Module 11: Constructions

Topic 1 Content: Constructing a Perpendicular to a Given Line from a Point on the Line Transcript

Hi guys. Welcome to Geometry. In this topic we're going to focus on how to use the compass and the straightedge to construct a perpendicular through a line and also through a point on that line. Now your knowledge of circles is going to come in handy for you during this topic.

You're ready to get started? Let's go.

Okay. Here we're give a line and we also see that point A is on our line, so what we're going to do is we're going to construct a perpendicular, so a line that passes through point A at a right angle, okay?

All right. Let me show you how you do that. Now, the first thing you want to do, is you want to set your compass up, so that the center of our circle that's right in here, is lined up right on top of point A, and then we want to set our compass width really to any setting that you want. Just make sure that whatever width you've set it to, that it still falls on your line, okay? Because the arc that we're going to draw, we want to make sure that it actually intersects our line, okay?

I'm going to have my width set right here and with a light hand I'm going to swing that compass up and I'm going to arc. Okay. Now, I'm going to keep the compass right where it is on point A, but I'm going to swing it around to the other side and I'm going to arc to the left of point A. Okay. So we have two arcs and we've intersected our line at two points. I'm going to darken those points of intersection so they stand out.

Now what I'm going to do is I'm going to line my compass up at that left that far left point. I'm going to set the width to just a little bit pass point A. It doesn't matter exactly where it's set, just make sure that it's set a little bit pass A. We'll swing our compass up and we'll arc above the line this time, okay?

Now I'm going to flip my compass around. Now I'm on that far right point, and I'm going to swing my compass up and I'm going to arc above the line. So notice there are two arcs intersected right above A. I'm going to darken that that point of intersection so it stands out. And I'm going to take my straightedge and I'm going to envision the line that I'm going to draw, that's going to pass right through point A and that point above my line. And there you go. We've constructed a perpendicular, that passes through our given line and passes through that given point that we started with, point A. All right?

Now what we're going to do is, we're going to take a few minutes and we're going to walk through the steps that justify this construction. We're going to see what it is that actually guarantees that this construction is true, all right? Okay, let's take a look.

What we have here is our completed construction. And what we're going to do is we're going to build on to this construction in order to justify that it's true, okay? Now, think back at first at how we started this construction. If you recall, we placed our in point of our compass on that given point, on the line, and we extended it, we set its width, just any set, any point, and we arched to the left and we arched to the right, okay? Those were the first steps we took.

Then we placed our compass at that left in point and we opened it a little pass to where our given point was and we arched above and we flipped the compass around keeping that same width, and we arched above. And we ended up right here, okay? Now keep all those steps in mind and look at this next step.

Now, what we're going to add to the figure at first is we're going to add this segment right here. And if you look, it's a segment that goes from that given point on the line, to that point where that left arc intersected the line. Now what I want you to really think about here, is what is special about this segment. And to help you really see that, I'm going to remove some additional things here. Let's just slide those arcs out of the way for a second. Okay.

Now think back again to what we just went over on that previous slide. We discussed how we made these two arcs at first. If you recall, the compass width was set to the same setting for each of these arcs. And if you look ... because our starting point on each of those arcs that we made, where we put our compass was the same. What that means is that these are actually arcs of the same circle. We use this point as the center each time and we arched to the left and we arched to the right. Those are actually arcs on the same circle.

This segment here that runs from that center of the circle to this point on the circle, is actually congruent ... I'm going to scroll down to this segment here. These are radii of the same circle. That means they have to have the same length. Those segments are congruent. I'm just going to add congruent marks in there, okay? And like I said, those are radii of the same circle, so those radii are congruent, okay?

Now let's focus on these red arcs for a second. I'm going to scroll down here. We're going to add a segment that runs from that left point of intersection there on the line, to the top of our perpendicular. And if you think about it ... I'm going to get that other red arc out of the way, that one on the right. This arc, we know it's part of a circle, right? So what we've added here is just a radius of that circle, all right? So this segment is just a radius of this circle. Keep that in mind and take a look at this.

