Module 10: Circles
Topic 4 Content: Calculating the Length of an Arc Transcript

Hi guys. Welcome to Geometry. In this topic, we're going to focus on how to calculate the length of an arc. Your knowledge of circumference, your knowledge of other relationships with circles and your algebra skills are going to come in handy for you during this topic. You ready to get started? Let's go.

Just to let you know what exactly it is we're calculating here. Now when you want to determine the length of an arc, I want you to think back to what you know about circumference. Remember that the circumference of a circle is the distance around a circle. It's almost like the perimeter of the circle. You can think about it that way. When you want to calculate the length of an arc or arc length, what you want to figure out is the length of just a portion of that circumference. Arc length is part of the circumference of a circle. There's a rule that we can use to figure out what the length of an arc of a circle is. Take a look at this here.

Here we're given circle P. You want to calculate in this case the length of arc MN. That's what we're asked or what this rule is setting up for us here. The length of arc MN over $2\pi r$, which if you remember, that's the formula for the circumference of a circle. So the length of that arc over the circumference of the circle equals the measure of the central angle over 360 degrees. I know this formula looks a little intimidating right now. This is a lot of different values going on here. We're going to break it down. You're going to see how it all just comes together.

What we have here if you look at this proportion, what we do here is there's a relationship between ... as you can see here, the ratio of the measure of the central angle to 360 degrees is proportional to the ratio of the length of arc MN to the circumference of the whole circle. That relationship allows us to set up this proportion which will give us an equation that we can use to solve for x. Let me actually show you that work solving for x then I think it'll really come together for you. Let me switch to my pen.

So we don't know the length of MN, that arc. Just to keep things a little more condensed, a little more simple, I'm going to refer to that arc as x. I'm going to let that be my unknown. I'm actually going to write x right here so that in my proportion, I'm going to use x to represent the length of arc MN. So x over $2\pi r$. Remember r stands for radius when you're dealing with the formula for circumference. So 2 times π [inaudible 00:03:08] our radius is 8 cm here. So that equals the measure of the central angle, which is 100 degrees. So 100 over 360.

I know it probably still looks like a lot but it's going to all come together here. So we have x over 2π times 8 equals 100 over 360. Now we're going to go ahead and cross multiply so that we can solve for x. I'm going to start on this side here, this diagonal part. You can start on either one. I just always have a habit of always starting where I'll have x on the left side when I can. So I'm going to just start cross multiplying right here. So 360 times x. So that's $360x$ equals ...

So we have 2 times π times 8 times 100. I know that's a lot of things you're multiplying together. Now with this case here, I want you to actually leave out π right now in the multiplication. I know you've learned earlier that π is approximately 3.14. That's going to

come into play in a little bit here. I want you for right now, though, to just carry π along because we're going to have our final answer at first in terms of π.

So when you cross multiply here, all I want you to cross multiply in the calculator is 2 times 8 times 100 and we're just going to carry π along. So 2 times 8 times 100. That is 1600. Let's go back to our work. On the right side we have 1600π because we're just carrying along that π. Now let's divide both sides by 360. So on the left side, that's going to cancel. We just have x. Again, we're just going to carry along that π for right now. In the calculator, all I want you to do is 1600 divided by 360. Let's go ahead and look at that. 1600 divided by 360. That is 4.4 repeating. Something I haven't showed you before that I'm going to show you now is there's a little trick in that calculator, a little application you can use in order to turn any decimal into a fraction. I know you learned how to do that in algebra I so I know that's a skill that you have. Here in geometry, I'm going to go ahead and show you how you can use some of the tools in the calculator to turn this decimal into a fraction.

You're going to press math, which is the button right underneath the alpha key. So math. It's the first option that we want to pull up. It gives us that little triangle and then fraction right after it. So hit enter. Enter one more time. We have 40 over 9. Basically what that means is that 4.4 repeating as a decimal is represented as 40 over 9 or forty ninths. Back to our work.

