Module 10: Circles
Topic 2 Content: Central and Inscribed Angles Transcript

Hi guys, welcome to Geometry. In this topic we're going to focus on central and inscribed angles. Your knowledge of arts and your background knowledge on circles and your knowledge on angle relationships is going to come in handy during this topic. Are you ready to get started? Let's go.

Okay. Let's begin by talking about a central angle. What a central angle is, is it's an angle that has it vertex at the center of a circle. The measure of a central angle is always equal to the measure of its intercepted arc. In this case, what that means ... Let's get the pen ... Is that X equals Y. The measure of central angle is always equal to the measure of its intercepted arc.

Let's go ahead and apply that knowledge in this first example. Given circle A, find X and Y. I notice that my angle here has its vertex at the center so that tells me that it's a central angle. I know the measure of this angle is going to be equal to the measure of its intercepted arc. Because this angle is a right angle, it measures 90 degrees, that means that X also measures 90 degrees.

I know here that X is 90. I'm going to go ahead and write 90 degrees in there. Now I'm left to figure out what Y equals. Notice that Y is just another arc on the circle. I know this arc measures 90 degrees, the remaining degrees of this circle is taken up by Y. The rest of what's left over after that 90 degrees is removed. Let's figure out what that is.

Y will equal 360 minus 90 so 360 minus 90, that's 270. Y equals 270 and if I just go ahead and bring that back to my circle, that means that this arc, 270 degrees and you're all done with that one. See how here, we used our knowledge of central angles to know that the measure of this arc would have to be the same as the measure of this angle. We figured out X was 90. Once we knew that this arc measure 90 degrees, 360 minus that 90 told us that Y had to measure 270 degrees. It contained those remaining degrees. Let's keep going here.

Your turn. Press "pause," take a few minutes work through this one. Press "play" when you're ready to check your work.

All right let's see how you did here. Here you given circle A and asked to find X and Y. I see here I have another central angle but I don't quite know yet what the measure of its intercepted arc is. I do know the measure of this arc right down here. I'll use the fact that I know this to figure out what Y equals. I know the sum of the arcs on my circle is 360 degrees, so Y will equal 360 minus 200. That's 160. Y is 160 and I'll bring that over to my circle. This arc is 160 degrees.

Now I can see I have a central angle and I do know the measure of its intercepted arc and because I have a central angle, it's going to equal the measure of its intercepted arc. That means that X is also 160 and the measure of that angle, 160 degrees and you're all done with that one.

See how here the first thing we had to do was figure out the measure of that arc using the measure of the arc we were given? Once we knew that was 160, we immediately knew the central angle was 160 degrees. Let's keep going and let's talk about an inscribed angle.
An inscribed angle is an angle that has its vertex on the circle, so how a central angle had its

vertex on the center, an inscribed angle has its vertex on the circle. The measure of an inscribed angle is equal to half the measure of its intercepted arc. In this case, what that means is that X would equal one half of Y. Or you could think about it as X equal Y divided by two. Whichever way you like to think about it.

Know that if you have an inscribed angle, the measure of that inscribed angle is equal to half of the measure of its intercepted arc. Let's go ahead and start applying that knowledge on the next example. Here, "Given the circle below, find X and Y." Let's see what we're working with here.

We know the measure of one of the arcs on the circle and we also know the measure of the inscribe angle. Let's think about what we just learned about inscribed angles. We know the measure of an inscribed angle is equal to half the measure of its intercepted arc. That means I took half of this arc and I ended up with 50 degrees. Just think about what I have to do then, because I know the measure of the angle is 50 degrees, I can double it and figure out the measure of the arc.

X would equal two times 50, which is 100. X equals 100 and I know the measure of this arc now, 100 degrees. Just a recap how we figured out that relationship. If the angle was 50 degrees, that means the arc had to be 100 because we took half of the measure of the arc to figure out the angle. We just doubled the measure of the angle to figure out the measure of the arc. There we have that X is 100 and now that we know that, we can figure out the measure of Y right here.

That is actually just a part of the arcs on my circle. I know that the sum of those arcs, 360 degrees. Let's figure out how much of that 360 is already taken up and then we'll see what's remaining for Y. Let's make sure I have the pen. We have 150 plus 100 so that's 250. 250 degrees are taken up by these arcs. Get some more work space here. 360 minus 250, that's 110. That tells us that Y equals 110 and if I bring that back to the circle that tells me that this arc measures 110 degrees. We're all done with that one.

I know that pulled together a lot of different things and we had to really think about how we could use that inscribed angle to figure out the measure of that arc, but I hope you were able to see how it all came together and we were able to figure out that the arc was double the measure of the angle and once we knew that that arc measured 100 degrees. We found the sum of 100 and 150 and 360 minus that sum led us to the fact that Y equaled 110. Let's keep going. It is your turn again so go ahead and press "pause," take a few minutes work through this one. Press "Play," when you're ready to check your work.

Let's see how you did here. Given the circle below find X and Y. Here, I don't know the measure of the inscribed angle or the measure of its intercepted arc but I do know the measures of a couple of arcs on the circle. I'll just start with what I know and I'll start by using these measures to help me figure out the measure of this arc right down here.

I'm using this to figure out X, so we have 120 plus 170 so that's 290. 360 minus 290, that's 70. This means that X equals 70. The measure of this arc right here, 70 degrees. Once we know that the measure of this arc is 70 degrees we can use that fact to figure out the measure

of our inscribed angle because we know the measure of the inscribed angle is half the measure of its intercepted arc. Our arc is 70 degrees, half of 70 is 35.

