Module 10: Circles

Topic 6 Content: Determining the Equation of a Circle Given the Coordinates of the Center and Point on the Circle

Hi guys. Welcome to Geometry. In this topic, we're going to focus on how to determine the equation of a circle when given the coordinates of the center and a point on the circle. Your knowledge of the standard equation of a circle is going to come in handy for you during this topic. You're ready to get started? Let's go.

All right. Just to review that standard equation for the circle, given a circle with center at h, k, and a radius of length r, that standard equation is given by this. X minus h, that quantity squared, plus y minus k, that quantity squared equals r squared. We're going to use this equation and our knowledge of circles and coordinate methods to get through the next few examples.

Let's take a look at this example. Here, we're asked to write the equation of the circle with center three, four that passes through five, one. To start this one, the first thing that we're going to do is just get a rough sketch of what this circle looks like on the coordinate plane. It's not going to be perfect. We're just going to roughly sketch.

This will just help us get a general idea of what this circle looks like on the coordinate plane. Let's get a circle here. We're told that it's center. It's three, four. That it passes through the point, five, one. What we're asked to do here is get that equation of the circle. I'm going to actually get that down here right underneath.

We are going to have to scroll a little bit in this example. X minus h squared, plus y minus k squared equals r squared. We know that h, k, represents the coordinates of the center of our circle. Here, we were told, if I scroll up a little bit, that our circle had a center of three, four. What that tells us here ... I'm going to switch colors.

Let me actually slide a rough sketch over just a bit. I'm going to leave that. Just a little bit. Just so we can get a little bit of information to the right of it. Like I said, if we know that the center is three, four, then that tells us that h, k is three, four. We know the h, k piece of our equation of this circle. What we still need to know if I scroll down is we need to know r. We need to figure out what the radius is of our circle.

Let's think back to what we know about a radius. We know that a radius is a segment that has one end point at the center of the circle and another end point on the circle itself. If we take a look at our rough sketch, we were given a point that's on our circle, five, one. If we can figure out the distance from the center of the circle to this point on the circle, then we'll know the length of our radius.

Let's think back to what we know about some of our coordinate methods. What formula do we know that we could use in order to figure out the distance between two points? The distance formula. That's what we're going to use here. We're going to use the distance formula to help us figure out the length of the radius of this circle.

Now, let's get a little more room to work up here. I think what I'll do, I'll extend this page some. Get a little work space and I'll shuffle a few things around up here. I still want to keep my equation in mind. I'll just bring it up here. We have a few different things we have

to work with in this example. Then I'll erase it down here and I'll go ahead and bring in the distance formula. That's what we need to use right now in order to figure out the length of the radius of this circle.

To start, let me just copy the points I'm working with right down here. Three, four and five, one. Then I'm going to go ahead and write down the distance formula. It's a square root, the x of two minus x of one squared plus y sub two minus y sub one squared. Let's get a little more space. We are going to have to scroll a lot with this one. Of course, on your notebook paper, you've got a lot of space to keep things nice and spaced out and organized.

Let me go ahead and label my points here. I'll label this x one, y one and x two, y two. Then let's substitute it into the distance formula. We need the square root, x of two minus x of one. That's five minus three. We're going to square that difference. Then we're going to add that to y sub two minus y sub one, so that's one minus four. We have that quantity squared. Scroll down some more.

Five minus three, that's two. This is two squared plus one minus four. That's negative three squared. Let's go ahead and clean this up. A little more space. Two squared is four. Negative three squared is nine. Four plus nine is 13. That means that the length of our radius, it's the square root of 13 units long. I'm actually not going to approximate this, because we know 13 is not a perfect square. I'm going to leave it as exactly the square root of 13. You're going to see why in a second.

Let's take that up to the top. Write that down. Scroll back up here. Now we know that r is the square root of 13. Now, we have all the pieces that we need in order to write the equation of the circle. Let's go ahead and get to that step. Let's get rid of a few things. Let's erase this. Of course, on your notebook paper, you're going to want to keep it, but I just want to get us a little more room up here to get our final part of this work done. Let's get rid of that and this last step here.

