Module 12: Perimeter, Area, and Volume

Topic 2 Content: Calculating the Surface Area and Volume of a Pyramid Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on calculating the surface area and the volume of a pyramid. Now, your prior knowledge of area and volume is going to come in handy for you during this topic. You ready to get started? Let's go. Okay, now before we dive into solving problems involving the pyramid, I want to just take a few minutes to get you familiar with a few different kind of pyramids we're going to focus on. One type of pyramid we're going to focus on is the triangular pyramid. Now, a pyramid is named by its base. On a triangular pyramid, the base is a triangle, and also the kind of pyramids that we're going to focus on in geometry will all have a regular base.

In the case of this pyramid, its base is an equilateral triangle. Also, in the case of a pyramid, its lateral faces are triangles, okay? On our triangular pyramid, the base is an equilateral triangle, and its lateral faces are triangles. Now another type of pyramid that we're going to focus on is the square base pyramid. On the square base pyramid, the base is a square, and its lateral faces are triangles, okay?

Now, the last pyramid that we're going to talk about is the hexagonal pyramid. Its base is regular hexagon, and its lateral faces are triangles.

Now that we've taken some time to become familiar with a few pyramids, let's learn about the formulas for volume, surface area, and lateral area of these pyramids. Now, these will be the pyramids that we're working with in this topic. The first one that we're going to look at is the triangular based pyramid. It's sometimes just called a triangular pyramid, just in case you see it like that a time or two. Now, if we look at the formula for volume here, let's get that out of our way. The formula for volume is that it equals 1/3, times the area of the base, times the height. Now, because it's a triangular based pyramid, we know that the base is a triangle. So to calculate B, we find the area of this triangle. And the height of the pyramid, that's a segment that has one end point at the vertex of the pyramid, and the vertex is that point where all of our lateral faces meet, and its other end point is at the base of the pyramid. The height runs perpendicular to the base.

That's the height of our pyramid, and that's part of our volume formula. Now, for lateral area, let's move a few things out of the way here. The formula for lateral area is that it's 1/2, times the slant height, times the perimeter of that triangular base. Now, slant height, what that is is that's the length along a lateral face. Whereas height was perpendicular, is represented by a segment that runs perpendicular from the vertex to the base, slant height is the length along a lateral face. So to calculate the lateral area, it's 1/2, times the slant height, times the perimeter of that triangular base.

Then, for surface area, we take a look at that formula, that's the sum of the lateral area, plus the area of the base. Now, these are the three formulas that we're going to be working with as we solve problems related to triangular base pyramids. Take a look at this one, the next one here. Now, for square based pyramid, let's look at that. The volume, it's 1/3, times the area of the base, times the height. In the case of a square based pyramid, the base is a square, so that capital B stands for the area of this square. Then, we can see that height of the pyramid, that perpendicular segment from the vertex of the pyramid down to the base.

Lateral area, that's 1/2, times the slant height, times the perimeter of the base. Here, you can see that slant height, it's that length along the lateral face. Then, the perimeter of the base would be the perimeter of the square. Then, surface area, that's a sum of the lateral area and the area of the base. Then, finally, take a look at this one, the hexagonal base pyramid. The volume, that's 1/3 ... I'm going to scroll a little bit for this one. 1/3, times the area of the base, times the height. Because our base is a hexagon, and we don't work too much with finding areas of hexagons, I wanted to make sure you had that formula here.

The area of the base, in this case the area of this hexagon, is represented by this formula, were S stands for the length of one of the sides. Recall that I said in this course, we're only going to focus on pyramids that have a regular polygon for the base. So this hexagon is regular. All of its sides are equal.

Now, for lateral area, that's 1/2, times the slant height, which we've said that's the length along one of those lateral faces, and then P is the perimeter of our base, so the perimeter of our hexagon. Then, surface area. That's a sum of the lateral area and the area of the base.

Now that you're a little familiar with those formulas, let's go ahead and work a few examples. All right, take a look at this one. I'm going to get my highlighter ready to start marking that important information. "A construction company plans to build a new sporting goods store in the shape of a square based pyramid. The slant height of the building is 100 feet, and the base has a length of 160 feet. How many cubic feet of air will fill the building? Round to the nearest hundredth." So we have a lot of different pieces of information here, so let's kind of pull it all together and see what we actually need to figure out here. We have a square based pyramid, the slant height is 100, the base has a length of 160, and we're asked to find how many cubic feet of air will fill the building.

Now, when you're talking in terms of solid figures, and you see anything that says cubic, or cubed, cubic inches, cubic meters, cubic centimeters, anything raised to the third power, that's a hint that you're being asked to calculate the volume, because volume is always measured in cubic units. Because by definition, what volume is, and I know you've heard this before in middle school, volume gives you a measure of how many cubes would fit into a solid. So any time you're asked to find the measure of cubic units, you're being asked to calculate the volume. So in this case, for this square based pyramid, we need to calculate its volume. Once we know the volume, we'll know how many cubic feet of air will fill the building.

