Module 12: Perimeter, Area, and Volume

Topic 2 Content: Calculating the Surface Area and Volume of a Cone Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on how to calculate the surface area and the volume of a cone. Now, your prior knowledge of area and volume is going to come in handy for you during this topic. You ready to get started? Let's go. All right, now before we dive into working some examples related to the cone, I first want to take a few minutes and just familiarize you a little bit with the cone itself, okay? All right. Now, here we have a cone, and it's filled almost completely to capacity with sand. A cone consists of a circular base, okay? So its base is a circle, and we can see here it also has a lateral face, this face that wraps around the cone. All right? So the lateral area would be the area of this lateral face. The surface area would be the lateral area plus the area of that circular base, and then its volume would be, like I said, its capacity, how much space is available inside this cone. And this one isn't filled to capacity with sand, but almost. All right?

So now that you're a little familiar with the cone itself, let's go ahead and work a few examples related to the cone. Okay. Now, first, you do need to know the formulas for volume, lateral area, and surface area, so let me get these out of your way and show those to you. The volume of a cone is one-third times pi times its radius squared times its height. That's the formula that we'll use to calculate the volume of different cones. The lateral area, that's a product of pi, the radius, and the slant height of the cone. You remember, slant height is the length along the lateral face. Okay? And then surface area, that is pi r squared plus pi times the radius times the slant height, and that tells us the area completely around the cone as well as the area of its base. It's a sum of those areas. Okay? All right, so now that you've got the formulas, let's go ahead and see a few examples. Okay, so let's take a look at this first example.

Let me make sure I've got my highlighter, and let's go ahead and pull out the key information that we need for this problem. So we have a cone. It has a base, the radius of 6 centimeters and a slant height of 10 centimeters. "What is the capacity of the cone? Give your answer in terms of pi." Okay, so we know we have a cone. We know the length of its radius. We know its slant height. We're asked to find the capacity in terms of pi. Now, any time you're asked to find the capacity of a solid, you're being asked to calculate its volume, so in this case, to find the capacity of this cone, we need to calculate the volume of this cone, and I've got that formula right behind this, so let me reveal this here. So that's the formula that we need to calculate the volume of a cone, so I'm going to go ahead and make a list of the pieces of information that we have so we can figure out what we may need in order to use this formula.

We know that the radius is 6. We know the slant height is 10, so if you look at the formula, we do have the radius, and we do need that, but we're missing the height. The height is still needed in order to find the volume, so I'm going to show you how you can use the radius and the slant height in order to figure out the height of this cone. In order to do that, I'm first going to just roughly sketch a cone. Okay? We know the radius of the cone is 6, and we're told that its slant height is 10. We need to know the height of the cone. Okay. So in this case, this is what's called a right cone. Part of the reason for that is because the height of our cone here, it's perpendicular to the base, so what happens in that case is, you have a right triangle kind of trapped inside the cone, you could say. One side of that triangle is 6. It's the radius.

The other side of that triangle's the height, which is unknown right now, and the slant height represents the hypotenuse of that right triangle. So what we can do is solve that right triangle, figure out what h is. That'll tell us the height of the cone, and then we can go ahead and find the volume of the cone. Okay? So let's go ahead, let's solve that triangle, figure out that height, so we can go ahead and get the volume of this cone. I'm just going to pull that triangle out of the cone so we can focus on it. We're going to need the Pythagorean theorem to solve this. I'm just going to label that triangle for us. Just remember, Pythagorean theorem, the hypotenuse is c, a and b are your legs, so c, that's 10 squared, equals 6 squared plus h squared. So 10 squared is 100, equals 36 plus h squared. Subtract 36 from each side, so that is 64, that's canceled, equals h squared.

