Module 12: Perimeter, Area, and Volume
Topic 4 Content: Solving Problems Involving Composite Solids Transcript

Hi guys. Welcome to Geometry. In this topic, we're going to focus on solving problems involving composite solids. Now, your knowledge of area and volume of solids is going to come in handy for you during this topic. You ready to get started? Let's go.

Now before we dive into this example, I just want to talk for a moment about exactly what a composite solid is. A composite solid is a composite three-dimensional figure. What that means is that it's one figure made up of different figures. So when you have a composite solid, you'll have a solid figure that is made up of two or more different types of figures.

For example, let's take a look at this one. The trashcan modeled by the figure below will be wrapped in plastic before it is shipped to distributors. How much plastic is needed? Round to the nearest tenth.

Okay, now here we have a composite solid, and it's made up of a cylinder and a hemisphere, so when I think about exactly what I need to do to answer this question, I have to think about how the parts of those individual figures, what role they play in this composite solid, okay?

Now, in this question ... I'm going to pull my highlighter out here ... we were told that this figure, this trashcan given by this model here, is going to be wrapped in plastic before it's shipped to distributors, so that means this entire solid is going to be covered in plastic, and we were asked to find out how much plastic is needed, and we're asked to round to the nearest tenth. Okay.

Now here, if I'm wrapping this entire solid in plastic, I have to think about exactly what parts of it are going to be covered. Well ... switch to our pen ... I know that definitely we're going to cover this hemisphere, right? And if we're talking about the outside of a hemisphere ... because we're only covering the top of it ... the base of the hemisphere, its circular base, is not going to be covered by that plastic because it's attached to the cylinder. So when I'm wrapping this trashcan in plastic, I know part of what I'm going to need to figure out is the surface area of this hemisphere.

Now, if we think back to how a hemisphere relates to a sphere, if I need to calculate the surface area of a hemisphere, then I actually need half of the surface area of a sphere. So I'm just going to start getting some notes down here so I can keep track of what I'm going to calculate once this is all said and done. So I know that I need half of the surface area of a sphere. That's the first thing I'm going to need.

The next thing I'm going to have to figure out here, thinking about how this plastic is wrapping around this trashcan, I know that it's going to completely cover the lateral face of this cylinder. So when trying to determine how much plastic is needed for this trashcan covering, I know that I'm going to have to figure out the lateral area of a cylinder. So that's the next thing I'm going to have to figure out, going to need the lateral area of a cylinder.

Finally, the bottom of this trashcan is also going to be covered. This plastic is wrapping all around it, the top of it and the bottom. The bottom of this trashcan is a circle, right? So I have to figure out the area of this circle in order to figure out how much plastic is needed to cover the bottom. So the third thing I need is I need the area of a circle.

So you see here, in order to figure out how much plastic is needed to cover this composite figure, which is actually us figuring out the surface area of this composite figure, we have to figure out what we need from its component parts. We're going to have to figure out half of the surface area of the sphere in order to figure out what's needed to cover that hemisphere, we're going to need the lateral area of a cylinder in order to figure out how much plastic is needed to around this trashcan, and then we're also going to need to know the area of the circle. That's going to tell us how much plastic is needed for the bottom of this composite figure, okay?

Now I'm going to scroll down because I actually have our formulas ready, and we can go ahead and actually get the calculations that we need here. Okay, so we said we needed half of the surface area of a sphere, so let's reveal this so we can get the formula for the surface area of a sphere, and we know that we need half of that. So the first part ... I'm just going to keep our numbering system here, so I'm going to number #1, because the first thing I'm going to do is find half the surface area of a sphere in order to figure out the surface area of that hemisphere. I'm going to scroll down here, and I'll go ahead and switch colors, just to help us stay organized.

Okay, so the first thing I need, I need half of this value. So I need 4π. Let's go ahead and get our radius, scroll up to the top. So the radius of this sphere is the same as the radius of this circle, so it's three feet. So I'm going to go down to my formula, I'm going to substitute 3 for r. Then I know that I just need half of this. Now, for right now, I'm just going to keep things in terms of π, and at the very end I'll go ahead and approximate the value. Just to make our calculations a little easier here, I'm going to keep things in terms of π for right now.

