Module 10: Circles
Topic 6 Content: The Standard Equation of a Circle Transcript

Hi, guys. Welcome to geometry. In this topic, we're going to focus on the standard equation of a circle. Now your background knowledge of circles and your skills in the coordinate plane are going to help you get through this topic. You ready to get started? Let's go.

Okay. Now before this topic, most of our work involving circles did not involve the coordinate plane. We spent most of our time away from the coordinate plane when we solved those problems involving arcs and segments and angles related to the circle. Now what we're going to do is we're going to work with graphs of circles and we're going to start out by using our knowledge of the coordinate plane and our knowledge of circles in order to derive a standard equation for the circle. All right?

Okay. Now let's dive in here and get this equation. Now what we have here is the graph of a circle with the center HK and we're just using H and K to reference the coordinates for the center and a radius of R. Now one thing we know about a circle is that all of the points on a circle lie the same distance away from the center and that distance is the length of the radius, right? Every point on this circle is the length of the radius that distance away from the center. So what we're going to do is we're going to come up with an equation that shows the relationship between all the points on the circle and the radius. All right?

Okay, so let's get going here. Make sure I've got my pen. So the first thing I'm going to do, we have our center here, HK, is I'm going to drop just a little dotted line here to show that that X-coordinate is H for the center and I'm going to drop a horizontal here and K is our Y-coordinate, all right? So we have here the vertical length is kind of what I've been trying to show. X is representing that H-coordinate and then we have the horizontal which is represented by the Y-coordinate, K.

Now here we have a point on the circle, I'm going to just kind of color over that in black and we're going to call this just X and Y. So I'm going to drop another vertical here to show that the X-coordinate related to that point, we'll just call it X. And I'm going to drop a horizontal here and we're just going to show that the Y-coordinate related to this point is Y. Okay? So far I haven't done too much, just showing you the H and the K for the center and the X and the Y for this point on the circle. All right?

Okay, so the next thing I want to do here is right in here, I want to figure out how can I describe the length of the radius using what I know about H and K and X and Y. So to do that, I'm going to drop a horizontal right here and a vertical right here. Because I know that every horizontal segment is perpendicular to every vertical segment, I know we have a right angle right here. So essentially what I've created is a right triangle. I'm going to use the Pythagorean Theorem in order to describe the length of the radius here, all right.

Okay, so come off to the side with me for a little bit just to review the Pythagorean Theorem. So we know we have legs A and B and hypotenuse C. We know that the Pythagorean theorem is A squared plus B squared equals C squared. So in relation to the triangle, the right triangle I kind of have going here, I'm going to use the Pythagorean Theorem to describe the length of the radius. So in the case of our triangle, when we want to describe A, this length, A corresponds to this length right here which is the distance between X and H, right? So I'm going to say that A is represented by X minus H and we're going to have to

square that because we're following the Pythagorean Theorem. So that's the distance or the length of that side of that triangle. And B is my horizontal side of my triangle which is represented by the distance between Y and K. So I can say, all right, Y minus K squared equals, so C is the hypotenuse of my right triangle, which on my triangle on my circle here, that's the radius, so C, that's like R, so R squared. And what we end up with here is the standard equation for the circle, X minus H, that quantity squared, plus Y minus K, that quantity squared, equals R squared.

And just to recap how we got that, the length of that bottom side of our triangle, that's X minus H; so there's our A and we need to square it. Y minus K, the length of our vertical side of our triangle, that's our B. So Y minus K squared equals and C, that's our radius, right? The hypotenuse is the radius; so R and then we have to square that. Okay? So that's the equation that we're going to use with the next few examples and just so you can know also you can write it formally that this represents a circle that has a center of HK and a radius of R. So any circle with center HK and radius R can be represented by a standard equation in that form. Okay? Okay so keep that in mind. I know it's kind of a lot to pull together but I'm going to show you exactly how to apply that in the next few examples.