Then last step, take the square root of both sides, so 8 equals h. So the height of our cone, now we know it, it's 8. So now that we have that, let's scroll back up to the top. We'll say h is 8. Now we have all the pieces that we need in order to calculate the volume of this cone or, like we were asked, to find the capacity of this cone, and give that answer in terms of pi. Okay? So what I'm going to do is, I'm going to scroll down here. I'm going to get rid of some of this work, just to get us a little more space here. Don't want things to start to get too crowded on us here. Okay, so we can [reuse 00:06:39] that. We know h is 8 from doing that work. Okay, let's go ahead and get the volume. So the volume equals one-third times pi times our radius, which is 6 squared, times our height, which is 8. So because we were asked to give our answer in terms of pi, when I find this product, I'm not going to approximate pi's value at 3.14.

I'm just going to find the product of one-third times 6 squared times 8, okay? And I'm just going to get that, so I can give my answer in terms of pi, so I'm going to switch to my calculator and find the product of one-third times 6 squared times 8. So one-third times 6 squared times 8, and that is 96, so let's go back to the work. This means that the volume of this cone in terms of pi is 96 pi, and I'm going to scroll back up to get our units, centimeters, so centimeters cubed. So the volume of the cone, 96 pi cubic centimeters. Okay? And just to recap, the work we did to get here. Recall, we had the radius. We had the slant height, but we didn't have the height, and that's where we did that work, that kind of scratch- work, and pull that right triangle out of the cone, and use it to figure out that the height was 8. Once we knew that, we did all the work necessary to figure out that in terms of pi, the capacity of this cone is 96 pi cubic centimeters. Okay?

All right, good job on that one. Go ahead and give this one a try. Let me get that out of your way so you can see everything here. Imight actually need to ... Let me bring this up for you a little bit so you can see your multiple-choice answers along with the question. This one's actually multiple-choice. Okay, so go ahead and press pause, take a few minutes, read through this, work your way through it, and go ahead and press play when you're ready to check your work. All right, let's see how you did on this one. Let me switch to my highlighter. All right, so we have the owner of an ice cream shop, and he plans, or she plans, to design a label to wrap around the ice cream cone. "The cone has a height of 5 inches and a base with a diameter of 3 inches. Which of the following is closest to the amount of paper needed for one ice cream cone?"

Okay, so we have an ice cream cone, and we're trying to design a wrapper, so something to wrap around the cone, so it's just covering its lateral face, and we want to know how much paper is needed for that wrapper, so to figure that out, we're going to need to calculate the lateral area of the cone, so let's go ahead and reveal this so we can see the formula that we're going to need. Okay, so for the lateral area of a cone, that's pi times the radius times the slant height. Okay? So I'm going to go ahead and just roughly sketch this cone so I can see what information I have and what I need, and then I'll know what I need to do to figure out the lateral area here, so I'm going to have to scroll back and forth for a second, so bear with me. Okay, so here is our cone. Let's get the radius in there and the height.

Okay, so we were told that the height is 5, and that the diameter is 3 inches, so if the diameter is 3 inches, then the radius is half of that, so it's 1.5. And the slant height, I don't know, but I need to know it in order to figure out the lateral area, so I'm going to go ahead and pull out this right triangle here. I'm going to go over it in black, just to make it stand out. I'm going to have to solve this right triangle and figure out the slant height. Then I can go ahead and figure out the lateral area. Okay, so I'm going to come down here and work out the Pythagorean theorem, go ahead and pull the triangle out of there. 1.5, 5, and then l. And I'm going to label the sides, so here's my hypotenuse, and then a and b, so Pythagorean theorem. C squared equals a squared plus b squared, so that would be l squared equals 1.5 squared plus 5 squared. A little more space here. I'm going to go to the calculator for this. 1.5 squared plus 5 squared.

I'll go ahead and get that in there all at once. So 1.5 squared plus 5 squared, so that is 27.25. Go back to our work. So l squared equals 27.25, and let's take that square root so we could figure out what l is approximately, so the square root of 27.25. So the square root, 27.25, that is, if we round to the nearest tenth here, we'll say that it's approximately 5.2, so let's go back to our work, and that tells us that the slant height is approximately 5.2. Okay? So now that we have that value, I'm going to scroll back up here, we have all the pieces that we need to figure out the lateral area, because we know that l is 5.2, and we know that the radius is 1.5, so I'm going to get us a little work space here. Let's get this cone out of the way, and then we can go ahead and figure out what the approximate lateral area is for this cone, which will tell us approximately how much paper is needed for the wrapper. Okay?