In the calculator, what I'm going to do is I'm going to find 4 times 3 squared, divide that value by 2, and then that'll tell us the surface area of our hemisphere. Let's go ahead and switch to our calculator. We need 4 times 3 squared, divided by 2. So 4 times 3 squared, let's see what that is. Then let's divide that by 2. Okay, so 18. So now I'm going to go back to my work, go full screen here. So for the hemisphere, 18π square feet. That's how much plastic is needed just to cover the hemisphere, this part of the trashcan.

Next, let's go ahead and figure out the lateral area of the cylinder. Let's scroll down and get that formula. Switch to the arrow here, our pointer tool. Let's get that. Okay, so the lateral area of our cylinder, 2πrh. Let's get the pen back, label that #2. So 2π, and let's go up to our figure to get the radius and the height. Okay, so the radius of the cylinder is right down here, it's the radius of the circle, so it's 3 feet, and the height of the cylinder is 4 feet. So r is 3, h is 4. Let's scroll down. So our radius ... make sure I still have the pen ... our radius is 3 and our height is 4. I think we can do a little mental math here. I'm going to keep things in terms of π for right now. So 2 times 3 is 6, and 6 times 4 is 24. So for the lateral area of the cylinder, 24π square feet.

The last thing for us to figure out ... and let's get up to our figure again ... we need the area of the circle right here on the bottom. If I go down, let's get the formula for the area of a circle.

Scroll down here, I know it's a lot of scrolling. Let's get that out of our way. Okay, area of a circle: πr squared. Let's go ahead and let's get that going, our third part. The area is π, and our radius ... I'll scroll back up to the top, but we've seen it several times so far ... we know the radius is 3. So back down to the work ... 3 squared. We can do some mental math here. We know that 3 squared is 9, so the area of our circle is 9π square feet. Okay?

Now, in order to get the surface area of this composite solid, we're going to want to add these pieces together. This is the point where we're going to go ahead and approximate π's value, okay? Let's get a statement here, a little room so we can get an intermediate step. We need 18π, and this is telling us the surface area of the composite figure. Because now we're at that final step where we can bring everything together. So we need 18π ... I'll switch to blue now ... 18π plus 24π plus 9π. Let's see what that sum is first just in terms of π. In the calculator, I'm going to do 18 plus 24 plus 9. All right, so 18 plus 24 plus 9. That is 51. Back to our work. That means that the surface area of this composite solid here, this composite figure, is equal to 51π square feet.

Now I'll go ahead and I'll approximate π's value. We were asked in the initial part here to round to the nearest tenth, so let's go to the calculator. We need 51 times 3.14. That is 160.14. Rounded to the nearest tenth, that'd be 160.1. Let's get that down. Scroll down here ... 160.1 square feet. We are all done with this one.

Now, I know this pulled together a lot of different pieces, and that's pretty much how composite solid problems go. You have to find out what do you need from those component parts, right? Once you know that, you use your formulas, you figured out all those different measures, and then you add together what's needed at the end.

So just to recap for this example, go over what we did here. We figured out at first that we needed to know the surface area of this composite solid. We figured out that it was made up of a hemisphere, a cylinder, and a circle at the bottom. Those were the shapes that we were going to work with in order to figure out our measures. So we used our formula, we figured out half of the surface area of a sphere.

We kept things in terms of π at first, just to keep our calculations simple. We figured out the lateral area of the cylinder, we figured out the area of the circle, we found the sum of those measures, figured out that it was 51π and the units were still square feet. Once we had this part here, then we went ahead and approximated π's value at 3.14 and were able to figure out that the surface area of that trashcan is approximately 160.1 square feet. So that's about how much plastic is going to be needed in order to cover that entire trashcan before it's sent out to distributors. Good job on that one.

Now I want you to take a look at this one. So press pause, take a few minutes, work your way through this one. I've given you a hint here that there's actually two formulas you're going to be working with for this composite solid. Once you've figured those out and you've done all the work, go ahead and press play to check your answers.

