Module 12: Perimeter, Area, and Volume
Topic 3 Content: Calculating the Surface Area and Volume of Spheres and Hemispheres Transcript

Hi guys, welcome to Geometry. In this topic, we're going to focus on calculating the surface area and volumes of spheres and hemispheres. Now, your prior knowledge of area and volume is going to come in handy for you during this topic. You ready to get started? Let's go.

Okay, now before we start solving problems, I first want to familiarize you a little bit with a sphere and a hemisphere. Okay, so here we have a sphere, which you could think about it as a ball, really, but geometrically, its technical name is a sphere. It consists of all the points in space that are equal distance away from the center, which I can't actually touch, but from the middle, the center point of the sphere. Then also, we have a hemisphere, which is half of a sphere. If you sliced a sphere in half with a flat surface, with a plane, the result would be two hemispheres, and here's one. A hemisphere consists of a curved surface, and it has a circular base, so this is our hemisphere.

Now that you're a little familiar with a sphere and the hemisphere, let's go ahead and work a few examples related to those solid figures. To do that, you're going to need to know the formulas. For a sphere, go ahead and reveal this, get this out of our way, the volume of a sphere can be found by finding the product of four thirds pi and the radius, cubed. That's the formula for the volume of a sphere, and a hemisphere ... I'm sorry, and the surface area, still working with the sphere, the surface area of the sphere is represented by the formula four times pi times the radius, squared. These are the formulas that we would use to find the volume of a sphere, and to find the surface area of a sphere.

Now that you're familiar with the formulas, let's take a few minutes and work a few examples. Let's take a look at this one first, make sure I've got my pen here. Let's get the highlighter first, so we can mark our key facts in this one. Here we have a decorative globe light bulb that has a diameter of 12 inches, so that's important to know, diameter is 12 inches. What is the volume of the globe? We're asked to give our answer in terms of pi. Okay, so the first thing I see from this problem is that I have a decorative globe, so I know I have a figure that's in the shape of a sphere, and the diameter is 12 inches. We're asked to find the volume, let's go ahead and get that formula here, let's get this out of our way, the formula for the volume of a sphere is four thirds times pi times our radius, cubed.

Now, if I'm told that the diameter is 12 inches, and I know that the radius is half of the diameter, then now I could go ahead and say that our radius is six inches. I'll go ahead and write down that r equals six. I can go ahead, because now I have all the pieces that I need to go ahead and use that formula, and calculate the volume. The volume equals, let's get a little more workspace here, four thirds times pi, times six cubed. Now because my answer is going to be in terms of pi, in the calculator, I'm only going to find the product of four thirds and six cubed. I'm going to leave pi out, so that my final answer will be in terms of pi.

Let's go ahead, let's get our calculator, and let's multiply four thirds and six cubed. All right, so four thirds times six cubed, let's get that in there. That is 288, so that means that our final answer, if we go back to our work, is 288 pi. Let's get a little more room here, all right, here we go. The volume of this decorative globe is 288 pi, and let's double check our units, inches, so inches cubed, and we're all done with this one. See what we needed to do here. We knew

that our diameter was 12 inches, so we know our radius is half the diameter, so we calculated that our radius would be six, did a little mental math for that. We used our formula for volume, we substituted the radius in there, because our answer was going to be in terms of pi, we only needed to find the product of four thirds and six cubed. We figured out that was 288, so that means that our final answer for the volume is 288 pi cubic inches.

All right, so now that we're done with this one, I want you to give this one a shot. I'm going to first reveal the formula for you for this, so you have that. All right, now press pause, take a few minutes, work your way through this one, press play when you're ready to check your work.

