Module 2: Angles Formed by a Transversal Intersecting Parallel Lines

Topic 1 Content: Angles Formed by a Transversal Intersecting Parallel Lines Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on angles formed by transversal intersecting parallel lines. Some of those angle relationships that you learned back in middle school are going to find their way back into this geometry lesson. You ready to get started? Let's go.

Let's think back a bit to some angle pairs that you learned about in middle school. You may or may not remember, but we're going to review them. Vertical angles. Vertical angles are pairs of angles whose sides form opposite rays. In this diagram, for example, angle 1 and angle 4 are vertical angles. I can trace over them a bit to let you know what I mean by their sides form opposite rays. Here's the sides of angle 1, and then here's the sides of angle 4. You see how their sides are opposite rays? The same goes for angle 2 and angle 3. That's another pair of vertical angles.

Then, if you remember, vertical angles are congruent. Let me erase this so I can talk about what I mean by *congruent*. *Congruent* means that they have the same measure. We're not given angle measures in this problem, but I know because angle 1 and angle 4 are a vertical pair that their measures are going to be congruent. If you remember, we mark congruent angles by arcs like that. Then, again, because angle 2 and angle 3 are another pair of vertical angles, their measures are also congruent. I could show that by using two arcs, and I'd use two to differentiate them from the other angle pair. I'd use two arcs for one, and then one arc for each of those, just to show that they're not all for the same measure, but that angle 1 and angle 4 are congruent, their measures are the same, and the measures of angle 2 and angle 3 are the same.

That'll jog your memory a little bit, hopefully, about vertical angles. Now linear pairs. If you remember, those are adjacent supplementary angles. What I mean by *adjacent* is that they share a side and they share a vertex. What I mean by *supplementary* is that their angle measures sum to 180 degrees. I'm actually going to use this little rectangle or square, sort of curved square shape to show you some of these angle pairs in this diagram. Angle 1 and angle 2, that's a linear pair right there. You see how they share this side right here and they share that vertex right in between them and they form a straight angle. Their sum is 180 degrees. Angle 1 and angle 2, that's a linear pair. I'm going to turn this a little bit to show you another pair, angle 2 and angle 4. That's another linear pair. See, again, it's rotated a little bit if you compare it to angle 1 and angle 2, but these angles are supplementary, they form a straight angle, their sum is 180 degrees, and they're adjacent, they share the side and they share a vertex.

In this diagram we have another, actually two more pairs of linear pairs. Angle 4 and angle 3, there's another linear pair. Again, same relationship. They are adjacent supplementary angles. Then, lastly, angle 1 and angle 3. That's another linear pair right there, another pair of adjacent supplementary angles. Again, I kept saying, "supplementary." Their sum is 180. Straight angle. That means that those linear pairs together each sum to 180. These pair of

angles would add up to equal 180 degrees, and then so on and so forth as you traveled around that diagram.

What we're going to do in this topic is use our knowledge of linear pairs and supplementaries to find some other angle pair relationships that occur when we have parallel lines intersected by a transversal. Keep this diagram in mind. I want you to think about what would happen if I sketched a line parallel to line *C*. Let me show you what happens. You essentially, to use our technology terms, copy and paste that same group of four angles that was formed by the intersection of *H* and *C* to the intersection of *H* and *D*. What I mean by that is *H* is called our transversal. A transversal, in this case it's just a line that intersects some other lines, crosses a path. Then I have my lines *C* and *D*. This symbol you may not be familiar with. It almost looks like an arrowhead. That symbol means parallel. When you see that symbol on a pair of lines, that means that that pair of lines are parallel.

What happened here is I had four angles that were created by the intersection of *H* and *C*, and when I drew a line parallel to line *C*, that same group of four angles has a matching pair that it corresponds to down here. What that means is that angle 1 up top corresponds to ... I'm calling it angle 1 right now, but we're going to change its name. Just so you understand what I'm saying. It corresponds right here. Their positions match. Angle 1 is in the top left here, and it's going to have a matching measure to the angle in the top left down here, the top left of that group of four. Angle 2 is in the top right. It has a matching pair that's in the top right of that bottom group of four, and so on. The same for here and here, bottom left/bottom left, and bottom right/bottom right. We call those corresponding angles because their positions match each other. They're in the same positions, except for one's in the top group and one's in the bottom group.