All right, let's see how you did here. Now on this one, you were asked to find the volume of this solid. So if I look here ... let's get this out of the way for a minute ... this solid figure consists of a rectangular prism ... I can see that ... and half of a cylinder. If you envisioned a cylinder sitting up tall, sitting on its circular base, and you sliced it in half vertically and then laid it down, that's how this cylinder is oriented on this rectangular prism.

Now, in order to figure out the volume of this composite solid, there's two things that I'm going to have to know. Start by making my list here of what I have to figure out. I'm going to first need the volume of this rectangular prism. So the volume of a rectangular prism, abbreviate that. Then I'm going to need half of the volume of this cylinder. So that's the second thing I'll need. Half the volume of the cylinder.

Now that I have my list, go ahead and take a peek at my formulas so I can go ahead and make those calculations. Okay, let's get that out of our way. Okay, so that's the formula for the volume of a rectangular prism. Here, I just need to figure out what the pieces are. I'm going to scroll down here, get us a little workspace. I'm going to just draw a line to mark this off so you know we're doing a different step right now, because I still want to keep my list so I can refer back to it.

The volume of that rectangular prism ... scrolling back up to the top ... it's length times width times height. In this case, that means it's going to be 18 times 10 times 9. Let me go down here. I need 18 times 10 times 9. Go ahead and put that in the calculator. So 18 times 10 times 9. That's 1,620. So back to my work ... go full screen here ... 1,620. Let me scroll back up to check our units here: inches. So this is inches cubed.

So I have the volume of the rectangular prism, now I need half the volume of the cylinder. Let's scroll up to the top, let's get that formula. Let's get this out of our way. Okay, so the volume of a cylinder is $\pi r^2 h$. Remember, I only need half of that for this composite figure. So I'm going to scroll down ... I know we're doing a lot of scrolling ... so I need half the volume. Let's see, let's represent that as ... let's do $\pi r^2 h$ and then we'll divide it by 2. Let's get π down first. Now we need the radius of the cylinder.

Okay, so look here. This, if you pictured that circle completing, this is the circular base of the cylinder. This 10 inches as well as being the width of the rectangle, it's also the diameter of this circle. So if its diameter is 10 inches, that means that its radius has to be 5, because we know the radius is half of the diameter. In the case of our cylinder, we know r is 5. Let's scroll down. So we know we have 5 squared.

Then for the height ... scroll back up to the top ... remember that the height of a cylinder is represented by the segment that connects those parallel bases. You can think about it like that, this perpendicular to those bases. In this case, the height of a cylinder is represented by this segment, so the height of this cylinder is actually 18 inches. Here we have the height of the cylinder at 18 inches ... let's scroll down ... so 18.

Then remember, we only need half of this, because we only have half of a cylinder. I'm going to go ahead and put our volume equals that there also. In the calculator, I'm going to leave π out for right now. Just going to get 5 squared times 18 divided by 2. So 5 squared times 18, and then let's divide that by 2. That's 225. Back to our work. That means that the volume of half of that cylinder ... because half of the cylinder is all that's represented in our figure ... is 225π cubic inches.

Now that I have those component parts I'm going to go ahead and add them together to figure out the volume of this composite solid. Now I'm going to do another line here just to show that we're doing a step three here, because I still need to refer to this. In order to get the volume ... I'll switch colors though ... the volume of the composite ... we're going to need to add those pieces together. So we need 1,620 plus 225, and we're going to go ahead and approximate π's value at 3.14. I'm going to switch to the calculator, and I need 1,620 ... 1,620 plus 225 times 3.14. We end up with 2326.5.

Let's go ahead and go back to our work. Go full screen here. Actually, let me check that answer one more time, make sure I remembered it right when I switch: 2326.5. Okay, so 2326.5, and let me just check our units one more time: cubic inches. You are all done with this one. Good job figuring out what those component parts were, what measures you needed from each of them, and then combining them to get the volume of this composite solid.

All right guys, you've reached the conclusion of this topic on solving for measures of composite solids. I hope you saw how your knowledge of area and volume helped you get through this topic. Bye.