# Module 12: Perimeter, Area, and Volume

## Topic 1 Content: Calculating the Surface Area and Volume of a Cylinder Transcript

Hi guys, welcome to Geometry. In this topic, we're going to focus on calculating the surface area and volume of a cylinder. Now your prior knowledge of area and volume is going to come in handy for you during this topic. You ready to get started? Let's go.

Okay, so before we dive into the problems relating to a cylinder, I first want to get you familiar with a cylinder to start with. Now a cylinder is a three-dimensional figure. It consists of two parallel bases that are circles, and one lateral face, which is a rectangle. Now by lateral, what I mean is it's the face that wraps around the cylinder. It's that curved surface, okay? Like I said, those faces, those circles, those actually lie on planes, which means they're flat surface. Now the height of our cylinder is represented by a perpendicular segment, which you can think about it as it connects those parallel faces. That's the height of our cylinder. Regardless of if our cylinder is oriented like this or like this, we know that this is still what we would focus on, this segment, to find the height of our cylinder.

Now as far as the volume and the surface area and the lateral area of our cylinder, let's talk about that for a few minutes. Now volume is capacity; it's the space available inside a three-dimensional figure. Now I've filled my cylinder up almost to full capacity. I still have a little bit of space left over in here, but just to give you an idea, the volume of the cylinder, that's where my sand is contained. Okay? Now the lateral area of my cylinder, that would be the area of this lateral base, this rectangle that we talked about a few minutes ago, and the surface area of my cylinder, that would consist of the area of this lateral face as well as the area of my two circular bases. Now that you're a little familiar with volume and surface area and lateral area and the parts of our cylinder, let's go ahead and work a few examples.

Okay. Now in order to work those examples, you have to know the formulas for volume, surface area, and lateral area. Let's take a few minutes, let's get familiar with those. Let's get this out of the way.

Okay, so the formula for the volume of a cylinder is the product of pi $r$-squared and the height. Really to break that down just so you can see where that really comes from, pi $r$-squared, if you remember from your prior knowledge in middle school, probably a little bit in elementary school, the area of a circle is represented by the formula pi $r$-squared. Scoot this down a little bit. When we're talking about the volume of our cylinder, we're really throwing a nod back to the area of the circle, that's where this part of the formula comes from, and then the $h$ represents the height of our cylinder. The volume of a cylinder is the product of the area of the circle, that circular base, and the height of the cylinder. You can always use that formula to calculate the volume.

Okay, now let's get into those area formulas, wave to the cylinder. Here we have a cylinder and we've noted the radius and the height. What I want you to do is kind of envision that you've cut along that lateral face. You cut right down that cylinder and you started to unravel it, okay? You want to unravel that lateral face and you've started to cut around those circles. That cylinder is starting to unravel. Keep that in mind. Now think here that you continue to unravel that cylinder until it is lying completely flat, okay? You see that rectangle, that lateral face lying completely flat, and those circular faces, those are also lying completely flat. All right? Keep that in mind.

Now let's talk about that lateral area. Think about this for a second: we have our rectangle here, and I'll actually draw it away from the cylinder for a second. Now this side of our rectangle, we know that that represents the height of our cylinder, that width of our rectangle is the height of our cylinder. Now the length of our rectangle, let's think about that for a second. We know that we unraveled this cylinder, so this rectangle, if you can kind of envision this, envision that you split the circle. You cut that circle and you've now unraveled that so it's lying flat along that side of the rectangle. The length of this side of our rectangle will be the same as the distance around our circle, because remember that rectangle was wrapped around that circle.

If we unravel it, that distance, that length of that rectangle, is the same as that distance around our circle. If we throw back to what we've learned probably in middle school and a little bit in elementary school, we know that the distance around the circle, that's its circumference. We can represent the circumference of a circle by this formula: circumference equals 2 pi *r*. What we could say is the length of our rectangle is equal to the circumference of this circle. I'm going to represent this length as 2 pi *r*. Okay? Now we're pulling a lot of things together here, but stay with me.

Okay, so now let's focus on this rectangle for a bit. We know probably from elementary and middle school that the area of a rectangle can be represented by the formula length times width. I think I'll actually write that all in cursive just so we don't get lost a little bit, just so we don't get that mixed up with the number one. Let's get that back there.

