Module 11: Constructions
Topic 2 Content: Constructing an Angle Bisector Transcript

Hi, guys. Welcome to Geometry. In this topic we're going to focus on how to use the compass and the straight edge to construct an angle bisector. Now, your knowledge of circles is going to come in handy for you during this topic. You ready to get started? Let's go.

Okay. Here we're given an angle, and we're going to use the compass and the straight edge to construct the angle bisector of this angle, okay? Here we go. The first thing we're going to want to do is line up the center of that small circle that's right here at the end of my compass, and we're going to set the compass width really to any given length that you want, okay? It doesn't matter how wide this is set to as long as when we arc the circle, so envision yourself arc ... I'm sorry, arc the angle, so envision yourself arcing that. You want to just make sure that the arc is going to hit both sides of your angle. You wouldn't want to slide it all the way out here, because you wouldn't actually touch the angle when we drew that arc, okay? I'm just going to keep it set just about right there, and with a light hand I'm going to arc both sides of my angle, okay?

Now, notice I intersected my angle here right at the top and at the bottom, so I'm just going to darken those points of intersection, and keeping the compass set at that same width, I'm going to go to that top point, hold my compass steady here, and I'm going to arc right here in the interior of the angle, right inside. Okay? Keeping that width set still at the same measure, I'm going to go to that bottom point and I'm going to arc again right in the interior of the angle. Okay? Now, notice that those two arcs intersected, so I'm just going to darken that point of intersection, and now what I'm going to do is I'm going to line up the straight edge of my compass starting right at the vertex of my angle, and then through that point of intersection in the interior of the angle, and I'm going to use that straight edge to sketch that angle bisector, and there you go.

This angle ... I'm just going to switch colors for a second with my marker. This angle right at the top is congruent to this angle at the bottom. We've constructed the angle bisector for this angle, okay? Now what we're going to do is we're going to take a few minutes and we're going to walk through the steps to justify this construction. We're going to see what it is that makes us know for sure that this construction is true. Okay? Just take a few minutes and let's walk through that together.

Okay. Take a look at here, we have the completed angle bisector construction. What we're going to do is we're going to build onto this construction a little bit in order to help us justify that it is valid, okay

Now, first I want you to think back to how we completed this construction. If you recall, we started out by setting the end of our compass at the vertex of the angle and then we opened our compass up just to any width as long as when we arced we would intersect the sides of our angle, and we did here, so we have this arc where we intersected both sides of our angle. Then we kept that same compass setting and we went to that top point of intersection, or that point of intersection on that top side of the angle, and we arced. Then we went to that bottom point of intersection and we arced again, okay? But our compass was set to the same width for each arc. That's going to be a very important piece to this justification, so keep that in mind, and take a look at this.

Now, here if you notice, we've added two blue segments that have end points at the vertex of the angle and then to each of the points of intersection. Now, recall that this arc that we made is really just part of a circle. We could've used our compass and completed that entire circle if we wanted to, but we just focused on the arc, and that arc is part of a circle. What that tells us is that these two segments are radii of this circle. They're segments that go from the center to a point on the circle, and because of what we know about circles, we know that these two segments must be congruent, because all radii on a circle have the same length. They're all congruent. I'm going to use that fact. I'm going to go ahead and just add congruent marks in there. We know that this segment's congruent to this one because they are radii of the same circle, okay? Keep that in mind. I'm going to scroll down, and I want you to take a look. We're going to focus on those other arcs in this construction.

First, let's focus on that blue arc. I'm going to move the red pieces out of the way for a second. Now, remember when we made this arc here we kept our compass at the same width that we had at this setting and we moved to that point of intersection and we made an arc, so this arc is part of a circle that we could've completed but we just made the arc. What that tells us here is that this is a radius of this circle, okay? Keep that in mind. I'm going to bring back those red pieces. Let's bring back that arc, try to get it exactly where it was, and then add this segment in here. We're going to ignore the blue pieces for a second. Let's take those out of the way.

Now, recall that compass was still set to that same width when we made this arc, and this arc is part of a circle, right? We could've completed it but we just made the arc. This segment here is a radius of this circle, okay? It's just a segment that goes from the center to a point on the circle. Now if I bring back those blue pieces, let's bring those back in here, try to line them up exactly how they were initially, basically what we have here, because our compass was set to the same width when we made each of these arcs, we really just copied the same circle over and over again. What that tells us about these segments is that they have to be congruent, because really this arc is part of a circle that's the same as this circle, and also the same as this one. We basically copied that circle three times, and each of these radii, each of these radius, radii I should say, are congruent. Each one has the same length, okay?

What we've created here is a parallelogram that has four congruent sides, and if you think back to what you know about parallelograms, if you have a parallelogram that has four congruent sides, it's a rhombus by definition, okay? Start thinking back to some of those properties about a rhombus. That's going to help us get through the rest of this justification. Okay? I'm going to move on to the next slide here.

I'm going to reveal what's behind number one. Okay. ABCD is a rhombus. We've proved that. We've shown that each of these segments are radii of congruent circles, so they're going to have to be congruent. We've created a parallelogram with four congruent sides, so by definition it's a rhombus. Thinking back to what you know about the properties of a rhombus, let's reveal this here. One of those properties allows us to prove that angle BAC is congruent to angle DAC. I'm going to pull the rhombus out of that diagram for a second just so we can focus on it and ignore the arcs. I'm going to make a rough sketch, try to get it as perfect as I can, but of course it's not going to be perfect, because it's a rough sketch. Okay. We've shown that four congruent sides here.

Now, really focus your attention on AC, this segment right here. Now, in regards to our construction, we know it's the angle bisector that we were trying to construct and that now we're trying to prove is really valid, but in regards to our rhombus, it's a diagonal of this rhombus. Think back to what you know about the diagonals of a rhombus. If you recall, the diagonals of a rhombus bisect the opposite angles, so what that tells us is that angle BAC must be congruent to angle CAD. These angles right here that share A as the vertex have to be congruent, because the diagonal of a rhombus bisects the opposite angles. It splits them in half. If these two angles are congruent, then that proves that our construction is valid, because in constructing AC we did indeed construct an angle bisector. We constructed a segment, switch to my pen here, that split our angle in half. I'll add two marks just to keep it ... make sure it matches. Okay?

We've really done all that we needed to do to prove that this construction is valid. All of the steps that we've taken have shown that we have a rhombus here, and that the diagonal in this rhombus bisected angle A, all right? Okay, good job pulling back to those properties that you learned of a rhombus and what you know about circles in order to justify this construction. All right, guys. We've reached the conclusion of this topic on how to use the compass and the straight edge to construct an angle bisector. I hope you saw how your knowledge of circles came in handy for you during this topic. Bye.