Module 12: Perimeter, Area, and Volume

Topic 5 Content: Comparing Ratios of Measures of Similar Figures Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on comparing ratios of measures of similar figures. Now, your knowledge of similar triangles and all that you've learned there is going to help you get through this topic. You ready to get started? Let's go.

Now here we're given two rectangles, and we're asked to determine if these rectangles are similar. If so, we have to find the scale factor. Now think back to when we cove red similar triangles, and even a little bit during your studies of geometry in middle school, but what you learned about similar figures. You know that if two figures are similar, then the ratio of their corresponding links will be the same. In order to figure out if these rectangles are similar, we're going to need to figure out if the ratio of their corresponding links is the same. Let's go ahead and let's get to work on that.

Here in this rectangle, like what we learned in triangles, it helps if you go ahead and label the smallest side, the medium side if you have three, and then your largest side. Here I know my rectangles do have four sides, but I know their opposite sides are congruent, so I'm just going to focus on these two dimensions here, the length and the width. In this rectangle, this is the smallest length, this is the largest length. In this one, this is the smallest length, this is the largest. I'm going to go ahead and set up my ratios now so I can see if my rectangles are similar.

I'm going to start with the smallest rectangle, but like you've learned before, you could start with the larger figure if you wanted to. I'm going to set up my ratios all beginning with the smallest one. The smallest in this to the smallest in this, that would be two to six. I have to see, is that equal? Right now this is a question mark to the largest length here over the largest length there. That's five over 15. Let's simplify those fractions and see if they are indeed equal. Two over six will reduce to one -third. Mental math, if you divide the numerator and denominator both by two, you'll end up with one over three. Five over 15 will also reduce to one-third. If I divide the numerator and denominator by five,I'll end up with one over three.

I see here that the ratios were equal. What that tells me is that these rectangles are similar, because the ratio of their corresponding links, in this case, they both equaled one -third. But the point is that they were both the same. Here, are the rectangles similar? Yes. Then the scale factor is one-third, all right?

We're all done there. We figured out the rectangles are similar. Their scale factor is one -third. If you recall how to interpret scale factor, what that means here is that this smaller rectangle is one -third the size of this larger one. Or you could think about that as this larger rectangle is three times the size of the smaller one. Keep all that in mind and take a look at this next example.

We're working with the same pair of rectangles, and we know for sure now that they are similar, and we're asked here, "What is the relationship between the scale factor and the ratio of the perimeters?" I'm just going to make a note here, let's get our work going. We already know that our scale factor, I'm just going to abbreviate that, is one-third. We're going to keep that in mind as we work through figuring out the perimeters of these rectangles. I know that in order to figure out the perimeter of the rectangle, I could add up all the sides, right? Or I

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could add two times the width plus two times the length, and that will also give me the perimeter. I'm going to focus first on this smaller rectangle, I'm just going to label it "small," and the perimeter of this one is two times two plus two times five, so that would be four, plus 10, which is 14. The perimeter of the smaller rectangle is 14 centimeters.

Now let's figure out the perimeter of the larger rectangle. We're going to have to scroll up and down a bit. I'll label this one "large," and let's get the perimeter of that one. Scroll up to the top. Two times six plus two times 15. Two times six plus two times 15. Two times six is 12. Two times 15 is 30, so 12 plus 30, that's 42 centimeters. Okay. Now I have the perimeter of the smaller rectangle at 14, the perimeter of the larger rectangle at 42. Let's go ahead and get the ratio of those perimeters. The ratio of the perimeters, that's 14 over 42. We can actually simplify this fraction. We can divide both the numerator and the denominator by 14, and that will actually give us one over three. Now notice here the ratio of the perimeters was actually equal to the scale factor. That's the relationship there. If you have two similar figures, the ratio of their perimeters will be equal to their scale factor.

