Module 11: Constructions
Topic 1 Content: Constructing a Perpendicular Bisector of a Line Segment
Transcript

Hi, guys. Welcome to Geometry. In this topic we're going to focus on how to use the compass and the straight edge to construct the perpendicular bisector of a line segment. Now, your knowledge of circles is going to come in handy for you during this topic. You ready to get started? Let's go.

Now, look here. We've got a segment. We're given a line segment, and I'm going to show you how to use the compass and the straight edge of the compass to construct the perpendicular bisector. Now remember, a perpendicular bisector intersects a line or a line segment at a right angle, and it splits that segment in half.

So to construct that, this is what we're going to do. First just make an estimate of where you think the middle of the segment is, just a little dash just to mark that. Then what you want to do is you want to line your compass up, and you want to set the width of that radius of the arc that we're going to draw, which is part of the circle, to be just a little wider than the mark that you drew. So just set that width, the end of it, to be a little past your estimate, and you're going to swing your compass up, and with a light hand, arc. Now flip your compass around and line it up at the other endpoint. Now, when I say line up I mean the center of the little hole, the center of the circle I have right here. There's a hole right at the end of my compass. I'm going to hold my compass here and I'm going to arc.

Now, notice that those two arcs intersected each other, so I'm just going to darken those points of intersection. I'm going to use the straight edge of my compass to line up right along those points, and I'm going to drop a vertical line. Now, this line is the perpendicular bisector of my line segment. So it intersected my line segment at a right angle, and this point here is the midpoint of the line segment. So this part is congruent to this part.

Now, that's how you complete the construction. Now let's take a minute and go over exactly why it's true. Let's justify these steps. Now, this is our completed perpendicular bisector construction. So what we're going to do is we're going to build onto this to help us justify this construction. We're going to prove that it's valid mathematically.

Now take a look at this. Remember that what we did when we first started this construction is we lined up our compass at the endpoint and we set the width a little past our estimate of the midpoint of the segment, and then we made an arc. Then we kept that same compass width and we made another arc. Now, remember an arc is a part of a circle. So in actuality, this red arc is part of one large circle that has this endpoint of the segment as its center. The same thing is true for our blue arc. Matter of fact, let me move our perpendicular bisector. Let's move that out of the way for a second. This blue arc is part of a large circle that has this endpoint as its center. Because these arcs were drawn with the same compass setting, we know that these circles that they're parts of are congruent. We basically have copies of the same circle.

So what we're going to do first is we're going to focus just on this red arc. We're going to move that blue one out of the way. We're going to focus on this one for a second to help us justify this construction. I'm going to scroll down. Now take a look at this. What we've done here is we've drawn a segment from the center of the circle, which remember it's the endpoint of this segment. We've drawn a segment from there to the bottom of our perpendicular bisector. We have one segment here. Let's scroll down again.

Now we have another segment, this time from the center of the circle to the top of our perpendicular bisector.

Now, if you really just focus on the arc and these two segments for a second, these are radii of this circle, right? This is a radius of our arc, of our circle, because it's a segment that goes from the center to a point on the circle. And this is also a radius of the circle because it's also a segment that goes from the center to a point on our circle. Because these are radii of the same circle, we know that they're congruent. They have the same length. So what I'm going to do is I'm going to go ahead and mark that they're congruent. I know that this radius is congruent to this one. So we've handled some stuff right there with the red arc.

Now let's take a look at the blue one for a second. Actually, what I'm going to do is I'm going to move that red arc and its radii out of the way. I'm going to move that perpendicular bisector out of the way just for a second, just to help you really focus on what we're going to be focusing on for the next few minutes, just this blue arc and some of its parts. So keep that in mind. We're going to scroll down.

What we're going to do here is we're going to draw a segment from the center of this circle, because remember that arc is just part of a circle. So we have a segment from its center to a point on the circle. We've drawn that and that's actually the top of our perpendicular bisector. We've got to scroll down and we have another segment here from the center to a point on our circle, which is the bottom of that perpendicular bisector. Recall, like we said, that this arc is part of a circle. So what I have here is actually two radii on the same circle. Because of what we know about circles, we know that these radii have to be congruent. They have to have the same length, right? So we can go ahead and we can mark those congruent as well. So you getting all those congruent parts that are forming here

Now let's bring it all together for a second. Let me extend the page a little bit so we can kind of get that figure up a little bit. So what I've done here is I've brought everything together. I have the red arc and the radii, and I have the blue arc and the radii there also, and then we've also dropped that perpendicular bisector back in there.

Now what I want you to focus on is this parallelogram that's formed by those segments that we drew or by those radii of the circle. Now because of what we said initially, that basically these two circles are congruent, because when we used our compass we drew them with the same compass setting, same compass width, and we also know because of that that these radii are congruent on this circle, and they're also congruent to the radii on this circle, because those circles are basically just copies of one another. So what that tells us is that all four parts, all four radii, are congruent.

Thinking back to what we know about parallelograms, if we have a parallelogram of four congruent sides, then what we actually have is a rhombus. So what we've formed here by these four congruent radii is a rhombus. So now start thinking back to those properties of a rhombus. Specifically, I want you to think back to the properties related to its diagonals.

Now, scroll along with me. Actually we're going to flip to the next page here. Now, when you have a rhombus, there's a few properties related to its diagonals, right? A couple of those properties are ... Let's reveal the first one. In a rhombus, its diagonals are perpendicular. So what that means is that they intersect to form right angles. Going to draw a right angle symbol right in there. Now, another thing that we know about the diagonals of a rhombus, let's reveal the second thing here, is that its diagonals bisect each other. So what that means is that they cut each other in half. So in regards to our construction here, because we constructed perpendicular bisector, which is what we're trying to prove AC, we know that AC bisected DB. So let's get some congruent marks there also.

Now because ABCD is a rhombus, then we know that this diagonal bisected this one. So this part of our diagonal in the rhombus is congruent to this part of the diagonal in the rhombus, and the same is true for the other diagonal, but we're just going to focus on that one for right now.

So now in regards to our construction, we basically proved what we needed to prove. We've shown here mathematically that AC is the perpendicular bisector of DB. AC intersects DB at a right angle because of the properties of a rhombus, and AC also bisects DB, also because of the properties of a rhombus. So what we've shown here is by creating this rhombus by those segments that we drew, we used its diagonal properties to justify our construction.

I know that pulled together a lot of old things that we've covered in geometry and some new things related to circles, so great job pulling all that together and proving this construction.

All right, guys, you've reached the conclusion of this topic on how to use the compass and the straight edge to construct the perpendicular bisector of a line segment. I hope you saw how your knowledge of circles and your knowledge of properties of quadrilaterals came in handy for you during this topic. Bye.