There are several angle pairs and several different relationships that develop from that intersection of the transversal through those parallel lines. One of the angle relationships that comes into play is that if two parallel lines ... Again I threw another parallel symbol in there. I'm trying to get you comfortable with the symbols. There are a lot in geometry. If two parallel lines are intersected by a transversal, then corresponding angles are congruent. Another symbol. That symbol means congruent. The corresponding angles, like I said on the last slide, those are the angles that are in the matching position of another angle. For example, let me get my pen out here, angle 1 is in the top left of this group of four, and angle 5 is in the top left of this group of four. Those angles are corresponding angles.

Because line *C* and line *D* are parallel, they're also congruent to each other, so they're going to have the same measure. Angle 1's going to be congruent to angle 5 because they're corresponding angles formed by parallel lines intersected by a transversal. 2 and 6, another pair of congruent corresponding angles. 3 and 7, see, bottom left/bottom left. Another pair. Finally, 4 and 8, your last pair of corresponding angles. Again, if two parallel lines are

intersected by a transversal, your corresponding angles are going to be congruent. For example, that means if angle 1 was 100 degrees, angle 5 would also be 100 degrees. Their measures are going to be the same.

Another pair, alternate interior angles. If parallel lines are intersected by a transversal, your alternate interior angles are congruent. Let's break apart alternate interior. *Interior*, that means inside of something. Think about this diagram as having an interior and an exterior part. Your interior would be right in here on the inside of those parallel lines. Your exterior ... I'm going to switch my highlighting color here if it'll let me. It's not going to let me I don't think, so I'll switch ink. Your exterior is going to be out here. That's your exterior. That's your exterior. If you thought about it almost like a sandwich, your interior is like the meat. If this were a burger, that's where the meat, the patty is, right there. Your exterior is like the bun. The angle's up here. Angle 1 and angle 2 would be in the exterior, and so would 7 and 8. Your interior are these group of four angles right here, 3 and 4, 5 and 6.

Alternate interior angles. Alternate. In this case, we're switching. We're switching positions along the transversal, switching sides. Alternate interior. Angle 3 and angle 6, that would be a pair of alternate interior angles that are congruent. Angle 4 and angle 5, another pair of alternate interior angles that are congruent. Again, if parallel lines are intersected by a transversal, your alternate interior angles are congruent.

Next pair, alternate exterior angles. Again, if you have two parallel lines intersected by a transversal, your alternate exterior angles are congruent. We saw from that last diagram that our interior was this group of four angles right in the center: 3, 4, 5, and 6. They're sandwiched in between our parallel lines. Our exterior angles are up top, here 1 and 2, and then at the bottom, 7 and 8. Our pairs of alternate exterior angles in this case, 1 and 8, that's a pair of alternate exterior angles, and 2 and 7, another pair of alternate exterior angles. In this case, if you have parallel lines intersected by a transversal, those alternate exterior angles, the pairs, they're congruent. Angle 1 is congruent to angle 8, and angle 2 is congruent to angle 7.

A couple more. Now consecutive interior angles. Where *alternate* meant "to switch sides," *consecutive* means "to stay on the same side." In fact, they're sometimes called same side interior angles. What that means is that we're going to stay in the interior of the parallel lines, so right in here, but this time we're going to stay along the same side of the transversal. Angle 3 and angle 5, that's a pair of consecutive interior angles, and their relationship is that they're supplementary. They're not congruent. I'm going to abbreviate supplementary. Their sum is 180. The sum of their measures will always be 180 degrees. Then angle 4 and angle 6, that's another pair of consecutive interior angles. If you have two parallel lines intersected by a transversal, your consecutive interior angle pairs are going to be supplementary.

The last pair, consecutive exterior angles. Again, consecutive, sometimes called same side, so you could think about this as the same side exterior angles. 1 and 7, and 2 and 8. Angle 1 and angle 7 are going to be supplementary, and angle 2 and angle 8 are going to be supplementary. Again, that's only true if you have two parallel lines that have been intersected by a transversal.