Take a look at this next one here. Dealing with those same rectangles, but this time we need the ratio of their areas, and we want to know its relationship to the scale factor. Let's make a note here that the scale factor is one-third, and let's go ahead and get the area of the smaller rectangle. Area of a rectangle I know is length times width, so two times five, that's 10. Then for our large rectangle, scroll down a little bit here, so for the large one, let's get its dimensions. Length times width, so that's six times 15. Scroll down here. Six times 15, that's 90. In the case of the rectangles, if I want to focus now on the ratio of their areas, make a note of that over here. I'll go ahead and switch colors, try to keep us organized. The ratio of the areas is 10 over 90, and we can actually simplify this fraction also. We divide numerator and denominator by 10, that's one-ninth.

Now look at the ratio of the areas and let's compare it to the scale factor. We know it's not equal to the scale factor, right? The ratio of the areas is one-ninth, and the scale factor is one-third. But if we squared our scale factor, so if we raised our scale factor to the second power, find our scale factor squared, we're going to square that scale factor, so one squared is one, three squared is nine. Now if we look here, notice this. The scale factor squared is equal to the ratio of the areas, and that's the same for all similar figures. The ratio of your areas will be equal to your scale factor squared, okay?

Take a look at this next one. Here we have rectangular prisms, and they're similar. We're asked to determine the relationship between the scale factor and the ratio of the volumes, okay? Here let's get our scale factor first, and we could just pick any pair of corresponding sides here. We see here this is the smallest length in our smaller prism, and this is the smallest length in our larger prism. This scale factor, just make a note here, and you may have already noticed it, two over six, one -third. Our scale factor is one-third for these prisms.

Now let's get the volume of that smaller prism. For the small one, the volume is length times width times height for a prism, so five times two times six. Five times two times six. We could do a little mental math. Five times two is 10. 10 times six is 60. Then for the large prism, we'll do a little work over here, we'll need 15 times six times 18. 15 times six times 18. Let's go to the calculator for that one. 15 times six times 18, getting that in the calculator. 1620. Let's go back to our work, go full screen here. 1620. That's the volume of the larger prism.

Now let's get the ratio of these volumes. Let's scroll down. The ratio of the volumes ... Let me just erase that little dot. Don't want you to think that it means anything there. 60 over 1620. Let's go to the calculator to reduce this. 60 over 1620, so we can do 60 divided by 1620. If you recall a few lessons back, we learned a trick to reduce a fraction, or in other words to change this decimal to a fraction. I'm going to hit "math," and enter twice, which is going to tell the calculator to make that decimal a fraction. Enter, enter, and that's one over 27.

Let's go back to our work. We have one over 27. Notice this ratio is not equal to our scale factor, because we know our scale factor is one-third. The ratio of the volumes we see is 127. But if we cube our scale factor, let's get that down. If we find our scale factor raised to the third power, that would be one-third raised to the third. One to the third power is one. Three to the third power is 27. That's the relationship between the ratio of the volumes of similar figures, will be equal to the scale factor cubed.

Let's keep going here, and let's just review just to bring everything all together. The ratio of the perimeters, given two similar figures, is equal to the scale factor. The ratio of the areas is equal to the scale factor squared, and the ratio of the volumes is equal to the scale factor cubed. We'll keep all that in mind and take a look at this example.

Here we're told that the ratio of the heights of two similar triangles is two to three. First thing we have to do is find the ratio of the perimeters. We know that the ratio of the heights, that's our scale factor there, because heights, we call it a linear measure. It's a measure in one dimension. The ratio of the heights, that would be the same as our scale factor, which in this case is two to three. The ratio of the perimeters we know is equal to the scale factor, so it's going to be the exact same thing, two to three. Because it's the ratio of the perimeters, we know that it's the same as the scale factor.

Now we're asked to find the ratio of the areas. Well we know that the ratio of the areas is equal to the scale factor squared. If we square two and square three, two squared is four, three squared is nine. The ratio of our areas would be four to nine, and you're all done here. As long as you keep those facts in mind, you'll be able to easily answer these kinds of questions.

