Module 10: Circles

Topic 6 Content: Determining the Equation of a Circle Given the Coordinates of the Endpoints of a Diameter Transcript

Hi guys, welcome to Geometry. In this topic we're going to focus on writing the equation of a circle when given the coordinates of the endpoints of a diameter. Now, your knowledge of the standard equation of a circle and your knowledge of coordinate methods, is going to come in handy for you during this topic. You ready to get started? Let's go. Okay. Now, just to review what that standard equation for the circle is, given a circle with center at H, K, and a radius of length R, we know that the standard equation of that circle is given by this. X minus H, that quantity squared, plus Y minus K, that quantity squared, equals R-squared. Okay. We're going to use this equation and some of our background knowledge on circles, to get through the next few examples. Okay. Here, given circle A, it has a diameter with endpoints 6, 5, and 2, -3, and we're asked to write the equation of the circle.

All right. I want you to first just think a bit about exactly what we have going on here. I'm going to just roughly sketch what this circle looks like on the coordinate plane. A little something like that. We know we have our diameter, and again, this is a rough sketch, but we know that one of the endpoints is 2, -3 and the other endpoint it's at 6, 5. We know to write our standard equation for our circle. There's two pieces of information that we have to know. We have to know the center and we have to know the length of the radius. What we have to think about here is how can we use the coordinates of the endpoints of the diameter to figure out the center, and how are we going to figure out the length of the radius. Okay, so let's think about this here. We know from what we know about circles, that the diameter is a chord that passes through the center of the circle, and because we know that the diameter is twice the length of the radius of the circle, that what we know here is if we could figure out what the center of the diameter is, then we'll know the center of our circle.

Let's think a bit about what we know about some coordinate methods. How can we figure out the center or the middle of a segment? How can we figure out that middle point? The Midpoint Formula, right. What we're going to do here is we're going to use the Midpoint Formula to figure out the coordinates of the center of the diameter, which is going to be the coordinates of the center of our circle, okay. That's going to be our first step. Let's do that first. We're going to scroll down a little bit to get a little workspace. There's going to be a bit of work associated with this one, so if you're it out on notebook paper, make sure you've got some room. Okay. What I'll do first is I'll go ahead and label X1, Y1, and X2, Y2, and then I'll write the Midpoint Formula, just to job your memory. To figure out the midpoint, we know we have to add the X-coordinates of our two points, divide by 2, then add the Y- coordinates, and divide by 2.

On this one here, X1 plus X2, that's going to be 2 plus 6. Let's get underneath this here, get our work going. 2 plus 6, I'll switch to black ink, that sum divided by 2. Then we need Y1 plus Y2, so that's going to be -3 plus 5, we need that sum, and then we'll divide it by 2. Okay. Let's simplify this. 2 plus 6, that's 8, so 8 divided by 2. -3 plus 5, that's 2, so 2 divided by 2. 8 divided by 2, that's 4, and 2 divided by 2, that's 1. The midpoint of our diameter, which is the center of our circle, is at 4, 1. Right now we know the center is at 4, 1, okay. We've got the first piece of information that we need to write our equation.

I'm going to go back to the coordinate plane here, our rough sketch, and let's get the blue right here. I'm going just mark it right here that now we know the center's at 4, 1. Again, just a rough sketch here. Now that we know the center is at 4, 1, let's think about how we can figure out the length of the radius of this circle.

All right. Well, let's see. We know that the length of the radius is the distance between the center and any point on the circle. What we can do, is we can use the distance formula to either figure out the distance between the center and 2, -3, or the distance between the center and 6, 5. It's completely up to you, because we know because of the properties of the circle, that the radius is going to be the same regardless of if we choose to use 2, -3, or 6, 5. Okay. What I'm going to do is I'm going to use the center and I'm going to use 6, 5, and I'm going to figure out the distance between those two points, okay. I'm going to scroll down a little bit. I'm just going to clean this up a bit. I'm going to get rid of this work here and I'm just going to take the fact that we know the center's at 4, 1. I'm going to write that right up here that we know the center is at 4, 1. Okay. Let's get a little more room here and let's figure out the length of the radius and then we'll be one step closer to writing the equation of our circle.

