Module 12: Perimeter, Area, and Volume
Topic 1 Content: Calculating the Surface Area and Volume of a Prism Transcript

Hi, guys. Welcome to Geometry. In this topic we're going to focus on calculating the surface area and the volume of a prism. Now, your prior knowledge of area and volume and solid figures is going to come in handy for you during this topic. You ready to get started? Let's go.

Okay. Now, before we dive into solving some actual problems what I want to do first is just get you familiar with the prisms that we're going to focus on. Okay, the first prism that we're going to focus on is the rectangular prism. Now, in the rectangular prism the bases are rectangles and its lateral faces are parallelograms. That's the same for all prisms. Their faces will be some sort of polygon. Their bases will be that, and their lateral faces will be parallelograms. In a rectangular prism the bases are rectangles and its lateral faces are parallelograms. Okay, so we're going to focus on this type of prism.

Another type we'll focus on, the triangular prism. Now, for the triangular prism, like it sounds, its bases are triangles and its lateral faces are parallelograms. Okay, so this is another one we're going to discuss. We will also discuss a hexagonal prism. In this case, we're only going to focus on hexagonal prisms where the bases are regular hexagons so each of those side lengths are going to be equal along the hexagon as well as its interior angles. Then, again, because it's a prism its lateral faces are parallelograms. All right?

Now that you're a little familiar with those, let's go ahead and talk about the formulas involved here. Okay. For the triangular prism the formula for volume, it's the area of the base times the height. That's actually what that capital B stands for. Go ahead and get that written down. Any time you see a capital B, that's referencing the area of the base. Get our pen here. That's the area of the base. Because in this case our bases are triangles, it's the area of the triangle. I'm going to abbreviate that. The area of the triangle, so that's what that capital B stands for. Then the h, that's the height of the prism.

Then the lateral area of a triangular prism, that formula is the height of the prism times the perimeter of the base. Any time you see that lowercase p, I'll write that down here, that's the perimeter of the base which in this case it's the perimeter of a triangle. I'm going to abbreviate that. Perimeter of the triangle. Okay, so that's the formula that we'll use for the lateral area of the triangular prism. Then for surface area, let's get that out of our way here. The formula for surface area, that is the height of a prism times the perimeter of the base or, again, that lateral area plus two times our base area or, again, the area of our bases. Okay? All right, so those are the formulas that we'll use when we're dealing with a triangular prism.

For the rectangular prism, you'll notice that the formula for volume on this one, let's reveal that, that's the length times the width times the height. That's what we'll use to figure out the volume of a rectangular prism. The lateral area, that's going to be the height of our prism times the perimeter of the base, so in this case, the perimeter of that rectangle. Then the formula for surface area, it's a long one but it's got just really basic pieces. It's two times the product of the length and the width plus two times the product of the length and the height plus two times the product of the width and the height. All right?

Let's keep going and talk about the hexagonal prism. Let's reveal the formulas for this one.

Again, the formula for the volume of the hexagonal prism, that capital B is the area of the base so it's the area of this hexagon times the height of the prism. Let's go ahead and just reference that here. This would be the height. Now, to calculate the area of a hexagon, that involves a formula that we haven't seen before. Let's go ahead and reveal that down here. I'm going to give that to you.

The formula for the area of a hexagon, it's 3 times the length of one of its sides squared times the square root of 3. It's all of that divided by 2. Remember, in our case we're only going to be dealing with hexagonal prisms whose bases are regular hexagons. Okay? All right, so that's the formula for volume. For lateral area, it's the height times the perimeter of the base. Remember that p stands for perimeter of the base. Then for surface area it's going to be the lateral area plus the area of the base. Let's scoot that right on over there. The surface area's the lateral area, which is our formula right here, plus the area of the base, which is our formula down here. All right?

Now that you've kind of familiarized yourself a little bit with those formulas, let's start solving a couple of problems. Okay, let's get our pen, or get a highlighter first, and let's highlight as we go here. A rectangular prism, that's important to know, has a length of 4 inches and a width 5 inches longer than its length. The height of the prism is 6 inches. Find the volume of the prism. Okay, so I'm going to need to know to volume formula for this one. Let's get that out of our way here. Volume is a product of the length, the width and the height.

Let's figure out what those pieces are, and then we'll go ahead and find the volume. The length, it tells us the length is 4 inches. The width is 5 inches longer than the length. Okay, so if the length is 4 and it's telling us the width is 5 inches longer than 4, then 4 plus 5, the width is 9. Then it tells us that the height is 6 inches, so h is 6. Now we have all the pieces that we need to figure out the volume of this prism. Let's go ahead and let's use...

We can use our workspace right over here to the right. I'll just switch colors here. Volume, that's length times width times height. Our length is 4, our width is 9, and our height is 6. Let's go ahead, let's use our calculator and find the product of 4, 9, and 6, so 4 times 9 times 6. That is 216. Let's go back to our work, let's go full screen here. The volume of this rectangular prism is 216, and let's get our units. We were measuring in inches for our units, so for volume that'd be cubic inches. Okay? You're all done with this one. You see how we highlighted our information, we figured out those necessary pieces, substituted into our formula, and we ended up that our volume was 216 cubic inches. All right?

