. Do a little mental math here, that is 65. Now I know that Y equals 65. Take that back up to the circle, go ahead and write that value in, scrolling a little more here, there we go, so 65 degrees.

Now let's look at X and Z. Those are a pair of vertical angles. I know that vertical angles are congruent. If X equals 115 degrees then so does Z. We're all done with that one. We have all of our values here. I'm just going to write them all off to the side. X equals 115 and then just review how we got that. That was half the sum of the measures of those intercepted arcs. Y was 65 and that's because X and Y formed a linear pair, so we knew they had to be supplementary. Z was 115, and that's because X and Z were a pair of vertical angles so they had to be congruent.

See how that brought together a lot of the old angle relationships that we covered earlier in the course and some new aspects that we covered here with circles. Good job pulling that all together. All right guys, you've reached the conclusion of this topic on angles formed by intersections in the interior of a circle. I hope you saw how your knowledge of angle relationships and your prior knowledge on circles helped you get through this topic. Bye.