Module 11: Constructions
Topic 3 Content: Constructing a Square Inscribed in a Circle Transcript

Hi guys. Welcome to Geometry. In this topic, you're going to learn how to use a compass and a straight edge to construct a square inscribed in a circle. You're going to see how your knowledge of circles and your knowledge of an earlier construction is going to come in handy for you during this topic. You ready to get started? Let's go.

Here we're given a circle. What we're going to do is we're going to construct a square inscribed in this circle. Now how we're going to start off by doing that, is the first thing that we're going to do, is we're going to sketch a diameter of this circle. Remember, a diameter is a chord that passes through the center of a circle, right?

I'm just going to line my compass up, just as accurate as I can get it, so that the line that I draw passes through the center of my circle. I've got my straight edge lined up pretty straight I think. Let's get that diameter. I'm just going to darken the endpoints of that diameter. Now what I'm going to do is, I'm going to construct the perpendicular bisector of this diameter, which basically allows me to accurately construct another diameter of this circle, okay?

I'm going to go ahead, I'm going to align my compass up right at the left endpoint of this diameter, and I'm going to make sure that the width is a little bit past the center of my circle. I'm going to swing the compass us and I'm going to arc. Okay? Now I'm going to flip the compass around so that now it's lined up with that right endpoint of the diameter and I'm going to arc. Okay?

Notice that my two arcs intersected. I'm going to darken the point right at the top where they intersected, and at the bottom. I'm going to line my compass up, the straight edge side of it, just to make sure that the next chord that I draw passes through all three of those points. Just get it as accurate as you can. That looks pretty accurate. I'm going to draw that perpendicular bisector, then I'm going to keep it as a chord, not going to draw a line, okay? I've got those endpoints.

Now notice that I have four points on my circle. Those are going to be the vertices of the square. Now what I'm going to do is I'm going to take that straight edge, and I'm going to construct this square. I'm going to actually get it down here. That's lined up pretty straight. There's one side of the square. Let's get the straight edge here, lined up as straight as we can. Let's get this side. Now let's get this third side of our square, get that lined up, and the fourth side of our square, all right?

You see we have a square, matter of fact, I'll go ahead and name it, ABCD, there's our square inscribed in our circle, okay? Now what we're going to do, is we're going to walk through the steps to justify this construction. We're going to see what's the math behind all the steps that we took that guarantees this construction is true, okay?

Let's take a few minutes and let's walk through that. What we have here is the completed construction of a square inscribed in a circle. What we're going to do, is we're going to justify this construction. We're going to pull back to what we know about parallelograms, one of our other constructions and circles in order to prove that this construction is actually valid, that we did indeed construct a square inscribed in a circle, okay?

Let's take a look at the first part of this here, what we're going to prove first. Let's get that out of our way. The first thing that we're going to show is we're going to show that the diagonals are congruent and that they bisect each other, okay? Now to do that, let's move a few things out of our way here. Let's get these arcs out of the way.

I want you to really just look at this parallelogram right here to start with. Now we have a parallelogram, and this parallelogram ABCD, has diagonals AC and DB. Now as well as those diagonals being diagonals of that parallelogram, there are also diameters of this circle. Now think back to what you know about the diameters of a circle.

Well one thing we know about them, is that they must be congruent. What that tells us, is that AC and DB are congruent, because they are diameters of the circle, as well as being diagonals of this parallelogram, they're also diameters of the circle. We've actually proved the first part of what we wanted to show here. I'm going to check it off. We've shown that the diagonals are congruent, okay?

Now the second part that we want the show is that they bisect each other. Let's focus on the fact that we know that these are diameters, like we've already shown. Well the definition of a diameter, is that it's a segment that has it's endpoints on the circle and it passes through the center of the circle. E, as well as being the center of our circle, is also the midpoint of our diameter.

That means that E is the midpoint of AC, and it's also the midpoint of diameter DB. What we can conclude from that, is because those diameters share that midpoint, we can show that AC bisects DB at E, we can conclude that. We've already taken care of the second part of this now. We've shown that these diagonals bisect each other, and we know that they bisect each other at E, the center of our circle, okay?

I'll go ahead and check that part off. We've taken care of that first statement. Now I'm going to scroll down, and let's look at the second part of what we want to show here. Let's take a look at this. Get this out of our way over here; see what's hiding behind there.

The next thing that we want to show is that our diagonals are perpendicular. Now I've got the arcs back here, because I want you to remember that we used the perpendicular bisector construction, in order to construct AC. When we use that construction, we guaranteed that AC would be perpendicular to DB. I'm going to switch to my pen here, so I can put our right angle symbol in there, okay? By using that construction, we've guaranteed that AC is perpendicular to DB. That tells us that our diagonals are perpendicular to one another. Now we've proven the second part of this here, got that taken care of, okay?

Now let's scroll down a little more, and let's look at this third statement, let's see what's hiding behind this one. Well we are actually at the conclusion that ABCD is a square. So why is it a square exactly? Let's think about that. I'm going to scroll up to review what we've already shown here.
I want you to think back to what you know about a square, about its properties. Now if you

recall, looking at our first statement, the diagonals of a square are congruent and they bisect each other, okay? If we have a parallelogram that has diagonals that are congruent, and bisect each other, think about that, and also, the diagonals of our parallelogram are perpendicular, then what we can conclude is that the parallelogram is a square, because those are all properties that the square has specifically. Its diagonals are congruent, they bisect each other, and they're perpendicular.

By showing that this parallelogram here holds all of those properties, we've shown that it is indeed a square. Now we could also show here that this square, now that we know it's a square for sure, that it's inscribed in this circle, because we can see here that its vertices do lie on the circle. That's the definition of something being inscribed in another figure. Its vertices lie on the figure, okay? We've done it here. We've shown that this construction is indeed a construction of a square inscribed in a circle.

We had to pull back from what we knew about the properties of a circle, the perpendicular bisector construction and the properties of a parallelogram to prove that. Great job pulling all that together in order to justify this construction. All right guys, you've reached the conclusion of this topic on how to use a compass and a straight edge to construct a square inscribed in a circle. I hope you saw how your knowledge of circles and your knowledge of the perpendicular bisector construction came in handy for you during this topic. Bye.