Okay, let's move on to the next one. Let's get this out of our way here. A decorative mirror is in the shape of a triangular prism. Let's go ahead and highlight that. We're asked to find the surface area of the prism. Okay, so we know what we're being asked to find here, we've highlighted our information. Let's take a look at our formula for surface area and then figure out what pieces we need to plug in. Get that out of our way. Okay, so for surface area we need to know first the height of the prism.

Let's go ahead, let's get that. Remember, the height, that's that perpendicular segment that connects the bases. You can think about it that way. The height of this prism is 6, so h is 6. The

perimeter of the base. Remember, the base is a triangle so we want the perimeter of the triangle in this prism. I see here at the bottom, I can see that two sides of the triangle are 3 and 4, so 3 plus 4. Then, up at the top I'm given that third side of the triangle. That's 3 plus 4 plus 5, that's 12. I know the perimeter of the base here is 12.

Now I need to find out the area of the base. I'm going to need a little space for scratch work here. Recall this from middle school, maybe a little bit in elementary school, but definitely in middle school, that the area of a triangle is represented by the formula 1/2 times the base of the triangle times the height of the triangle. In this case, I'm going to pull this triangle out of our prism just so we can focus only on that triangle for a second.

Let me get that just roughly drawn right over here. We're told it's a right triangle. I see up at the top there the right angle symbol. We know that its sides are 3, 4, and then 5 right up there at the top. Let's get those here. In the case of this triangle, lowercase b, the base of the triangle, is 4. The height of the triangle is 3. Here, the area of this triangle is 1/2 times 4 times 3. Just doing a little mental math here, that's 1/2 of 12 because 4 times 3 is 12, so half of 12, that's 6. The area of this triangle is 6 inches squared.

Taking that information back to my problem, going to scroll back up to the top that means capital B, the area of the base, is 6. Okay? See how we had to kind of step off to the side and do a little scratch work to figure out the area of that triangle, and then once we had it from this formula we knew that our B is 6 in relation to our problem that we're really trying to solve here. All right? I'm going to go ahead, I'm going to get this scratch work out of our way. If you're working on notebook paper of course keep it down. I just want to get a little more room for us to do some work up here.

All right, so we have all the pieces that we need to figure out the surface area of this prism so we can go ahead and perform the calculations. Surface area, that equals we know it's the height times the perimeter of the base. 6 times 12 plus 2 times the area of the base, so 2 times 12. I'm sorry, 2 times 6. The height times the perimeter, 6 times 12, plus 2 times the area of the base, so 2 times 6. Do a little mental math here. 6 times 12, that's 72. 2 times 6, that's 12. 72 plus 12, that's 84. The surface area of this triangular prism is 84 inches squared.

You are all done with this one, okay? Just to recap what we did here, we highlighted the key information, we figured out what we needed to do, we figured out what our dimensions were that we needed to plug into our surface area formula. Remember, we had to come off to the side to figure out the area of that triangle, figure out that that capital B was 6. Once we had all of that, we substituted the appropriate values into the formula and we ended up at the surface area at 84 inches squared. All right?

Okay, let's keep going here. I want you to go ahead and try this one. Press pause, take a few minutes, work through this one. Press play when you're ready to check your work. All right, let's see how you did here. Let me get my highlighter first, start getting my key information here. Okay, so we have a hexagonal prism. The base is a regular hexagon. Each side of the base is 10 centimeters. The height of the prism is 8 centimeters, and we're asked to find the volume to the nearest tenth. Okay, so we have our pieces of information that we need.

Let's take a look at our formulas here and see what we need to do next. Let's reveal those. Let's get that volume formula out of the way, that volume covering out of the way. Volume equals the area of the base times the height. Because our base is a hexagon, we also need that formula. Get that out of our way here. Our keyboard's kind of doing something different here. All right, let's get that out of our way. The area of our base is represented by this formula. Before we can actually figure out the volume, we really need to figure out the area of that hexagonal base, okay?

Let's start actually by using this formula to figure out B. Get a little work space. Okay, going to extend my page a little bit. All right, here we go. The first thing I'm going to focus on is figuring out B. That's going to be my step one. I need to know S, which if you recall from our work earlier, S represents one side length of that hexagon. If I scroll back up to the top, we were told that each side of the hexagon was 10 centimeters, so I know that S is 10. That's really the only piece of information that I need in order to use this formula, so I'll go ahead and start substituting the other values in.

B equals 10 squared. I'm sorry, 3 times 10 squared. Let's not forget that. That's definitely an important piece. 3 times 10 squared times the square root of 3, and all of that is divided by 2. Okay, so let's get this into the calculator, see what this is. We have 3 times 10 squared times the square root of 3. 3 times 10 squared times the square root of 3. Okay, so we have all of that. Remember, that was all divided by 2. Okay, so let's see here. Let's see what we're actually asked to approximate to. We're asked to round to the nearest tenth.