I'll go ahead and just show you the work down here just so you can see it. To get Y, we need 70 divided by two which is 35. Y equals 35 and you're all done with this one.

Just a recap just to pull it all together, we use those given arc measures that we had to figure out that the amount of the degrees that was left over for X was 70 degrees. We added 170 and 120, we took that sum away from 360, figured out that X was 70. Once we knew the measure of this arc was 70 degrees, we took half of that measure to figure out the measure of that inscribed angle, so half of 70, 35. We landed at the fact that Y equals 35 degrees. Let's keep going.

Now what I want to discuss is a few special relationships that exist with inscribed angles. One of those relationships is this one. "Two inscribed angles that intercept the same arc are congruent." Let me show you want I mean. I'm going to highlight one of these inscribed angles here. I'm going to highlight this one. Notice that this angle that I highlighted in yellow, and then this inscribed angle that's still in black, both of those inscribed angles intercept the same arc right up at the top. I'm going to trace over that in blue.

Both of those inscribed angles intercept this arc. That means that the measure of each of these angles is going to be the same. These inscribed angles are going to be congruent. They're each going to be half the measure of this intercepted arc. It just so happened that you had a pair of inscribed angle that intercepted the same arc. Those inscribed angles are congruent. They're each going to measure half the measure of that intercepted arc. Keep that in mind and I want you to try this next one. Press "pause," take a few minutes, work through it. Press "play" when you're ready to check your work.

All right let's see how you did on this one. Notice here we had a pair of inscribed angles that each intercepted the same arc. Each of those inscribed angles intercepted that arc that measures 100 degrees. I know that these angles, they're going to be congruent and they're each going to be half the measure of that intercepted arc. Half of 100 or 100 divided by two, that's 50. Each of these angles measures 50 degrees. X equals 50 and Y equals 50 and you're all done with that one.

You were able to see here that special relationship in action. Take a look at the next one. Here, another special relationship. "An angle inscribed in a semicircle measures 90 degrees." Let me show you what I mean by that. I see here, I have a circle, its center is marked A, so this is circle A. I have a chord that passes through that center. That tells me that this chord is a diameter. What's special about a diameter, because it passes through the center, is it splits your circle in half. You have 180 degrees right up top here and you have 180 degrees right on the bottom. The diameter splits your circle into two semi- circles.

I'm going to focus on this semi-circle on the bottom. I'm going to focus on the fact I know this is a semi- circle so this arc, I'm going to trace over it in blue, measures 180 degrees. If I look at this inscribed angle right up here, this one that's inscribed in a semi-circle, notice that this angle intercepts this arc. It intercepts that semi-circle. I know the measure of this angle is

going to be half the measure of this intercepted arc. That tells me, switch back to my pen, is the measure of that angle is one half of 180 degrees, or in other words, 90.

This right here, that's a right angle. The measure of that angle is 90 degrees. That's what's meant by, "An angle inscribed in a semi-circle measures 90 degrees." This inscribed angle intercepts this semi-circle so it's half of its measure. Half of 180 degrees, it's 90 degrees. Keep that in mind and I want you to try this next one. Press "pause," take a few minutes, work through it. Press "play" when you're ready to check your work.

Let's see how you did on this one. "If the measure of angle ADC equals 90 degrees, than arc CEA is ..." and it's multiple choice. Is it a minor arc, a major arc or a semi-circle? Let's look at exactly what angle we're talking about here. Angle ADC, that's this angle right here, 90 degrees. If this angle is 90 degrees, then that means that the arc that it intercepts ... I'm just going to extend the lines of this angle just to show it, extend the sides of that angle ... And notice that it intercepts this arc right here. I'm going to trace over it in blue.

I know that the measure of this angle has to be half of the measure of this arc. If this angle is 90 degrees, that means that I took half of 180 to figure out this measure is 90 degrees. If arc CEA measures 180 degrees, then that means that arc CEA is a semi-circle. Good job pulling all that together. Take a look at this one.

"If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary." What I mean by a quadrilateral inscribed in a circle is I mean that the vertices of that quadrilateral lie on the circle. Here we have quadrilateral ... we started here ... DABC. Each vertex lies on the circle so I can say that this quadrilateral is inscribed in this circle, and because I know that, that tells me that its opposite angles are supplementary. I want you to keep that in mind and I want you to figure out the measures of these two angles. The measures of angle DCB and of angle ABC.

Knowing the fact those opposite angles are supplementary, go ahead and find the measures of each of these angles. Press "pause," take a few minutes, work through that. Press "Play," when you're ready to check your work.

All right let's see how you did here. The measure of angle DCB. Let me first find where that is so DCB. That's this angle. It's opposite from this angle so I know these are going to have to be supplementary. If this angle has a measure of 110 degrees that means angle DCB ... Switch to black ink ... is going to have a measure of 70 degrees. 180 minus 110 is 70. Angle DCB is 70 degrees.

Now the measure of angle ABC, so ABC is this angle. It's opposite from this one. I know these angles are also going to have to be supplementary so if this angle measures 80 degrees, that means angle ABC has to measure 100 degrees because 180 minus 80, that's 100. Good job on that one. Good job pulling all those facts together to get that one solved.

All right guys you've reached the conclusion of this topic on Central and Inscribed angles. I hope you saw how your knowledge of arcs and the parts of a circle and angle relationships all came in handy for you during this topic. Bye.