All right. Got some more workspace here. Let's scroll back up. Okay. Like I said, I'm going to find the distance between 4, 1, and 6, 5, and I'm going to get the radius that way. 4, 1 and 6, 5, are going to be the points that I use. I'm going to go ahead and get the distance formula written down here. I know the distance, square root, and like I said, you may have this memorized by now, because we've used it a lot throughout geometry, but if you don't, go ahead and refer back to your notes. Okay. I'm going to go ahead and label my points, X1, Y1, and X2, Y2, and I need the square root. X2 minus X1, that's going to be 6 minus 4, and I'm going to have to square that. Y2 minus Y1, 5 minus 1, I'm going to have to square that. Scroll down. Okay. 6 minus 4, that's 2, so 2-squared, and 5 minus 1, that's 4, so 4-squared. 2- squared, that's 4 and 4-squared, that's 16. 4 plus 16, that's 20. The radius is the square root of 20. I'm not going to simplify this any further, because I know when I write my equation of my circle, I'm going to have to square this.

I'm just going to leave this right now at the fact that the radius is the square root of 20. I'm going to scroll back up to the top and I'm going to write that right off to the side right here, that my radius was the square root of 20. Now that we know the coordinates of the center, which if we recap, remember we got that by using the Midpoint Formula with the endpoints of our diameter and we got the center. Once we knew the center, I went ahead and used these two points, the center and I used 6, 5, and I figured out the length of the radius of my circle. Now that I know the center and the length of the radius, I have enough information to write the equation for this circle. Let's go ahead, let's get a little more room. Like I said, there's a lot of work associated with it. The calculations aren't very complex that you're performing, but there's just different pieces that you have to do in order to get to that equation. Okay.

All right. I'm going to write that general form first so I can see exactly what I'm going to substitute and where. Okay. I've got that. Now I know my center is at 4, 1, so I know this is my H and my K. Let's go ahead and let's substitute that in there. X minus 4, we're going

to square that quantity, and Y minus 1, and we're going to square that, equals R-squared. Then thinking back to what we learned in algebra I, I have the square root of 20 and I need to square that, right? Just to do a little scratch work, I'll do that right underneath. The square root of 20-squared, equals the square root of 20, times the square root of 20, which is just 20. Okay. Going back to my equation, that R-squared piece, I'll just have 20, and you are all done with this one. Okay. Good job pulling all that together, okay, because I know there were a lot of different pieces, like I said, that you had to get through in order to get to that equation.

Remember, if I just scroll back up to the top, essentially to solve this type of problem, we need to use the Midpoint Formula to figure out the coordinates of your center, and then once you know the center, use the distance formula so that you can figure out the length of the radius. Just get that distance between the center and one of those endpoints of your diameter. All right. Good job on that one. Now I want you to try this one. Go ahead and press pause, take a few minutes, work your way through this one. Press play when you're ready to check your work. All right. Let's see how you did here. Here you were again given the coordinates of the endpoints of your diameter and you were asked to write the equation of the circle, then you kind of had a bonus question here too. I'll also ask you to tell me the length of the diameter of that circle. Okay. Let's work this one out.

The first thing I'm going to do is I know that I have to figure out the length or the midpoint in order to figure out the coordinates of the center of my circle. I'm going to use the endpoints of my diameter and the Midpoint Formula to figure out the coordinates of the center. I'll go ahead and write A, because all this first bit of work is going to be associated with that first answer. -4, -1, and 4, 5. I'm going to use that Midpoint Formula to get the coordinates of the center. Go ahead and label those points. I'm going to write my Midpoint Formula down here so that I'll know exactly ... I'm going to abbreviate that. I'll know exactly what I'm plugging into. I'm just adding our X's, dividing my 2, and then adding our Y's, and dividing by 2. For the X's, we'd have -4 plus 4, so -4 plus 4, divided by 2. Then for the Y's, -1 plus 5, so -1 plus 5, divided by 2. Get a little more room here. -4 plus 4, that's 0, so 0 divided by 2. -1 plus 5, that's 4, so 4 divided by 2. 0 divided by 2, that's 0, and 4 divided by 2, that's 2.