This is something that someone working in terms of say heating and cooling this building would need to know this kind of information in order to make sure that the air in that building is heated properly and cooled properly when it needs to be. So let's go ahead and get the pieces of information that we need for this formula, and we can go ahead and find out the volume of this square based pyramid. Have to scroll back and forth a little bit, just to get everything down. Okay, so we were told here that the slant height of the building is 100. So we know that L, that's 100. The base has a length of 160 feet. So we will call that ... Let's get that out of the way. We're going to call that S for one of the sides of our square, 160. Then, we're being asked to find how many cubic feet of air will fill the building, so we need that volume.

Now, we have enough information here to figure out the area of the base, because we know what one side of our square actually looks like. Just so you kind of have an image to go off of here, I think what I'm going to do here by the magic of technology, I'm going to flip back to that square based pyramid that we touched on a few minutes ago, and I'm going to copy it, and let's go ahead and just paste it right here along with our work, so you can really see what exactly you're being asked to figure out here. You know one side of that square is 160. I'm actually going to go ahead and label that right on our figure here, 160. To find the area of a square, you know you can just square one of its side lengths, raise it to the second power. 160 squared, let's go the calculator for that, it's going to be pretty large number. 160 to the second power, that is 25,600.

Okay, so back here, let's kind of scoot this over a little bit. So 25,600, that's our B. Now, in order to figure out H, the height of this pyramid, you're going to have to do a little bit of work here for that. So right here, this length, this height, this segment representing our height, we said that it's perpendicular to the base. So what happens is when it drops down to hit the base, it intersects that base at a right angle. What we can do is we can form a right triangle, which is made up of the height of the pyramid. It's the length of half of the side of the square, and then the slant height here. So let's go ahead, let's get that right triangle drawn. I'll actually pull it out of the figure so you can really focus on it.

Because basically what happened here is that height is also referred to as an altitude, which in other terms, is a perpendicular bisector. So when this segment dropped down to hit the base, it intersected the base at a right angle, and it intersected dead center, like right in the middle of that base. I know that every side of this square is 160, so if that height is in the center of that square, then just half of the length of that square will give me the bottom side of this triangle here. So if this entire length is 160, then what that tells me is this part of this triangle is 18, it's half of that. So the bottom part of my triangle, that's 80. We were told, if I scroll back up to the top, that the slant height's 100. Let's get that down here. L's 100, and we're trying to solve for H.

So notice this triangle, that's what we're focusing on here in order to figure out the height of our pyramid. Since we have a right triangle, we know we can use the Pythagorean Theorem to do that,

Okay, so I'm just going to scoot it over to the left and do the work right beside it. So C squared equals A squared plus B squared. So C, if I go ahead and just label it, just to keep up with our practices we established, that's our hypotenuse, that's C, A, and B. So C squared, that'd be 100 squared equals H squared, plus 80 squared. Let's go to the calculator, because we have some pretty large numbers here.

So 100 squared, that is 10,000. So let's go back. So 10,000 equals H squared plus, let's go ahead and get 80 squared. Going to the calculator. So 80 squared, that's 6,400. Back to our work. So 6,400. Get a little more space here. Let's go ahead, subtract 6,400 from each side. So we need 10,000 minus 6,400, so let's go to our calculator. So 10,000 minus 6,400, that is 3,600. Back to the work. So we have 3,600 equals H squared. Go ahead and take the square root. And from pattern recognition here, I can tell the square root of 3,600, 60. So the height of our pyramid is 60. So now we know the height is 60, and if I scroll back up to the top, we know that the area of the base is 25,600.

So now we have all the piece of information that we need in order to figure out the volume of this pyramid. Okay, so I'm going to scroll up to the top just to get that formula one more time. So the volume of a pyramid is 1/3, times the area of the base, times the height. So I'm going to scroll down, a little more space. So the volume equals 1/3, times the area of the base, times the height. So 1/3, the area of the base, I'm going to use parentheses, 25,600. And we were told that our height, or we figured out that our height was 60. So definitely let's use the calculator for this. So 1/3 times 25,600, times 60. 1/3, times 25,600, times 60. We have 512,000, very large number.

Let's go ahead and get that down. So 512,000 cubic feet of air. That's how much air will fill this building. Now, if we recap, because I know this had a lot of pieces, just to pull it all together, just review what we did here. We first highlighted our given information, the important pieces that we needed, and we knew that the slant height was 100, and one side of our square base was 160. So first we figured out the area of the base, the area of that square. We got that information. Then, we found that hidden right triangle in here made up of our slant height, the height of the pyramid, and half the length of one of the sides of that square base. We pulled out that right triangle, and we used the Pythagorean Theorem to figure out the height. And once we had that, we had all the pieces that we need to calculate the volume.