So the lateral area equals, we're going to go ahead and approximate pi's value at 3.14, since we were told this is an approximation, times the radius, 1.5, times the slant height, 5.2. So in the calculator, we need 3.14 times 1.5 times 5.2, so 3.14 times 1.5 times 5.2. So round to the nearest tenth, that would be approximately 24.5. Okay? So back to our work, let's get off of this calculator for a minute, so the lateral area is approximately 24.5 inches squared, and if we look at our answer choices, that's D. Okay? All right, good job with that one. I know you saw again that, in this case, you had to pull that right triangle out of the cone to get a piece of missing information, and then use it to figure out the lateral area. All right? Now take a look at this next one.

Okay, so here, a cone has a surface area of 216 pi square centimeters, or square inches, and a radius of 9 inches. "What is the slant height of the cone?" Okay, so here we're told the surface area is 216 pi inches squared. The radius is 9, and we're asked to find the slant height. So let's go ahead and reveal the surface area formula, so we can see what we need to substitute where to figure out what the slant height is in this case. Okay, so let's start plugging in our information. We're told the surface area is 216 pi, so 216 pi equals pi, the radius is 9, so 9 squared, plus pi, again the radius is 9, times the slant height. Okay, so let's simplify this a little bit, and then we'll be able to solve for l.

So 216 pi equals, 9 squared is 81, so 81 pi, plus pi times 9 times l. We really could just rearrange that, just to make it look familiar, 9 pi l. Okay? So let's go ahead, let's keep working this algebra to solve for l. Subtract 81 pi from each side, so let's go to the calculator for this. I'm just going to ignore the pi for a second and subtract 216 and 81, so 216 minus 81, that is 135, so back to the work. 135 pi equals, that's canceled out, so 9 pi l. Last step, divide each side by 9 pi. Scroll down a little bit here. So on the right, the 9 pis, those are canceled out, and you're just left with l.

On the left, the pis will cancel out, so we need to figure out 135 divided by 9. So to the calculator, 135 divided by 9, that is 15, so back to our work. I know we have to keep flipping back and forth here, so the slant height is 15, and let's go ahead and check our units, scrolling back up to the top. Inches, so 15 inches, so see in this case, this problem was a little different. We were actually given the surface area, and we had to figure out some of those smaller dimensions related to this cone. Okay? So go ahead and take a look at the next one and give that a try, and I'm going to go ahead and reveal this formula to you, because you're going to need it. Okay, so press pause, take a few minutes, work through this. Press play when you're ready to check your work. All right, let's see how you did here.

Let me switch to my highlighter, so our cone has a volume of 150 pi cubic centimeters and a height of 18 centimeters. We're asked to find the radius of this cone. Okay, so let's go ahead and plug in what we know. We know that the volume's 150 pi, so 150 pi equals one-third times pi, the radius is what we don't know, so we'll leave that as r squared, times our height, which is 18. So I'm going to simplify this right side a little bit here, so 150 pi equals, I'm going to go ahead and multiply one-third times 18, which, if I use mental math, I can think about that as 18 divided by 3, which is 6, so this is 6 pi r squared. I'll divide both sides by 6 pi, get a little more work space. On the right side, the 6 pis will cancel out, so you're just left with r squared.

On the left side, the pis will cancel out, so we need to figure out 150 divided by 6, and let's go to the calculator for that. So 150 divided by 6, that's 25, so let's go back to our work, get that 25 in there. So 25 equals r squared, so if I take the square root, the square root of 25 is 5, so that means my radius here, if I check my units, 5 centimeters. That's the radius of this cone. Okay? And you're all done with this one, so see how, again, on this one, we had that volume, and we had a little more given information, but we had to use it to figure out one of the smaller dimensions of this cone. All right, good job on that.

All right, guys, you've reached the conclusion of this topic on how to calculate the surface area and the volume of a cone. I hope you saw how your prior knowledge of area and volume came in handy for you during this topic. Bye.