Okay, let's take a look at this one. Determine the coordinates of the center and the length of the radius and diameter of the circle given by the equation below. Okay, so we're actually asked to find three different things here, the coordinates of the center, the length of the radius, and the diameter and then we're given the equation. So what I'm going to do here is write that standard equation and then we can just figure out exactly what was plugged in where and get the coordinates for the center, the length of the radius, and the diameter. So because I know that HK is the center, right, our center is HK; for this problem two was substituted for H and because this is now a plus, I know that I must have substituted a negative number. So let me show you what I mean by that. Just do a little scratch work off to the side.

If now this is Y plus 5, that means that I must have substituted a negative five in there, so that I got a double negative which turned that into a plus, right? So if this part right here is Y plus five, I know my K was actually negative five. So H is two and K is negative five, okay? So we have that piece. So now let's go ahead and get the radius. So we know the radius is given by R and for our equation we know that R squared equals sixteen. So in order to figure out what R is, we're just going to need to go ahead and take that square root. So let's take the square root of both sides and we have that R, get a little room here, R equals four, right? So so far we know the center is two negative five; we have this part, the radius is four, and we were also asked to figure out the length of the diameter. Now knowing what we do about circles, we know that the diameter is twice as long as the radius. So if the radius is four, going to use D for diameter or I'll go ahead and write out diameter, that means our diameter is two times four which is eight.

So now we have all the information that we were asked to determine here. We've got that our center is at two negative five, our radius is at four, and our diameter is eight units long, okay? All right. Let's keep going here.

Let's go ahead and let you try one. So what I'm going to do before you get started is that I'm

going to go ahead and write that standard equation for you, just in case you haven't committed it to memory yet because I know it's still early on that you're working with it. All right, so go ahead and press pause, take a few minutes, work your way through this one. Press play when you're ready to check your work.

All right. Let's see how you did here. So here again you were asked to determine the coordinates of the center, the length of the radius, and you weren't actually asked to find the diameter here, so just two things. So here we've got our H. If this is now a plus inside of our parentheses here we know that we must have substituted a negative number, right? So because we know our center is HK, H must've been negative 1 and Y minus K, this is still in the form Y minus and it's Y minus three, so K is three. And now so we know the center is negative one, three. Our radius, so right now we know that our squared is 25. So if we take the square root, R equals five. And we've got everything we need. The center is at negative one, three and the radius is five units long. And you're all done with this one. All right? Okay good job on that one. Let's go ahead and take a look this one.

So given a circle with center 5,5 and radius of twelve, write the standard equation of the circle. All right. So let's go ahead and just write down that standard equation, the form, so we'll know what exactly what we're going to substitute where. So here we're told that the center is 5,5. So that's our H and our K and we know the radius is twelve units, so that's R. So let's just go ahead and substitute H, K, and R in the appropriate spots and then we'll have our standard equation. So X minus H is five, so five squared, plus Y minus K is also five, so five squared, equals R squared and twelve squared, do a little scratch work off to the side, is 144.

So right here you could either write 144 or you could write twelve squared. You wouldn't be wrong if you wrote twelve squared, like literally twelve with exponent of two. That wouldn't be wrong but I just want you to be familiar with the fact that more often than not, you'll see that R squared already given to you. So instead of seeing twelve squared, more likely than not you'll see 144. Okay? All right.

So our equation of the circle here with the center at five, five, and a radius of twelve, is X minus five squared plus Y minus five squared equals 144 and you're all done with that one. All right?

Okay, take a look at the next one and go ahead and give this one a shot. So press pause. Take your time. Work your way through this one. Press play when you're ready to check your work.

All right. Let's see how you did here. So I'm going to first go ahead and write that standard equation, that form so that I'll know what I'm plugging into here. And here we were given a circle with a center at two, six, so that's H and K, and we were given a diameter of eighteen. So let's think about that for a second. If our diameter is eighteen, we know the relationship between our diameter and our radius, the radius is half the length of the diameter. So if the diameter is eighteen that means our radius is nine, half of eighteen. Okay? So now that we have those pieces that we need for our equation, let's go ahead and substitute and get that equation written out.

So if H is two then it's X minus, we're about to write that H again. So it's X minus two, that quantity squared, plus Y minus six, squared, equals, our radius is nine so I'll go off to the side over here this time, nine squared is 81. Okay? And then you're all done with this one. All right. Good job on that one.