So 512,000 cubic feet of air will fill that building. All right? Okay, good job on that one. Now, it is your turn. I'm going to get that out of your way. So go ahead and press pause, take a few minutes, work through this. Press play when you're ready to check your work. And before you actually do that, let me go ahead and reveal this formula to you, so you don't have to look back for it. There's the formula for the lateral area. All right, so press pause when you're ready to check that work.

All right, let's see how you did here. So you needed to figure out the lateral area of this pyramid. And that formula's given by the lateral area equals the slant height, times the perimeter of the base. So in this case, the slant height is 9 inches. Make sure I have my pen. So L equals 9. Perimeter of the base, I know that my base is a regular hexagon. So that means each of these sides is 10 inches. So a quick way to get the perimeter, we could 10, plus 10, plus 10, plus 10, and continue on till we added up all six sides. Or, for a short cut, 10 times 6. So that's 60. So in order to get the lateral area of this pyramid, we need to figure out, let's get a little more room for work, the lateral area equals 1/2, times the slant height, which is 9, times the perimeter of the base, which is 60.

So let's go ahead, let's get that in the calculator. 1/2, times 9, times 60. Let's see what that is. 1/2, times 9, times 60. That is 270. So go back to our work here. That means that the lateral area of this pyramid is 270 square inches. And you're all done with that one. Okay, good job on that one. Now, let's take a look at the next example. So here, let me get that out of our way. Get my highlighter. We have a square based pyramid that has a surface area of 1,296 square centimeters, and a lateral area of 720 square centimeters. We're asked to find the length of each side of the base.

So let me get this out of our way here so we can get that surface area formula, because in this case, we were actually given the surface area. We were given the lateral area. We were asked to find one of those smaller pieces, the length of each side of the base of the square based pyramid. So let's use our formula here, and let's figure out first what the area of the space is. Let me get my pen. So the surface area is 1,296. So 1,296 equals the lateral area, that's 720, plus the area of the base, so that's unknown. So first, I'll just use my algebra skills, and I will solve for the area of the base.

Subtract 720 from each side. So looks like that's going to give us 576 equals the area of the base. Okay, so now let's think about how we can use the area of the base to figure out the length of one of the sides of the base. Now, recall this is a square based pyramid. So that means that its base is a square, right? So basically, what we figured out here is we know that the area of that square base is 576 centimeters squared. I'm just going to focus on the number for right now. So we know that to figure out the area of a square, we can just square one of the side lengths. So if i know the area, and want to figure out the side lengths, I can simply take the square root of that area, and then I'll know what each length of the side of the square is.

So all I need to figure out ... Okay, and just to show you what I mean by that. If the area of the base equals a side squared, then that means that the square root of that area will tell me the length of one of those sides. So I'm going to figure out the square root of 576. Go to my calculator, get that. So the square root of 576 is 24. So back to my work. That means that the length of each side of the square base is 24. Get the units again, I believe it was centimeters, just to double check. Yes, centimeters. We got that because the square root of 576 was 24. You're all done with this one, okay?

So see, just to recap what we did. We used that given information and we plugged it into that surface area formula, and we figured out that the area of that square base was 576. Once we knew that area, we took the square root of that area in order to figure out the length of each side of that square. We found that each side of that square was 24 centimeters. Now, I want you to go ahead and try one. So press pause, take a few minutes, work through this one. I will go ahead and reveal this for you so you have that formula. Go ahead and get all that work done, and press play when you're ready to check your work.

All right, let's see how you did here. Let me get my highlighter. So we have a triangular based pyramid, and we know that the lateral area's 480 square centimeters. The slant height's ... I'm sorry, square inches. And the slant height is 20 inches. We're asked to find the length of each side of the base. And then, we're given a little hint here that that base is an equilateral triangle. Let's fill in what we know here, and figure out what we need to know. So we know that the lateral area's 480. So I'm going to go ahead and substitute that in there. So 480 equals 1/2. We know the slant height's 20. And it's the perimeter of the base that we're solving for here.

Going to go ahead in simplify the right side. So 480 equals 10, 1/2 of 20 is 10. So 10 P. I'll divide each side by 10. So 48 equals the perimeter. Okay, now this is where you have to kind of think back and recall that the base is an equilateral triangle. So that means that we have a triangle, all of its sides are equal, and its perimeter is 48. So if I divide 48 by 3, split that perimeter up equally three ways, then I'll know the length of each side. So 48 divided by 3, that's actually 16. So that means that each side of that trigonal base is 16, scroll back up to the top to get the units, inches. So 16 inches, that's the length of each side of that triangle. All right? Okay, good job on that. All right guys, we've reached the conclusion of this topic on how to find the surface area and the volume of a pyramid. I hope you saw how our prior knowledge of area and volume came in handy for you during this topic. Bye.