I'm going to extend this page a little bit, just to get that in focus. Okay. Now we've added in the other arc and we've added in this segment here. Now, this segment runs from the top of that perpendicular to that point of intersection on the line. And think about it. That arc, is part of a circle, right? So we've just added a radius of the circle onto our diagram here.

And because we know that these red arcs were drawn with the same compass setting, that tells us that these circles, these red circles that we're envisioning, are basically copies of one another. Those circles are congruent. That means that they're radii also have to be congruent.

We have another pair of congruent parts here. We know that this segment is congruent to this segment. And we proved earlier that this segment was congruent to this one, okay? Now what we've done is we've created a few different triangles. And we're going to think back to what we know about congruent triangles, in order to help us justify this construction, okay?

Right now keep that in mind and take a look at this.

Okay. Here's our first step in using what we know about congruent triangles to prove this construction. The first thing we're going to prove is that triangle ACB is congruent to triangle ACD by side, side, side. Okay?

Now, just so we can focus on those triangles and don't have to worry about sorting out those arcs, I'm going to redraw the triangles here, just help us focus, I'm going to roughly sketch what those triangles look like. All right, and right down the middle here. And we have A, B, C and D. Okay.

Now, we proved on the previous slide that AB was congruent to AD, right? Because those are radii of congruent circles. That means I can mark AB congruent to AD. And we also proved that BC was congruent to CD, because those were two radii on the same circle, on this blue circle that we're envisioning here. That tells us I can add that to my figure over here. BC is congruent to CD. Now, because of what I know about the reflexive property, I know that I can say that AC is congruent to itself. AC is congruent to AC by the reflexive property.

We've proven here that triangle ABC is congruent to triangle ACD by side, side, side. We've shown those three pair corresponding sides, that are congruent. Okay? Now, keep that in mind, there's one more step to this. I'm going to scroll down. Let's get this out of our way here. I'm going to have to do a little scrolling up and down to get through this last part.

Our next step is to show that angle ACB is congruent to angle ACD and that they're also supplementary, okay? I'm going to scroll up here for a second. Because of what we know, what we've shown here, that these triangles are congruent, that means that their corresponding parts are also congruent. So here ... I'm going to switch colors ... angle ACB must be congruent to angle ACD, because those are corresponding parts of congruent triangles. And because we know that corresponding parts of congruent triangles are congruent, that tells us that these angles must be congruent. We've shown they have to be congruent.

The second part of this is to show that they're also supplementary. Now think back to what you know about linear pairs. If I take a look at these angles here, I know that they're supplementary, because they lie on a straight line, on a straight angle, you could say 180 degree angle, and they're adjacent, they share a side. So if angles are adjacent and supplementary, we know they form a linear pair. Okay? And because we can see here they're a linear pair, we know that they have to be supplementary. We've proven the second part of this, that these angles are also supplementary.

Now think about what that means. If you have two angles that are congruent and supplementary, that means that the measures of each of those angles is going to have to be 90 degrees. That's the only way that both of those statements can be true at the same time, okay?

I'm going to write that just so you can really see what I mean by that. Let's get a little space. Here we go. The measure of angle ACB must be 90 degrees, and the measure of angle ACD must also be 90 degrees. And I will scroll back up to the picture. Here, this time I'm going to focus on the construction. The only way that these angles are both congruent and supplementary, is if they both measure 90 degrees. That's the only way that that can be true.

And if you think about it, we've really proven what we set out to prove at this point. Because if we know these angles are 90 degrees, then that tells us that AC must be a perpendicular. This segment must be perpendicular to this line, if these angles measure 90 degrees, okay?

And we're really all done there. We've used what we know about congruent triangles and circles in order to prove that we did indeed construct a perpendicular to this line, okay? Now, good job pulling back to all those geometry skills that you learned earlier in the course, to prove that this construction is valid.

All right guys, we've reached the conclusion of this topic, of how to construct a perpendicular through a given line and through a point that's on that given line. I hope you saw how your knowledge of circles came in handy for you during this topic. Bye.