Module 11: Constructions
Topic 3 Content: Constructing an Equilateral Triangle Inscribed in a Circle Transcript

Hi guys, welcome to Geometry. In this topic, you're going to learn how to use a compass and a straight edge to construct an equilateral triangle inscribed in a circle. Now, your knowledge of circles is going to come in handy for you during this topic. You ready to get started? Let's go.

Okay, so here we've been given a circle. What we're going to do is use the compass and the straight edge to see how to construct an equilateral triangle inscribed in the circle, okay? What you want to do starting off is you want to make sure that your compass width is set to the width of the radius of the circle. Here, I'm just making sure I've got the center lined up at the center of my circle, the center of the little hole in my compass that is, lined up to the center of my circle. My width is set open to the length of that radius, okay?

Now what I'm going to do is just make a starting point. I'm going to just start right here at the top. I'm going to line my compass up with that point, and I'm going to begin to arc around my circle. Right here, I'm lined up at my starting point, I'm centered there, and I'm going to arc. I have a point of intersection here. Now I'll move to that point of intersection, lining up my compass and I'll arc again to intersect my circle. Now, I'm going to line up at that point, okay, and I'm going to arc again.

Now, I'll line up at this point and arc again. I'll line up here and arc again. Have that point, then one more time. I've lined up here and I'm going to arc. Okay, so see what we have here, we have six points that are on our circle. Now what we're going to do is we're going to draw chords that connect non-consecutive points. That's what's going to let us set up our equilateral triangle. For example, if I decided to start at this point, then I would sketch my triangle like this. Like skipping that next point.

Now, if I decided to start, say, at this point, then I'd set up my triangle a little something like that, right? You just want to make sure that you skip the consecutive points, so that you can go ahead and form your equilateral triangle. I'm going to start right up at the top, and I'm going to get this chord. I'm going to line up my straight edge so that I pass through those points, and there's a chord. Now, I'll line up my straight edge here along those points, and we've got another chord. I'll line up my straight edge right here along these points, and we have another chord so there you go. We have an equilateral triangle inscribed in the circle, okay?

Now, that we have this construction complete, what we're going to do is we're going to take a few minutes and we're going to walk through the steps that justify this construction. We're going to see what the math is behind all of this that ensures us that this construction is true. Take a look at this.

Now take a look at our completed construction of an equilateral triangle inscribed in a circle. What we're going to do is we're going to use our knowledge of circles and our knowledge of triangles in order to justify this proof, okay? Take a look at this next slide. Now if you recall when we completed this construction, the first thing that we did is we set our compass open to the width of the radius. Then we went around our circle and we made these six arcs. Now because our compass was set to the same width, each time we made those arcs, that means that we split our circle into six congruent parts, right? Each of these arcs is going to have the same measure. Keep that in mind.

Now because of what we know about circles, which is that the sum of the arcs equals 360 degrees, when we split our circle into six congruent parts, we split that 360 degrees up into six equal parts. That tells us that each arc measures 60 degrees, so I'm going to scoot that over a little bit, make sure we don't run off there. Okay, so that's how we got this here, that each of these arcs has a measure of 60 degrees. Keep that in mind, that's going to become a really important piece here. Going to scroll down again.

Now, we're going to bring our triangle into the mix here. Once we had our arcs split up there, once we had our circle split up into those arcs, we went ahead and we drew our equilateral triangle, which we're trying to justify that it is actually equilateral. What we did is we picked an arc to start at, or picked a point of intersection to start at. Here, we started at the top. Then we skipped an arc, and here's where we drew that side. We kept that pattern going to make that triangle, okay?

Now this triangle is inscribed by definition because its vertices lie on the circle. If you have a polygon that has its vertices on a circle, then you can say that polygon is inscribed in the circle. Here, we do have a triangle inscribed in a circle so our next step is going to be to prove that it is actually equilateral, okay? To help us do that, notice those arcs that we've been discussing a lot here, that each measures 60 degrees. We're going to pair those down a little bit, and just focus on a few 120 degree arcs.

I'm going to scroll up and down just a bit to show you where these measures came from. What we did here is we combined these two arcs in order to show that this one large arc is 120 degrees, just added those measures together. That's how we got this arc right here. Then we did the same thing all around here. We paired these arcs, we combined their measures and we combined these two arcs here, their measures. Instead we referenced two 120 degrees arcs over here. We have this large arc, that's 120 degrees, and this one and then the one that we discussed earlier, okay? Now, we're going to use these arcs to help us work our way into the angle measures of this triangle. Keep that in mind, and take a look here.

Now think back to what you know about an inscribed angle of a circle. If you recall, an inscribed angle is an angle that has its vertex on the circle. Here when we have this triangle, I'm first going to just focus on this vertex of the triangle and this angle of it. Going to just highlight here just so I can focus my attention right there. If I want to figure out the measure of this angle, I can reference the measure of this arc. If you guys recall that an inscribed angle is half the measure of its intercepted arc. If this arc has a measure of 120 degrees, that means that that angle has a measure of 60 degrees, because 120 divided by two is 60. That means this angle measures 60 degrees, okay? Now scroll down with me here. Let's get the rest of the angles of this triangle.

Went a little too far, let's bring it back up. There we go. Now I'm going to focus on this arc, and use it to help me get the measure of this angle. If I highlight again, just focus on just this angle here, the measure of this angle will be half the measure of this intercepted arc. Again, half of 120, that's 60. The measure of this angle, 60 degrees, okay? Let's keep going here, one more angle in this triangle, that angle right at the top. I'll highlight again so we can focus on that angle. Again, the measure of this inscribed angle will be half the measure of its intercepted arc. Half of 120, again that's 60 degrees.

If I look at what I have here, I have a triangle where each of its measures is 60 degrees, right? Think back to what you know about triangles, I'm going to extend this page a little bit so we can focus right up there, okay, so thinking back to what you know about triangles, if you have a triangle that has three congruent angles, that means that it also has to have three congruent sides. If a triangle is equal angular, that means it also has to be equilateral. This triangle that we have here inscribed in the circle, it must be an equilateral triangle. If all of its angles are congruent, that means that all of its sides have to be congruent. If you look, we've done it.

We've used what we knew about circles and about triangles in order to prove that this is indeed an equilateral triangle inscribed in the circle, okay? Good job on that. All right guys, you've reached the conclusion of this topic on how to use a compass and a straight edge to construct an equilateral triangle inscribed in a circle. We saw how your knowledge of circles came in handy for you during this topic. Bye.