That means we have here that x equals 40π over 9. I believe our units here were centimeters, so 40π over 9 centimeters. Now just to recap, let me review a little bit what we did here. We used our rule, the length of the arc over the circumference of the circle equals the measure of the central angle over 360. So we have that x equals 2 times π times 8 because that was our circumference of our circle equals 100, that's our central angle, over 360.

We cross multiplied here. I'll just show that. We cross multiplied. So 360 times x. We said to first hold off multiplying that π right now so we did 360 times x for the left, 2 times 8 times 100 for the right side. Carried along that π. Divided by 360. We have that x equals 40π over 9 centimeters. I know that seems kind of like how long exactly is that, 40π over 9? Because we don't always talk in terms of π when we're talking about length. But I want you to see how if you were asked to give an answer in terms of π, you could.

But now I want to show you how to actually approximate that value so that you can really get an estimate of how long is that exactly. This is where this fact, which I know you've covered in middle school that π is approximately 3.14. We're going to use that value to estimate this value. So 40π over 9 would be approximately ... we're going to do that work right here. There's where we're going to approximate. Just shorten that.

40 times 3.14 divided by 9. We're just going to figure out what that is. That'll give us an approximation of what this length is. So 40 times 3.14 divided by 9. Go back to our calculator. So 40 times 3.14 gives us that. Let's divide that by 9. That is 13.95. You see that keeps on for a while also. We're going to go ahead and round to the nearest whole number for this one. So 13.9555 and on and on and on is approximately if you round to the nearest whole number 14. Let's go back to our work. So x is approximately 14 centimeters. Here's

our answer given in two forms here. In terms of π the length of that arc or the length of arc MN is 40π over 9 centimeters, which is approximately 14 centimeters. Good job pulling all that together and taking all that in. Keep all that work in mind and I want you to go ahead and try this one. I've given you the formula so you have that for reference here. Go ahead and press pause. Take a few minutes. Work your way through this one. Press play when you're ready to check your work.

Let's see how you did here. Let's go ahead and read through this. The portion of the circle shown below represents a curve on a local highway. Find the length of the arc rounded to the nearest whole number. Let's see what we have here. Now I have here the length of arc MN because that was the arc we were working with in our circle on example one. But here we have it represented by x, so x over 2πr. So 2 times π times here our radius is 100 feet, so times 100, equals measure of central angle. So that's 45 over 360. Let's go ahead and cross multiply here. Get a little more work space. So x times 360. Let me just show that so you remember that's exactly what I'm doing right now. I'm cross multiplying. So x times 360, that's 360x, equals 2 times π times 100 times 45. We're going to go ahead and leave π out for right now. So I'm going to go to the calculator and multiply 2 times 100 times 45. Let's go to our calculator. Let's just clear that. 2 times 100 times 45, so that is 9000. Back to our work here. Scroll down here.

All right, so 9000 on the right side. Get my pen. 9000π because we're still carrying along π. Let's divide each side by 360. So x equals ... and we're still going to carry along the π right now so we'll just do 9000 divided by 360. So 9000 divided by 360. That is 25. So back to our work. X equals 25π. And we're working with feet here but if you recall for this one you were asked to round to the nearest whole number. That means we're going to have to approximate. So we'll need 25 times 3.14 to get that approximation. Back to the calculator. 25 times 3.14. That is 78.5. round to the nearest whole number, that's 79. Back to the work here.

So x is approximately 79 feet. You're all done with this one. What that means, if we go ahead and recap and take it back to our circle, we used our formula to set up our proportion. So x over 2 times π times 100 equals 45 times 360. We cross multiplied carrying along π along the way and we ended up at x equals 25π. We approximated π's value at 3.14 and found that x is approximately 79 feet. Or in the terms of our word problem here, what this means is the length of this arc, the length of this curve is approximately 79 feet. All right? Good job on that one.

All right guys. You've reached the conclusion on this topic of how to determine the length of an arc. I hope you saw how your knowledge of circumference and of solving proportions all came in handy for you during this lesson. Bye.