If we go back to our calculator, if we round this to the nearest tenth we're at about 259.8. We'll use that value for the area of our base, 259.8. Let's go back here. B equals, extend our page. B is approximately 259.8. Now that we have that value, we can go ahead and figure out the volume of our hexagonal prism. Let's scroll up to the top. Volume is capital B, the area of the base, times the height of the prism. We're told that the height of the prism is 8 centimeters, so let's scroll down and do part two, which is actually to go ahead and use that volume formula. Volume equals the area of the base times the height and the area of our base, 259.8, and the height we saw was 8.

Now let's go ahead and get that in the calculator and we'll have the volume of our prism. We need 259.8 times 8, so 259.8 times 8, and that is 2,078.4. 2,078.4. That's all approximated. In our units here, if I scroll back up to the top, centimeters, so the volume centimeters squared. I'm sorry, centimeters cubed. We are all done with this one, okay? See, this one took a couple of steps also. We had to first figure out the volume or the area of our hexagon before we could figure out the volume of the hexagonal prism.

Once we used our formula for the area of a hexagon and we had that value, we went ahead and plugged it into our volume formula and figured out that the volume of the prism was approximately 2,078.4 centimeters cubed. All right? Okay, good job on that one.

Take a look at this one. A [rectangular 00:19:06] prism has a length of 7 centimeters and a width of 5 centimeters and a surface area of 142 centimeters squared. Let's highlight all of

that. That's all things we're going to need to know. We're asked to find the height of the prism. Okay. In this problem we're given the surface area and we're given some of the dimensions of this rectangular prism and we're asked to actually solve for its height, okay?

Let's get this out of our way so we can get the formula for surface area. Let's fill in what we know and we'll solve for what we don't. The surface area is 142. That equals 2 times the length is 7 and the width is 5, plus 2 times the length times the height, so 2, the length is 7, the height's unknown, plus ... I'm going to run out of space here. I see it coming, so let's go ahead and scoot this over just a bit. There we go. I don't want you to think that that is a negative 7, so let's just go ahead and get that. Let's fix that little piece right there. That was just a 7.

All right, and we have 2 times the width times the height. 2 times 5, and then again, our height's what we're solving for here. That's h. Let's go ahead, let's clean this up and let's see what we end up with here. On the left side we have 142 equals, we have 2 times 7 times 5, do a little mental math here. 7 times 5, that's 35, so 2 times 35, that's 70. We have 2 times 7 times h. 2 times 7 is 14, so $14h$ plus 2 times 5 times h. 2 times 5 is 10, so 10h. Let's simplify some more here. We can combine like terms, so 142 equals 70 plus $24h$. We'll go ahead, we'll subtract 70 from each side, so minus 70, minus 70. 142 minus 70, 72 equals 24h, and we'll go to the calculator for this part.

Our last step is to divide both sides by 24. 72 divided by 24, divided by 24, gives us 3. Back to our work here. That tells us that in this case the height is 3. Our units here, scroll back up to the top, see what we were working in. Centimeters, so that means that the height of our prism is 3 centimeters. You're all done with this one, okay? See how in this case we were given different pieces of information, different measures of this rectangular prism. We used that surface area formula, we figured out what we needed and what we had, we did our substitution, and then we used our algebra skills to solve for the height and found that the height was 3 centimeters. All right? Okay, good job on that one.

Go ahead and give this one a shot. Press pause, take a few minutes, work through this one. Press play when you're ready to check your work. All right, let's see how you did on this one. Here we have a triangular prism has a height of 6 inches and a volume of 720 cubic inches. We're asked to find the area of the base of the prism. Okay. If you recall, the formula for finding the volume of a prism, volume equals the area of the base times the height. Here we're given some pieces of information. We're going to substitute what we know into this formula and then we'll be able to figure out what's unknown here. We'll be able to solve for that. Okay?

We know that the volume is 720 cubic inches, so I'm going to substitute 720 in here. The area of the base is unknown right now, so I'll leave that as B. The height of the prism is 6 inches, so I'll substitute 6 in there. Then I'm just going to rearrange the right side, just so it looks a little more familiar. Get a little more work space here too. I'm going to write this as 720 equals 6B. Now I'm going to solve for B by dividing each side by 6 and I'm left with 120 equals B. What that tells me here, if I scroll it back at the top, find out the units. The area of the base of this prism is 120 square inches, and you're all done with this one. Okay? See how we highlighted our key information, we used our formula for finding the volume of a prism, we substituted the values that we knew, we solved for what was unknown, and we found out that B equals

120, which tells us that the area of the base is 120 square inches. Okay? Good job pulling all those pieces together.

All right, guys. You've reached the conclusion of this topic, on finding the surface area and the volume of a prism. I hope you saw how your prior knowledge of area and volume helped you get through this topic. Bye.