Module 11: Constructions

Topic 1 Content: Constructing a Perpendicular to a Given Line from a Point Not on the Line Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on how to use the compass and a straight edge to construct a perpendicular through a given line and through a point not on that given line. Now, your knowledge of circles is going to come in handy for you during this topic. You ready to get started? Let's go.

So here we're given a line, and we're given point A, which is not on the line. What we're going to do is we're going to sketch or we're going to use the compass to construct a perpendicular, so a line that passes through point A and intersects our given line at a right angle. Let me show you how we do that. The first thing we want to do is we want to set up our compass. Go ahead and put the center. There's a tiny circle right here and it's clear plastic, so I'm sure you probably can't see that, but there's a small hole in there where I'm lining that up right along with my point. I'm going to set the width of my compass so that it stops just underneath my given line. So I'm right underneath the line. Going to hold my compass steady right here, and with a light hand I'm going to arc. All right? Now, notice that arc intersected our line at two points, so I'm just going to darken those points so they stand out for me.

Now what I'm going to do is I'm going to estimate the midpoint between these two points. So I'm just estimating where's about halfway between these two points? Going to make an estimate that it's about right there. Now I'm going to line my compass up at that left endpoint. This time I'm going to set the width to be just a little bit past my estimate. I'm going to swing my compass down underneath the line, and I'm going to arc.

Now I'm going to keep that same compass setting and I'm going to go to my right endpoint. Going to hold the compass. Got to do a little crisscross with my arms this time. I'm going to hold it right at that endpoint and I'm going to arc on this side. Now, notice that our arcs intersected, so I'm going to put a point just right there, just so that point of intersection stands out.

Now I'll use my straight edge just to line up, kind of imagine, envision that line that's going to pass through point A and that point of intersection. Then I'll go ahead and get it drawn. And you're all done.

This line that I sketched passes through my given line at a right angle and also passes through point A. I've constructed a perpendicular through a given line and also through a point not on the line.

So now what we're going to do is we're going to take a few steps to justify this construction. You're going to learn how is it that this construction is actually true. Now take a look at this. Here's our completed construction where we construct a perpendicular from a point not on our given line. What we're going to focus on first is just a segment of that perpendicular line that we constructed. Just focusing on the segment is going to help us really justify the steps of this construction.

So recall when we first constructed. Our first step in this construction was holding our compass vertically. Remember we held it straight up and down and we set the width to a point a little below our line, our given line, and then we arced. That arc intersected our circle at two points. Keep that in mind and take a look at this.

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Now, the first step we're going to take in this justification is we're going to draw a segment from the top of our perpendicular to that right point of intersection. Now, recall that an arc is just a part of a circle, right? We just made an arc, but this arc is part of a circle that we actually could have drawn. So this segment that we've added right here is actually a radius of this circle, because here is the center of that circle and we've drawn the segment from its center to a point on the circle. So really we've just added a radius to this figure here.

Now look at this next step. Here we've added another segment from the center of the circle to another point on the circle. So actually what we've done is we've just added another radius, and because we know that all radii on a given circle are congruent, we know that these two segments are congruent. So what I'm going to do is I'm just going to add those congruent marks. We know that these are radii of the same circle, these two segments, so they have to be congruent. Keep that in mind.

Scroll down with me and take a look at this. Now what we're going to do is we're going to focus on those red arcs, and for a second I'm just going to move the blue arc and its radii out of the way here. So recall when we drew these segments our compass was set to the same width for each of these. We put the end of our compass here on that left point of intersection and we set our width to a point past what we estimated as the midpoint between these two points. We swung down below and we arced. Then we flipped our compass around, kept that same setting, and we arced again. So here, if you focus your attention on just this red arc here, what we have is a radius of this circle because this arc is just a part of a much larger circle, or of a circle. So we have one radius here of the circle on the right.

And take a look here. Let's scroll down again. We added the blue part here back into the picture.

So what we've done here is we've focused on the left arc, that left red arc down at the bottom, and we've added a radius to this circle. So because we know that these arcs were drawn with the same compass width, that tells us that these are congruent circles that they're part of, so the radii in each of these circles is congruent. Because basically we've just copied that circle, so the radii are going to be congruent. So I'm going to add congruent marks there also.

So I'm going to add those other ones that we had up top at first too. What we have here is we've created several different triangles in our process of adding those segments to the figure. So now what we're going to do is we're going to pull back to what we know about congruent triangles in order to help us justify this construction.

Now take a look at this. Our first step in using our knowledge of congruent triangles to justify this construction, let's get that out of the way, is we're going to prove triangle ADC congruent to triangle ABC by Side Side Side. So what I'm going to do to help us really focus on those triangles is I'm going to pull them out of this construction so that we don't have to worry about trying to sort out the arcs and really focus on the triangles. I'm just going to redraw those triangles over here to help us just focus on that for a second. I'm just going to give it my best sketch. It's not going to be perfect. Just going to roughly sketch those triangles. Let's scoot that up a bit. I'm starting to run into our congruent statement there. So we have A, D, C, B. So we're trying to prove triangle ADC, so I've pulled that triangle out right here, congruent to triangle ABC. I pulled that triangle out right here. We're using Side Side Side to prove these triangles congruent.