Okay, so the length of our rectangle, like we said before, that's this edge here and it's represented by the circumference of that circle. That's 2 pi *r*. The width of our rectangle, that's represented by the height of our cylinder. That's *h*. The area of this rectangle can be represented by the formula 2 pi *rh*. That is actually the formula for the lateral area of our cylinder. Let's scroll back up to the top and reveal that up here. The lateral area of our cylinder, delete that, give the computer a second to catch up to me. There you go. 2 pi *rh*, this is the formula that we can use to find the lateral area of a cylinder. Okay, let's keep moving.

Talk about surface area for a second. Like we said, we know from our work just in that prior example, that the area, the lateral area of our cylinder, that's represented by the formula. Switch back to my pen here. Lateral area is 2 pi *rh*. We discussed before that the surface area of a cylinder consisted of that lateral area, that area around the cylinder, as well as the area of the faces, the top and the bottom circle. We know that the area of a circle is pi *r* squared. Right? The area of our circles, together, let's get that written down here, we'll call that the area of the bases. I have pi *r* squared plus pi *r* squared, so I'll save that two pi *r* squared. Basically now what I can say is since I know that the surface area of my cylinder is a measure of that lateral area plus the area of my bases, I can represent the surface area of this cylinder by this formula. Let's get that out of the way. 2 pi *r* squared, which we said that's the area of our bases, plus 2 pi *rh*, which is the area of our rectangle. Okay?

Now that you know where that formula comes from, you don't have to do this work every time to derive it, to figure it out. We could just use this formula when we have to solve problems involving the surface area of a cylinder. All right? Okay, so now that you're a little

warmed up with the formulas, let's go ahead and get into a few examples.

Okay, let's take a look at this one. A cosmetics company plans to print its logo along the curved surface of a cylindrical tube of lip balm. If the width of the tube is half an inch, and its height is three inches, approximately how much area is available for the logo? Okay, there was a lot going on in that word problem, so what we want to do first is let's just highlight that key information so we can really see what we need to focus on to solve this problem. Okay, so I'm going to go ahead and get my highlighter out.

Let's pull up that key information. We have a cosmetics company that plans to print a logo along the curved surface, that's important to know, of a cylindrical tube. That's telling us the shape we have.

Now if the width of the tube is half an inch, that's important, and its height is three inches, approximately how much area is available for the logo? Okay. Let's focus on this and figure out whether we need to calculate the surface area, the lateral area, or the volume of this cylinder. Because they didn't give us a picture or a graphic, it may help to just get yourself a quick sketch. Just roughly sketch what that cylinder would look like, because when you're given these kind of problems, you're not always going to be given the picture. Just roughly sketch a cylinder, doesn't have to be perfect, just get an idea, and let's drop that information in there. The width of the tube is half an inch. That's telling us how wide the cylinder is. Now, putting that into what we know about geometry, if in this problem they're saying the width of the cylinder is half an inch, then they're really telling us that the diameter of this circle is half an inch.

Let's just go ahead and let's switch colors here. Let's just roughly sketch. Let's get that a little neater there. All right, let's get that diameter up there. All right. The diameter, half an inch. We know that that's from end to end. It tells us the height of the cylinder is three inches. Okay. Let's go ahead and get our units along with it. Okay. Then they're asking us to figure out how much area is available for the logo. Now recall up at the top that they told us the logo was only going to be printed along the curved surface of the tube. If we're only covering the curved surface of the tube, is that surface area, lateral area, or volume? Think back to what those mean. All right. That's going to be lateral area, because we're not going to cover the bases, we're not talking about filling the tube, we're just talking about the area around that tube. We need to calculate the lateral area of this cylinder, all right?

Okay, so let's get that going. Going to reveal our formula here now that we decided that we need the lateral area of the cylinder. I'm going to scroll down a little bit just to get a little work space. All right, probably have to scroll back and forth for a second, so bear with me. For the lateral area here, it helps when you're doing these type of problems if you kind of get a list going of what the radius is, what the height is, and then you can just easily substitute into your formula. Here I know that my diameter is half an inch. In order to figure out my radius, I need to divide that diameter by two. Because it's a decimal, I'll go ahead and go to the calculator. I won't do mental math. Let's pull our calculator up. We need to figure out half, one half divided by two. All right, let's get that calculator going.