Scroll back up to the top. Here we go. Now, let's go ahead and substitute the appropriate values in the right spots and we'll be able to write the equation of this circle. Let me make sure I've got my pen. X minus h squared. I know that three is h, four is k. X minus h squared will be x minus three. That quantity squared, plus k is four, so this will be y minus four squared equals r squared. I know that r is the square root of 13.

I'm going to come off to the side, do little scratch work for that. What we need to figure out is the square root of 13 squared. For this, think back to a little bit of algebra one. The square root of 13 squared. That means the square root of 13 times the square of root of 13. The square root of 13 times the square root of 13, that's just 13. In our equation here, the equation of our circle, when we have to insert our squared, our squared is actually just 13. That's because the square root of 13 squared is actually just 13. You're all done with this one.

Just to recap, just to review what we did here. We started out by getting our rough sketch down so we could just get a general idea of what this circle look like on the coordinate plane. We figured out that we knew h, k, it was three, four, but the piece that we needed to figure out was the length of the radius. We use the distance formula to figure out the distance between these two points and that led us to find out that our radius was a square root of 13 units long.

Once we have the coordinates of the center and the length of the radius, we're able to just substitute everything into our formula. We landed at x minus three quantity squared plus y minus four quantity squared equals 13. You're all done with this one. Good job with that.

Now, I want you to go ahead and try this one. Press pause, take a few minutes. Work your way through this one. Press play when you're ready to check your work. Let's see how you did here. Here, you were given that you needed to write the equation of a circle that had a center at negative one, two and that pass through the point seven, eight. Automatically, I know h, k.

I know that the center of my circle here is at negative one, two. What I need to do is figure out the distance between the center and this point seven, eight, then I'll know the length of the radius. I'm going to go ahead and get the distance formula written down here. We know that distance formula. We've used it several times throughout the course. You may have committed it to memory, or if you're referring back to some old notes, that's fun too.

Here, we're trying to determine the distance between negative one, two and seven, eight. What I'm going to do here just so we don't get too crowded, I'm going to shrink that just a bit. Move that off to the side. I'm going to move this down a little bit here. Just so I can focus on my two points, negative one, two and seven, eight.

I'm going to label x one, y one, x two, y two. Let's get a little work space. Let's get this work taken care of. X two minus x one, that's seven minus negative one squared. We have a double negative in here. We'll go ahead and clean that up now. Then y two minus y one, so that's eight minus two squared. Let's keep moving through this.

Seven plus one, that's eight, so eight squared. Eight minus two, that's six, so that's six squared. Eight squared is 64 and six squared is 36. 64 plus 36, that's 100. The square root of 100, that's 10. Now, from the distance formula, we know that the length of our radius is 10. R equals 10, and I'm going to go ahead and scroll up to the top just to get the coordinates of the center one more time.

Our center is at negative one, two. I'm going to scroll that down to the button so I can write down that h, k. It is negative one, two. Now, I have what I need to go ahead and write the equation for this circle. I'm going to write just the standard form first so I can see really clearly what I'm going to be plugging into.I knew my h, k, that's negative one, two. I have x minus negative one. That quantity squared. I have a double negative. I'm going to go ahead and clean that up now. Then y minus two, that quantity squared equals r squared. Because I know r is 10, 10 squared, that's 100. You are all done with this one.

You see how we use that given information. If I scroll back up to the top, we knew the center. We used the distance formula to figure out the distance between the center and the point that was on our circle. Once we knew that, scrolling on down. We use the distance formula. We figured out the length of the radius and we had all the information that we needed to write the equation for this circle. Good job on that one.

All right guys, you've reached the conclusion of this topic on how to write the equation of a circle when given the coordinates of the center and the point on the circle. I hope we saw how your knowledge with the coordinate plane and that standard equation for the circle helped you get through this topic. Bye.