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Now, because of what we learned in our process just on that previous slide, remember we proved that those were radii of the same circle, so we knew that these segments were congruent. So now what that tells us here is that AD is congruent to AB. We also proved when we focused on those red arcs that these were radii of congruent circles. So what that tells us here is that DC is congruent to BC. So we can add those marks to our figure over here.

Now, if we're trying to use Side Side Side, then we need one additional pair of congruent sides here, and if you look, notice our triangles share side AC, so that means that I can use the reflexive property to prove that AC is congruent to AC. If you look, we've done it. We've proved that triangle ADC is congruent to triangle ABC by Side Side Side. So we're one step closer to justifying this entire construction.

Take a look at the second step. Going to scroll down. Going to have to scroll up and down just for a minute with this. Now, the next step here is that we need to show that triangle DAE, so I'm going to scoot my figure down here so we can see the statement and the figure at the same time. So we need to prove that triangle DAE, so this triangle here, is congruent to triangle BAE, so that's this triangle here. We're going to use Side Angle Side to prove that these two triangles are congruent.

So what I'm going to do first is I'm going to copy the triangles over here just so we can focus on the triangles and don't have to sort out those arcs. So just my best sketch. I'm going to get that down over here and I'm going to scoot that congruent statement up a little bit because I'm starting to run into that. Let's add our letters here. We have A, D, E, and B. So we need to prove triangle DAE congruent to triangle BAE.

Now, Side Angle Side is what we're using here, so I know I'm going to need a pair of congruent angles in order to show these triangles are congruent. Now I'm going to scroll up for a second and go back to what we just proved about that other pair of congruent triangles. Now because these two triangles are congruent, I know that all of their corresponding sides are the same, have the same length, they're congruent. And all of the corresponding angles are congruent. So I'm going to use a pair of corresponding angles that must be congruent, because I've proved these triangles are congruent, in order to help me with this next step that I'm currently working on.

This is what I'm going to use. Because these triangles are congruent, I know that angle DAC is congruent to angle BAC. So basically the angles that share A as their vertex, I know that they must be congruent because they're corresponding parts of congruent triangles. So now if I keep that in mind and look at the triangles that I'm currently working on, here are those two angles that share A as their vertex. So what that tells me here is that angle DAE is congruent to angle BAE.

Now, I know you probably noticed that in the triangles up at the top, if I scroll back up there, that I refer to these as angle DAC and angle BAC, and now I'm referring to them as angles DAE and BAE. That's okay because it's still the angle that's the same. I just have another stopping point, basically, on that other side of that angle. But I'm still referencing the same angles here. The angles that share A as their vertex are congruent. So in this pair of triangles, they were named, like I said, DAC and BAC. Those were the angles' names. But here there's just another stopping point on the side of that angle. So angle DAE is congruent to angle BAE because corresponding parts of congruent triangles are congruent, right? We use the fact that we knew those angles were congruent parts on another pair of congruent triangles.

Now we've already proven that DA and AB are congruent, so I can use that piece of information here. DA is congruent to AB. And then here, these two triangles share AE. So I can use the reflexive property to prove that AE is congruent to AE. If you look, we've done it here. We've proven that triangle DAE is congruent to triangle BAE by Side Angle Side. We have our two pair of corresponding sides congruent and our one pair of included angles congruent. So keep that in mind and we're at the last step to justify this proof.

Going to scroll down here and extend this page a little bit, just to kind of get that up at the top. Now let's reveal what's behind this. So here we're told that angle AED and angle AEB are congruent and supplementary. So we need to figure out why that is actually true. Let's focus on angle AED and angle AEB. Well, if I look, those are two pair of angles that are in those pair of congruent triangles that we just worked on. If I scroll back up to the middle section here, here are angle AED and angle AEB. Now, because these triangles are congruent, that tells me that their corresponding parts also have to be congruent. So angle AED must be congruent to angle AEB because corresponding parts of congruent triangles are congruent.

I'm going to scroll back down. So we've proven that they're congruent. We know that this part is true. Now, it also says that these two angles are supplementary. So now think back about what you know about a linear pair. Here we have two adjacent angles. They share a vertex and a side and they lie on a straight line, on a 180 degree angle. So that means these angles must be supplementary. Their sum must be 180 degrees. So we've proven they're supplementary.

Now let's put those pieces together and think about what that means. If these angles are both congruent and supplementary, that means that they each must measure 90 degrees. That's the only way that both parts of this statement are true. So what that tells us is that the measure of angle AED is 90 degrees and the measure of angle AEB is also 90 degrees. That's the only way that these angles can be both congruent and supplementary. If each of these angles measures 90 degrees, then that means that AC must be a perpendicular. AC must intersect this line at a right angle if these two angles are each 90 degrees.

There, we've actually proven what we set out to do. We've shown that AC must be a perpendicular of this line. It must intersect it at a right angle. Now, we pulled back and we used what we knew about congruent triangles and circles to prove what we needed to do here. Good job pulling all those pieces together and justifying this proof.

All right, guys. We've reached the conclusion of this topic on how to construct a perpendicular to a given line from a point not on the line. I hope you saw how your knowledge of circles and congruent triangles came in handy for you during this topic. Bye.