Now go ahead and try this one. Press pause, take a few minutes, work through this. Press play when you're ready to check your work. All right, let's see how you did here. Here you're asked to find the ratio of the volumes of these similar cones, and you're told that the ratio of the volumes of these similar cones, and you're told that the ratio of their radii is nine to four. The length of a radius, that's a one- dimensional measure, so it's linear. When you find a radius, its length, and you make a ratio of corresponding radii in this case, really what you've found is the scale factor. Here, when you're told the ratio of the radii of these similar cones is nine to four, we're really told that the scale factor is nine to four.

When we're asked to find the ratio of the volumes of these cones, we know that the ratio of the volumes is equal to the scale factor cubed. We're just going to need to cube our ratio, and then we'll know the ratio of the volumes of these cones. We're going to need to figure out nine to the third power, and four to the third power. Let's go to our calculator, let's get that. Nine cubed, that's 729. Four cubed, that's 64. Back to our work. 729 to 64. You're all done with this one. We've cubed our scale factor, and we have the ratio of our volumes. Good job with that.

Now take a look at this one. The ratio of the areas of two similar triangular prisms is 121 to 64, and the first thing we're asked to do is find the ratio of their heights. Now we're told here that this is the ratio of the areas of these prisms, so what we can learn from that is that we have squared our scale factor in order to figure out the ratio of these areas, because as you recall, the ratio of the areas is equal to the scale factor squared. If we take the square root of the ratio of our areas, and the square root of what that is, we'll know what the scale factor is. Taking the square root of the ratio of the areas will help us figure out our scale factor in this question.

I'm going to scroll back up to the top, and we're told here that the ratio of our areas is 121 to 64. I'm going to take the square root of 121, and the square root of 64. The square root of 121, that's 11. The square root of 64, that's eight. This ratio, this is our scale factor. That's what this ratio is, so now that we know that, we can go ahead and answer these questions. If we scroll back up, the ratio of the heights, that would be the same as our scale factor, right? It's a linear measure. The ratio of the heights, 11 to eight. The ratio of the volumes, well we know that's our scale factor cubed, right? What we're going to have to figure out ... Let's get some of this work out of the way here. Let's get rid of this. We're going to need to cube our scale factor, so cube 11 and cube 8, and then that will tell us the ratio of the volumes. In our calculator, we're going to raise 11 to the third power, and eight to the third power. Let's see what that is.

11 cubed, that's 1331. Eight cubed, that's 512. Back to our work. 1331 to 512. That's the ratio of the volumes of these triangular prisms. Good job on that. Go ahead and give this one a try. Press pause, take a few minutes, work through this. Press play when you're ready to check your work.

All right. Let's see how you did here. Here you were given that the ratio of the volumes of two similar square-based pyramids is 64 to 125. You're asked to find the ratio of the lateral areas. If the ratio of the volumes is 64 to 125, that means that I've cubed my scale factor in order to get to this answer. If I find the cubed root of 64 and the cubed root of 125, then I'll know my scale factor. Once I know that, I'll be able to figure out the ratio of these lateral areas. The cubed root of 64, I'm just going to do a little mental math here. That's four. The cubed root of 125, that's five. Here, this ratio, this is my scale factor. Now that I know this, I can go ahead and figure out the ratio of those lateral areas. Because I know if I square my scale factor, I can find the ratio of areas, so I'm going to square the four, and square the five. Just draw a line here to show this is our second step. Four squared, that's 16, and five squared, that's 25. You're all done with this one, okay?

Just a recap to pull together what we did here. We were given the ratio of volumes, so we found the cubed root in order to figure out what our scale factor was. We figured out it was four to five, and once we had that, we squared it to figure out the ratio of our lateral areas. We ended up at 16 to 25. Good job with that.

All right, guys. You've reached the conclusion of this topic on using the ratios of the measures of similar figures in order to solve problems. I hope you saw how your knowledge of similar figures in general helped you get through this topic. Bye.