Now let's use this knowledge and complete a couple of examples here. Let's get this out of our way. I see from my diagram that I have two parallel lines, because we see that looks like a little arrowhead that we talked about before. If the measure of angle 4 ... Again I'm throwing another symbol at you. When you see that *m* followed by the angle symbol, it just means "measure of angle" and then whatever angle name follows it. If the measure of angle 4 equals 150 degrees, then the measure of angle 6 equals ... That's what we have to figure out.

Let's figure out what angle pair we have here. 4 and 6. I see they are in the interior of my parallel lines and they are on the same side of my transversal, so I'm just going to make a note to myself: interior, same side. These are consecutive interior angles. I know that if I have two parallel lines, like I do, intersected by a transversal, that my consecutive interior angles should be supplementary. If angle 4 is 150 degrees, then 180 minus 150 = 30. I've gotten my answer. The measure of angle 6 has to be 30 degrees. To justify that ... Because you're a lot of times, more times than not, asked to justify your answer by some definition, some fact that you know from geometry. If two parallel lines are intersected by a transversal, then the consecutive interior angles are supplementary. That's the logic that I used to figure out that the measure of angle 6 has to be 30 degrees.

Let's try another one. If the measure of angle 2 is 71 degrees, then the measure of angle 7 equals ... That's what we have to figure out. You remember in that last problem I started out by figuring out what angle pair I was working with. Let's see what we have here. I have 2 and 7. I'm just going to circle those in red, make them stand out. I notice here I'm on the exterior of my parallel lines and I'm on opposite sides of that transversal. Exterior and I switch sides along the transversal. That means these are alternate exterior angles.

What I know is, because this transversal's intersecting parallel lines, that these alternate exterior angles are congruent. If the measure of angle 2 is 71 degrees, then the measure of angle 7 also has to be 71 degrees. Then we justified right along the way, but just to show you, because you know we're probably going to have you do that when you do the one you do on your own, if two parallel lines are intersected by a transversal, then alternate exterior angles are congruent. That's why the measures of angle 2 and angle 7 had to be the same.

One more together. If the measure of angle 1 is 100 degrees, then the measure of angle 5 equals ... Let's see. Remember, first, figure out which angle pair you're working with here. I'm going to circle them, 1 and 5. Now notice here how 1 is on the exterior and 5 is on the

interior. When you end up with an angle pair like that, double check their positions. See if their positions match, if their positions correspond to one another. I see in this top group angle 1 is the top left angle in this group of four, and in the bottom group angle 5 is the top left angle in this group of 4. Their positions do match, so that means that they are corresponding angles. Abbreviate that. Then remember what you learned. If parallel lines are intersected by a transversal, then corresponding angles are congruent. If the measure of angle 1 is 100 degrees, then the measure of angle 5 also has to be 100 degrees. Then down here we just wrote out that justification just so you could see it written out.

I do believe that it's your turn. I'm actually going to give you a diagram. Some of the text is in blue and the blue text actually has errors, so I want you to use the knowledge that you just acquired from going through the concepts in this topic and I want you to correct the errors in those statements. Press pause and take your time and fix the errors in the blue text.

Let's see the corrections that you made. I see I have a diagram, two parallel lines intersected by transversal *J*. This says, "If the measure of angle 3 equals 120 degrees, then the measure of angle 8 equals 120 degrees." It's 8 that's in blue, so that's the error. Let's see. Angle 3 and angle 8. Well, those are on the exterior of the parallel lines and they're on the same side of the transversal, so they're not alternate interior angles like it says down here. Those are actually consecutive exterior angles. Let me write that, make sure I've got enough room here to show it to you. Those are consecutive exterior angles, and they should actually be supplementary. Get the pen here. This thing has a mind of its own sometimes. Let's get rid of some of that. Here we go. Those are consecutive exterior angles, and they are supplementary, they're not congruent. If angle 1 or angle 3 is 120 degrees, that means that angle 8 should be 60 degrees, because 180 minus 120 is 60. That is the correction for that error right there.

I believe we have reached the conclusion of this topic in geometry. I hope you saw how your prior knowledge of angle pairs, those vertical angle pairs and linear pairs, how that knowledge helped you further your understanding of angles formed by a transversal intersecting parallel lines. Bye.