Now I know that the center of this circle is at (0, 2), that's the center. Okay. I'm going to make a note of this up at the top, so that I can erase this work and do the second part of this. Okay. I know the center, go back up to the top, even up a little further there. Maybe I'll do this, I'll erase this first and then I'll go ahead and write that I know my center is at (0, 2). Okay. Now let's erase this work and let's do the work that we need to do in order to figure out the length of our radius. We know to do that, we just need to determine the distance between the center and one of the endpoints of our diameter. I know my center's at (0, 2), and I think I'll use 4, 5. I'll get the distance between those two points. Again, my center, (0, 2), and I'm using the endpoint 4, 5. This will help me figure out the length of the radius. I'm going to label X1, Y1, X2, Y2, and this time I'm using the distance formula so that I can figure out the length of the radius. Let's get that written out here.

A good idea if you're trying to memorize different formulas is just to write them over and

over again anytime you have to use it. That will really help you commit it to memory here. Because you see that distance formula, you have to refer to that a lot during the course. All right. X2 minus X1, that's 4 minus 0, squared, and Y2 minus Y1, that's 5 minus 2, and we're going to square that quantity also. 4 minus 0, that's 4-squared, and 5 minus 2, that's 3-squared. Let's keep going. 4-squared, that's 16. 3-squared, is 9. 16 plus 9, that's 25. The square root of 25 is 5. Okay. Now we know the distance between the center and that endpoint and that tells us the length of our radius. Now we know the radius is 5 units long, okay.

We have the two pieces of information that we need in order to write the equation for this circle. I'm going to scroll back up to the top and just write beside center, or actually, let's go ahead and start erasing, because we know we're going to need this space. We're going to write that our radius is 5, okay.

Let's get this out of the way and let's go ahead and write the equation for this circle. Get this last bit of work here and I think I missed just a little bit right here. There we go. Now we know the center, we know the length of the radius. I'm just going to go ahead and write that general equation so that I can just easily substitute where I need to. Okay. I know my center is 0, 2, so that's my H and my K. X minus 0- squared, plus Y minus 2, that quantity squared, equals R-squared, and I know my radius is 5, so 5- squared, that's 25. You could stop here, right? If you stopped here, you would be correct, but I just want to show you how you may see this more often. You see we have X minus 0, which is just X, right?

That quantity in the parentheses, so you'll probably see this written as X-squared, plus Y minus 2, that quantity squared, equals 25. This is just a more simple way to write that equation, because X minus 0 is just X, so that quantity is just simplified even further, okay.

This was your answer for part A. The equation for this circle is X-squared, plus Y minus 2, that quantity squared, equals 25. All right. Now let's take a look at part B and for that one you were asked to determine the length of the diameter. Well, now that you know that the radius is 5, right? You know your diameter is double the radius. Scrolled a little too far there. There we go. Our diameter is 2 times 5, which is 10. The length of our diameter is 10 units long. Okay. All right. Now before we leave that, I want to just cover really quickly, if you were only given the coordinates for the endpoint of the diameter, or the endpoints of the diameter and you were asked to determine its length, you could've jumped right in and just used the distance formula to figure out the distance between these two points.

Because we'd already done the math to determine the radius, we used that property of the circle, that the diameter is double the radius, and just quickly did the math that double our radius here would give us 10. Okay. Just so you know a couple of different methods to find the length of your diameter. All right, guys, we've reached the conclusion of this topic of how to write the equation of a circle given the coordinates of the endpoints of its diameter. I hope you saw how your knowledge of that standard equation of the circle and coordinate methods helped you get through this topic. Bye.