Here we go. One half divided by two. That is .25, or one quarter. Let's go back to our work. Let's go full screen here. Now we know our radius is .25. I'll just start making a list, my radius, 0.25. I know the height of my cylinder, that's three. Now that I have the information that I need, I'm going to go ahead and fill into the formula. Now because in the question it asked me to approximate the area, I'm going to go ahead and approximate pi at 3.14. I'm going to calculate the lateral area. It equals two times 3.14 times our radius, which I'm going to run into our cylinder a little bit, so let's scoot this down, get a little more space. Like we said before, when math starts to get crowded, it can start to get a little confusing. Let's bring that down. All right, get our pen back. We have our lateral area, equals two times 3.14 times our radius at .25 times the height of three. Now we need to put this into our calculator, so let's switch back.

Get our calculator up. We need two times 3.14, get that part in there first, times our radius, which is 0.25, and our height of three. Okay, so 4.71. Let's go back to our work. Go full screen here. The lateral area of this cylinder is approximately 4.71, and because we're talking about area, you know we're going to have, I believe our units were inches, so inches squared. Area's a square unit. Okay? Then you're all done with this one.

Now just to recap, just to pull it all together, we started out by highlighting the key information from our problem. We've roughly sketched the cylinder and we've labeled the dimensions. We figured out that we needed lateral area because we were talking about the area around that cylindrical tube, and we did our necessary calculations, and we've figured out that the lateral area was approximately 4.71 square inches. All right? Good job with that one.

Now this one is your turn. Press pause, take a few minutes, work through this one. Remember our strategy of highlighting the key information, getting a rough sketch of the figure, labeling those dimensions and figuring out what you need to go ahead and calculate. Get that work done, and then go ahead and press play when you're ready to check your work.

All right, let's see how you did here. Go ahead and switch to our highlighter. It looks like on this one you were still working with that tube of lip balm, so we know the width of the tube is half an inch, the height's three inches, and this time you're asked to approximate the amount of plastic needed to enclose the entire tube in plastic. Okay, so let's think about that. Let's get that sketch down then figure out what we need to calculate. We have our cylinder, just roughly sketching that. We know that the width of it is half an inch. That's the diameter. We know that the height, that's three inches. Get our units in there also. Now if this time we're talking about covering the entire cylinder in plastic, then this time it's not lateral area, because it's not just the area of the lateral face. It's the area of the lateral face as well as the area of the bases. We want to cover an entire solid, take out every piece of that area, that's going to be surface area. Let's reveal that over here.

That's the formula that we're going to need in order to solve this problem. Let's go ahead and let's figure out what's the radius, what's the height, which we know from the previous example because it's the same cylinder. Let's go ahead and just get that written down so we could just easily fill into our formula. We know that the radius, and I think this time I'll write it over here just to keep my work space free on the left. We know that our radius is 0.25, and

we know that our height's three. Let's go ahead, let's fill into our formula for surface area. Two, we're approximating pi at 3.14, the radius 0.25 squared, and we are still running out of space. Let's go ahead and shift a few things here. We can get rid of our sketch since we've gotten what we needed from it. Now we can run into that area. Okay.

We handled that first part. Plus two, pi is 3.14 approximately, our radius, 0.25, and then the height's three. Let's get this into the calculator and then we can approximate the area, the surface area of this cylinder. Okay. Going to have to flip back and forth for a little bit to get all this in there. First we need two times 3.14 times one quarter squared. Two times 3.14 times 0.25 squared, plus, and let's flip back, we need two times 3.14 times 0.25 times three. Two times 3.14 times 0.25 times three. We see we have 5.1025. Let's go back to our work, and let's go ahead and we'll actually approximate this to the nearest tenth, since you weren't asked exactly, you weren't told exactly what to approximate to, we can safely approximate to the nearest tenth here. We'll say this is approximately 5.1. Okay, so let's go back here.