Okay, let's take a look at this one. Now we're actually on the coordinate plane. So here we're asked to determine the coordinates of the center and the length of the radius and the diameter of the circle.

Then we're asked to write the equation of the circle. Okay? Okay, so let's see how we can get that information from the graph of the circle without actually being given those coordinates and everything already spelled out for us.

Okay, so here we have the center marked. So let's go ahead and figure out the coordinates of the center and it looks like our center here is at (1,1). I'm going to switch to blue ink. Our center is at (1,1). So now to get the length of the radius, what we can do here is because we know the radius is the distance between the center and any point on the circle, the easiest thing for us to do here is just to count. So we can count horizontally in either direction or we could count vertically in either direction and then we could go ahead and get the length of our radius. So I'm going to count horizontally. I'm going to get the length of this radius here which I see is two units. So I know my radius is two. Okay?

So what I'm going to do now is go ahead and start a list going and that'll help me very easily be able to write my equation once I'm all done. So I know my center, I think I'll write it underneath here just so we don't get too crowded on the graph, just going to have to bear with me with the scrolling. So our center is at (1,1). So remember that's our H and our K and we know our radius is two units, so the radius is two, and we were also asked to find the length of the diameter, even though we don't need that information to write our equation, so the length of our diameter, we know the diameter is twice the length of the radius so two times two, our diameter is four. So now that we have the pieces that we need to write our equation, let's go ahead and write that out, going to put it right underneath here. All right, so the standard form, just so you can really see exactly what we're substituting where, let's get that written down. So we know our center is at (1,1). So that's our H and our K. So X minus 1 squared plus Y minus 1 squared equals R squared, the radius is two, so two squared, that's four. All right? And you're all done with this one.

So just to recap, I'm going to box actually both pieces of this in here because we were asked to determine both of those groups of information. We figured out the center, just got those coordinates right from the graph. We counted out a horizontal length from the radius to a point on the circle but we did say we could've counted vertically in either direction or horizontally to the left and we used that to get the length of the radius. We doubled the length of the radius to figure out that our diameter was four units, right? So we have all that information right here and then once we had that, we used our standard form for the equation of a circle, substituted the appropriate values in the appropriate spots, and we got our equation all written out. All right? Okay, good job on that one.

Now go ahead and give this one a try. So press pause. Take a few minutes. Work your way through it. Press play when you're ready to check your work.

All right. Let's see how you did here. So here again, let's see what we were asked to determine. I'm going to highlight it this time. We need to figure out the coordinates of the center, the length of the radius, and the diameter and then we're going to write the equation of the circle.

All right, so let's look at this graph here. Now in this case, it looks like our center is right at the origin. So our circle, I'm just going to write it off to the side because our circle is kind of small, our center is at (0,0). That's our H and our K. And our radius, I'm going to count a vertical length this time. I'm going to count the length of this radius here and it's just one unit long. So R equals one. And because we know the diameter is double the radius, one times two is two. So the diameter is two units long, right?

So now we have enough information here to actually write the equation of the circle. So I'm going to write just that general form first just so we can see what we're going to substitute where. Okay, so our H and K are both zero. So I'm going to show you the zeroes right now and then I'm going to show you how you'll probably normally see this written out when your center is at the origin. But we have X minus zero squared plus Y minus zero, that quantity squared equals R squared. And we know R is one, one squared, that's just one. So this equals one. Now if you stopped here you would absolutely be correct, but I want to show you how you'll normally see an equation written out when its center, the center of that circle, is at the origin.

Because X minus zero is just X, you'll normally see just X squared. And because Y minus zero is just Y you'll normally see just Y squared, and then equals one. So either one of these would've been correct but more likely than not, when that center is at the origin, you'll see that first part of that equation written out like that. All right? Okay, good job on that one.

All right, guys. You've reached the conclusion of this topic on the standard equation of a circle. I hope you saw how your knowledge of circles and your knowledge of the coordinate plane helped you get through this lesson. Bye.