All right, let's see how you did here. I'm going to pull out the highlighter, we have a sphere with a radius of seven centimeters, we're asked to find the surface area of that sphere, and we're asked to round to the nearest tenth. Okay, so I was given the radius, I'm going to go ahead and substitute it in here, so the surface area equals four. Because I was asked to round to the nearest tenth, that tells me that I need to approximate pi's value at 3.14. Anytime you're asked to round or to approximate, that's a hint to go ahead and use 3.14, your answer's not going to be in terms of pi. Then seven is my radius, so seven squared. In the calculator, four times 3.14 times seven squared, so four times 3.14 times seven squared. 615.44, so round it to the nearest tenth, that would be 615.4. Let's go ahead, go full screen here, so 615.4 square centimeters. That is the surface area of this sphere, all right? Okay, good job on that one.

Now take a look at this one. I'm going to switch to my highlighter so I can pull out that key information. A hemisphere has a radius of nine centimeters, and we're asked to find the volume of the hemisphere. We're also asked to give our answer in terms of pi. Okay, so when you're working with a hemisphere, what you want to do is figure out how whatever you're being asked to find relates to what you would be finding in a sphere. What I mean by that, here you're asked to find the volume of this hemisphere. Well, how does the volume of a hemisphere relate to the volume of a sphere? Okay, let's think about that for a second. If a sphere, when it's cut in half, it's split into two hemispheres, then the volume of one of those hemispheres is equal to half the volume of the sphere.

If we find out what the volume of the sphere is that has a radius of nine centimeters and cut it in half, then we'll know the volume of this hemisphere. Let's go ahead and do that first. I'm going to go ahead and write "sphere" here, just so we remember as we're working that the first thing we're doing is finding the volume of the sphere, and then we're going to cut that in half to figure out the volume of this hemisphere. We know that our radius is nine, so four thirds times pi times nine cubed, and we're asked to give our answer in terms of pi, so what I'm going to do is I'm actually going to just find the product of four thirds and nine cubed in the calculator, I'm not going to multiply by the approximation of pi. Here we go, four thirds times nine cubed that is 972.

I'm going to go back to our work, and what that tells us is that the volume of a sphere that has a radius of nine centimeters is 972 pi centimeters cubed, and remember that's for the sphere, but we want to know the volume of a hemisphere that has a radius of nine centimeters. All we're going to need to do is cut that volume in half. Need a little more space here, so for the

hemisphere, we need that volume, that 972 pi, divided by two. Let's go ahead and let's get that into our calculator, but ignore the pi and just find 972 divided by two. 972 divided by two, and that is 486. Back to our work, and that means that the volume of this hemisphere is 486 pi centimeters cubed, and we're all done with this one.

Now that's how you want to attack any kind of hemisphere problem. You want to think about how what you're being asked to find relates to something on the sphere, like in this problem, if I scroll back up to the top, the first thing we did is we figured out how the volume of this hemisphere related to the volume of this sphere. Once we thought about that, we figured out that we needed to find half of the sphere's volume to figure out the volume of this hemisphere. We found that volume of the sphere, found that it was 972 pi, we divided it by two, and figured out that for the hemisphere, the volume was 486 pi cubic centimeters. All right, good job on that one.

Now take a look at this one, get that out of our way. We're working with the same hemisphere on these next few problems. Here, same hemisphere with the radius of nine centimeters, and we're asked to find its lateral area. Okay, so let's think about this for a second. If we had a sphere, and we wanted to find the surface area of that sphere, matter of fact if I back up a few slides here and go back to the beginning so we can actually look at that sphere, the surface area of the sphere is the area of that curved surface all around the sphere. Now, in this problem, we're being asked to find the surface area ... Or I'm sorry, let me scroll back to it, flip back, we're being asked to find the lateral area of this hemisphere.

Now, how that relates to our sphere is that it actually relates to the surface area, because if we want the lateral area of this hemisphere, then we only want the area of the curved surface, and that curved surface is actually the surface area of the sphere. If we find half of the sphere's surface area, then we'll know the lateral area of this sphere. All right, so let's reveal that formula, and let's figure out the surface area of a sphere that has a radius of nine centimeters, and then we'll find half of that surface area, and figure out the lateral area. Again, I'm going to first write down that this is relating to the sphere, this first part of our work, and we know the radius is nine centimeters, so four times pi times nine squared. I'm going to leave this in terms of pi, so in the calculator I'm only going to find four times nine squared, let's get that. Four times nine squared, that is 324.