We can say that the surface area of this cylinder is approximately 5.1 and we actually have inches, get that unit right, inches squared. We are all done with this one, okay? See how this one was different from the last one, we were calculating a different measure of our cylinder. We needed the surface area, but our process is still the same. Highlight that key information, get that sketch, get those dimensions, figure out what you need to solve for, which formula you need, and then go ahead and do your calculations. All right? Good job on that one.

Now take a look at this one. Okay, example two. A cylinder has a volume of 96 pi cubic centimeters, and a height of six centimeters. What is the length of the radius of the face of the cylinder? Okay. Now this one's a little different because we were actually given the volume of the cylinder. We were given it actually in terms of pi. Notice that our volume still has 96 pi, and we've dealt with measures in terms of pi before with our earlier work with circles, but just notice here it wasn't an approximate volume. It's exactly 96 pi. We were given that, we were given the height, and we were asked to find the length. Now in this case, really what you need to do, going to reveal our formula here. This problem really becomes kind of like an algebra problem, because we have some information for this formula, and we're just solving for a little missing piece. I'm going to go ahead and I'm going to start substituting what I know.

Going to highlight it here. I know the volume and I know the height, and I'm asked to find the length of the radius of the base.

I'm going to go ahead; I'm going to copy this formula right here just so I can have it above what I'm filling into. Now I know the volume's 96 pi. 96 pi equals pi, the radius is actually what I'm solving for, so right now that's still unknown, so $r$ squared. I do know that the height is six. Okay. I'm going to clean this up a little bit, I'm going to clean up the right side, really. 96 pi, and I still have pi times $r$ squared times six.

Really all I'm going to do is move that six to the front. Six pi $r$ squared, just so it looks a little familiar to me. Now what I need to do is I'm solving for the radius, so I need to make sure that the radius is the only term on the right side. Since six is being multiplied times pi and times $r$ squared, I'm going to divide both sides by six pi. That's going to help me eliminate everything

on the right side, except for that *r* squared. That six pi, that's canceled out. The only thing left on the right is *r* squared. Make a little more space here.

Now on the left side, the pi's cancel out. Let's go ahead and just divide 96 by six. Go to the calculator for that one. 96 divided by six, that is 16. Back to the work. 16 equals *r* squared. Because I actually need to know what *r* is, what I need to do here is take the square root of each side. The square root of 16, that's four. The square root of *r* squared is *r*. Now I know that *r* equals four. I'm just scrolling back up to the top so I can get my unit of measure here, we were talking in terms of centimeters. The length of the radius of this cylinder is four centimeters. You're all done with this one, okay? See how we attacked this one a little differently, we needed to substitute the values that we had, if I scroll up to the top here. We used our formula, we substituted in the values that we had, and we used our algebra skills to solve for the unknown. Okay? All right, good job with that one.

Now go ahead and try this one. Press pause, take a few minutes, work your way through this one. Press play when you're ready to check your work. All right, let's see how you did here. Go ahead and switch to my highlighter. We have a cylinder and its lateral area is 80 pi square feet. It has a radius of eight feet, and we're asked to find the height of the cylinder. Going to go ahead and reveal that formula for lateral area. I'll switch to my pen, going to copy it over here so I can just look at it as I'm substituting, see it right above. I know the lateral area is 80 pi square feet. On the right side I'll have two, I'll have pi, I know my radius is eight. My height is still unknown.

Let's go ahead, let's simplify that right side. On the left I'll still have 80 pi. Here I can multiply my whole numbers here. I'll go ahead and multiply two and eight, so that's 16 pi. I still have *h* unknown. Now go ahead and divide both sides by 16 pi. On the right, 16 pi, that cancels out. I'm just left with *h* on the right. On the left, the pi's will cancel out. All we need to do is divide 80 by 16. That one I know from mental math that that's going to be five. In this case, *h* equals five. If we scroll back up to the top just to get our unit of measure, the height of this cylinder is five feet. We can go ahead and write our final answer, five feet. All right? Good job on that one.

All right, guys. We've reached the conclusion of this topic on calculating the surface area and the volume of a cylinder. I hope you saw how your background knowledge of area and volume and circles and rectangles all came in handy for you during this topic. Bye!