Module 12: Perimeter, Area, and Volume

Topic 5 Content: Determining How Changes in One Dimension Affect Perimeter, Area, and Volume and Vice-Versa Transcript

Hi guys. Welcome to Geometry. In this topic, we're going to focus on how changes in one dimension affect perimeter, area, and volume, and vice versa. Now, your knowledge of solid figures is going to come in handy for you during this topic. You ready to get started? Let's go.

Let's take a look at this example and then we'll see how we kind of have to work our way through these types of problems. The radius and height of a cylinder are each multiplied by three. How does this change affect the lateral area of the cylinder? Now, let's reveal the formula here. Here's our formula for the lateral area of a cylinder. Now, what's happened here is we're told that the radius and the height were each multiplied by three. Now the radius is three times what it was originally and the height is three times what it was originally. We need to figure out how this change affected the lateral area, or in other words, how many times larger will this new lateral area be when compared to the original lateral area?

Let's use the formula and let's get to that answer. We know that lateral area can be represented by this expression, two times pi times the radius times the height. Now, what's happened, kind of draw a line to kind of show this as our second step, is we still have two times pi, but now our radius is three times as large and the height is also three times as large. Let's simplify this expression so that we can get a new expression to represent the lateral area of this cylinder that's been manipulated a little bit. I'm going to use some mental math and I'm going to keep the answer in terms of pi. I'm going to find two times three times three. Two times three is six, and six times three is 18. I'll have 18 pi r h. The new lateral area could be represented by this expression, 18 pi r h.

Now what I need to do is figure out how this expression compares to the lateral area of the original cylinder. I'm going to divide. I'm going to do 18 pi r h divided by two pi r h. That will tell me how many times larger this new lateral area is. I'm going to draw a line again. I'm going to show that this is our third step here in this process. Now we'll have 18 pi r h divided by two pi r h. What happens here is the height cancels, the radius cancels, pi cancels and we're just left with 18 divided by two. Now, 18 divided by two, that's nine. What that tells us is that this new lateral area will be nine times as large as the original. That's how those changes to the radius and the height affected the lateral area of this cylinder. The lateral is nine times as large as what it was originally.

All right. Now, keep those steps in mind as you work through this one. Press pause. Take a few minutes. Work your way through this one. I will go ahead and reveal this for you so you have the formula while you're working. After you have all that work done, go ahead and press play to check your work. All right, let's see how you did here. Here you were told the radius of a cylinder is multiplied by five. How does this change affect its volume? All right, let's get our pen. Basically what we have here is our radius now is five times what it was originally. We want to figure out how this is going to affect the volume of the cylinder. Let's go ahead and let's use this expression to represent the volume, pi r squared h. Draw a line to show this is our second step here. Pi times five r squared times the height.

Let's simplify this. We'll have pi five r, that quantity squared. That's going to be 25 r squared. Then again it's times the height. Let's just clean this up a little bit, which really just means I'm going to bring that coefficient, that 25 as a coefficient to the front. 25 pi *r* squared *h*. Now that I have an expression to represent the volume of this cylinder ... I just scrolled back up to double check what we were working with here. Now that I have this expression to represent that volume, what I'm going to do is I'm going to divide. I'm going to divide 25 pi *r* squared *h* by pi *r* squared *h*, and that'll tell me how many times larger the volume of the new cylinder is when compared to the volume of it originally. Let's get that division going.

We have 25 ... Let's fix that. Forgot the five there. We have 25 pi *r* squared *h* and we're dividing it by pi *r* squared *h*. The *h*s cancel. The *r* squareds cancel. The pi's cancel. All we're left with is 25. What that tells us is that the volume of this new cylinder will be 25 times as large as it was originally. That's how that change to the radius affected the volume of this cylinder. All right, now keep all those processes in mind and take a look at this next example. Here, the lateral area of a cone is 30 times as large as its original lateral area. We're asked, is the following statement true or false? If the slant height is five times as large, then the radius is six times as large. All right. In order to figure that out, we're going to need to take a look at the formula for lateral area.

Here, the lateral area of a cone is pi times *r* times the slant height. What we need to figure out is if the slant height is five times as large, and the radius is six times as large, does that really result in a lateral area that's 30 times as large? Let's figure that out here. We know that our slant height, *l*, is now five times what it was originally, and the radius is now six times what it was originally. Let's see, does that really result in that change in the lateral area? The expression that represents the lateral area, let's check which solid we have here, of a cone is pi times the radius times the slant height. Here we'll have pi times six *r*, because that's our new radius, times five *l*, because that's our new slant height. Let's clean this up. We can use a little mental math.

Six times five, that's 30. 30 pi *r l*. This expression will represent the new lateral area. We need to figure out is it actually 30 times as large as the original lateral area. You may actually see it right now, but I'm going to go ahead and divide, just to make sure that it's clear. The new lateral area 30 pi *r l* divided by the original, pi *r l*. We can see here that the lateral areas will cancel, *l*'s cancel, *r*'s cancel, pi's cancel. You're just left with 30. It does look like that statement is true. If we scroll back up to the top, just to recap, the lateral area of the cone, if it is 30 times as large as it was originally, then that means if the slant height is five times as large, then that radius is six times as large. We figured that out by doing our substitution to represent those changes in the lateral area and the radius. We figured out our new expression to represent that lateral area, 30 pi *rl*. We did our division and we found that actually it is 30 times as large.

All right. Our answer here was true. Now, keep all of that work in mind, all those steps we took to get through this one as you work your way through the next one. Press pause. Take a few minutes. Work your way through this. I will actually reveal the formula you'll need for this one. Go ahead and get that worked on and press play when you're ready to check your work. All right. Let's see how you did on this one. Get my highlighter out here. We have the surface area of a sphere and we're told that it's 64 times as large as its original surface area. Then we're asked to figure out if the following statement is true, that the radius must be four times as large. All right. Let's do the work to figure that out. In this case, we're told, we're asked to figure out if it's true if the radius is actually four times as large if the surface area is 64 times as large. Let's figure that out.

We know that our radius is now four times what it was originally. Let's figure out how that change affects the surface area. Now, for our expression here, four pi r squared, we have four pi, and we have the product four r squared. Let's clean this up. This is going to be four times pi. Four r, that product squared would be 16 r squared. Let's keep cleaning this up. Four times 16, that's 64 pi r squared. This expression would represent the new surface area. Now what we need to figure out is this expression 64 times as large as the original. You may see it right now, but just in case, let's go ahead and divide here. 64 pi r squared divided by four pi r squared. Let's see what cancels out. Our r squared's cancel and our pi's cancel. We're just left with 64 divided by four. That is 16. What that means is that the statement was actually false.

If we scroll back up to the top to see what that means here, that means if the surface are of this new sphere is 64 times as large as it was originally, that does not mean that the radius is four times as large. There had to be a different change. It wasn't four times the length that it was originally. This statement, as we see when we scroll down through our work here, that statement was false. All right. Good job on that one. All right guys, you've reached the conclusion of this topic on seeing how change s in one dimension affect the perimeter, area, and volume, and vice versa. I hope you saw how your knowledge of solids helped you get through this topic. Bye.