Back to our work, that means that the surface area of a sphere with nine as the radius, let me get that number again, and that was 324, make sure I was remembering that right. 324 pi centimeters squared, that would be the surface area of the entire sphere, but because we know that we only want the lateral area, which is that same curved surface of the hemisphere, like we figured out at first, we need half of this measure here. For the hemisphere, half of the surface area of the sphere, just to write it out so you can see it, equals the lateral area of the hemisphere. We need one half of 324 pi, or in other words, 324 pi divided by two. Let's go to the calculator, and we need 324 divided by two, so 324 divided by two, that is 162.

Back up to our work, 162 pi centimeters squared, and that's the lateral area of this hemisphere. Now good job pulling all that together. I know that one may have taken a little

more than the volume did, as far as figuring out how the lateral area of a hemisphere relates to a sphere, but the lateral area of a hemisphere is half of the surface area of a sphere. All right, good job on that.

Now I want you to go ahead and try this one. I'm going to get this out of your way so you can see that formula, and to give you a little hint on this one, there's one more formula that you're going to need in order to find this answer, so take that as a hint. Press pause, take a few minutes, work through this, press play when you're ready to check your work.

All right, let's see how you did here. Let me go ahead and switch to my highlighter, and again we're working with that same hemisphere that has a radius of nine centimeters, and you're being asked to find the surface area of the hemisphere. Now, the surface area of the hemisphere, it involves two different surfaces here. It has the curved surface, we're going to need to figure out what the area of that curved surface is on the hemisphere, but it also has that base. The base of a hemisphere is a circle, and if we want to find the surface area of the hemisphere, we're going to need to involve all of those pieces. We're going to need to figure out its lateral area, as well as the area of the base.

Now in our prior example, we already figured out what the lateral area of this hemisphere is, so let me just go ahead first and write down what we're solving for here. We need the surface area of the hemisphere, just to spell out exactly what it is that you were working with here. In order to find the surface area of the hemisphere, we needed to figure out what its lateral area is. We need the lateral area of the hemisphere, plus the area of the base, so the area of this circle. Figuring out those two pieces will help us figure out what the surface area of this entire sphere is. We need its lateral area, plus the area of that circle. Now the lateral area, you already did the work to figure that out, so use that math that you already have in order to use in this problem also.

Let's flip back to the work that we just did, because we did all the work to figure out that lateral area, and we found that the lateral area of that hemisphere was 162 pi centimeters squared. Let's use that information in our new problem, because we're going to need it. 162 pi, and then we also need to know the area of that circular base, so let's get a little more space here. We know we have a circle at the top of that hemisphere, and its radius is nine, just to get a little rough sketch there. The area of a circle, pi r squared, and the radius is nine here, so pi times nine squared, nine squared is 81, so 81 pi. This is the area of the base of the hemisphere. Let's go ahead and get it in our problem, 81 pi. Once we find this sum, the sum of the lateral area of the hemisphere and the area of that circular base at the top, then we'll know the surface area of the hemisphere.

In the calculator, I'm going to go ahead and do 162 plus 81, so 162 plus 81 that is 243. Back to the work, let's get that here. 243 pi centimeters squared, I'm getting a little crowded here, so let me just get rid of our scratch work down here, when we found the area of that circle. All right, and we have our final answer, 243 pi centimeters squared. Again, see how on this problem, in order to figure out the surface area of the hemisphere, we had to think about how that measure related to the sphere. We knew, okay to find the surface area of this hemisphere,

we need its lateral area plus the area of its base. The lateral area, we had found in our previous example that it was 162 pi, and the area of our circular base, we used our formula for the area of a circle to figure out that was 81 pi, found the sum of those measures, and found that our answer here was 243 pi centimeters squared, all right? Hey, good job on that one.

All right guys, you've reached the conclusion of this topic on how to find the surface area and volumes of spheres and hemispheres, where we saw how your prior knowledge of area and volume came in handy for you during this topic. Bye.