Module 2: Angles Formed by a Transversal Intersecting Parallel Lines
Topic 3 Content: Using Angle Relationships to Prove Lines Are Parallel Transcript

Hey, guys. Welcome to Geometry. In this topic, you're going to use angle relationships to prove lines are parallel. Get ready to apply your logic and reasoning skills, and your knowledge of angle relationships, to get through this topic. You ready to get started? Let's go.

Okay, so just to jog your memory a bit, take a look at this diagram. We're given that if *c* is parallel to *d*, then angle 1 is congruent to angle 2. We know that to be true right? Because angle 1 and angle 2 are corresponding angles and if *c* is parallel to *d* then we know that our corresponding angles are congruent. In this topic, you're going to spend time going the opposite direction with that frame of reasoning. You're going to be doing things such as, if angle 1 is congruent to angle 2, then line *c* is parallel to line *d*. We're going to use that knowledge that we have of angle relationships to prove that we've actually been given parallel lines. We're going to attack that in a few different formats.

The first way is going to be based off of a diagram. Here we're given a diagram and a few angle measures within it. What value of *y* will guarantee line *m* is parallel to line *n*? Notice again, I keep throwing those symbols at you for geometry. It didn't say "line" here, but how I knew I had a line, whenever you're given a lowercase cursive letter that's a sign that you have a line, okay? You can also be given two capital printed letters and know that you have a line as long as that line symbol's up top. In this case, two lower case cursive letters, I'm dealing with lines here. Okay?

Here I have to figure out what does *y* have to be in order to guarantee that line *m* is parallel to line *n*. Now you're given four lines here so it'd probably help if you traced over or highlighted lines *m* and *n* because that'll just help you focus in on the lines that you're actually working with to really solve this problem here. Line *m*, going to highlight that, and line *n*. I've been asked to find a value for *y*. I notice *y* is up here at the intersection of line *c* and line *n*, so I'm going to go ahead and trace line *c* right here. Even though I've been given a lot of different lines, I'm going to focus my attention on lines *m* and *n* and my line up here, line *c*.

I'm really going to not pay any attention to what's going on in the lower part of this diagram. All my attention is focused right up top. In fact, I'm going to circle it to really help us just focus in right there. I see up here the only angle measure I've been given is this one right here at 120 degrees. I need to figure out some relationship between this 120 degree angle and angle *y*, or the angle that has *y* right there as the variable. I see 120, I'm going to circle it, I see *y*. Okay, and I'm focusing in here. I notice that I'm in the interior of these parallel lines and I'm on opposite sides of my transversal. A pair of angles in the interior and opposite sides of the transversal, I'm working with a pair of alternate interior angles. We know that if I have alternate interior angles that are congruent, that means that I have a pair of lines, parallel lines that have been intersected by a transversal. If this angle measure here is 120 degrees, *y* is going to also need to be 120 degrees. Okay? Because what that will do is force those alternate interior angles to be congruent. *y* is going to have to equal 120 and my justification, if alternate interior angles are congruent, my transversal, or in this case line *c* intersected parallel lines *m* and *n*. Okay?

Alright, so apply that logic and I want you to try this one. Here I want you to figure out what value of *x* will guarantee that line *c* is parallel to line *d*. Remember our strategies, we highlighted the lines that we were really focusing on, figured out what angled pair we were working with and then made out justification and answered the question. Okay? Go ahead and press pause and take a few minutes to work through that.

Alright, let's see how you did. Going to switch to my highlighter and I'm focusing on lines *c* and *d* this time. I'm trying to find a value for *x*. I notice *x* is up here at the top at the intersection of *c* and *n*. Here we go. I'm going to keep working through this. Let's see, let's try to find a relationship between and let's even circle that really we're just focusing right in here. Let's try to find a relationship between the 60 degree angle and *x*. Alright, so I'm going to circle those angles just to help me focus in on them.

x is in the exterior, that 60 degree angles in the interior so let's check their positions and see if they match. Angle *x* is the top right angle of this group of four. The 60-degree angle is in the top right of this group of four. That means I have a pair of corresponding angles. We know that if corresponding angles are congruent, our transversal intersected parallel lines. If this angle is 60 degrees that means *x* is also going to have to be 60, or that angle's going to have to be 60 degrees. To answer this question, *x* is going to have to equal 60 and our justification, if corresponding angles are congruent then the transversal intersected a pair of parallel lines. Okay? Good job on that.

Let's take a look at this one. Now geometry is famous for its two-column proofs, okay? We're going to tackle this two-column proof applying our knowledge of parallel lines and angle relationships. Okay, so let's read through it first. Given that line *t* is parallel to line *p*, prove that angle 4 and angle 6 are supplementary. We're given a statement of given information. We're given something to prove. We have our diagram and then we have this table. The table here that we have is giving a statement, we actually need to just fill in the reasons, which are going to be the justifications for each of these statements.

It's a two-column proof but with a little help because you've actually been given the statements you don't have to come up with those as well. Alright, so let me make sure I've got the blue pen here. Alright, so the first statement is that line *t* is parallel to line *p*. How did we know that? That was given information. The reason we know that is because it's given. The second statement, angle 4 and angle 8 are supplementary. Take a look at your diagram. Angle 4 and angle 8, they form a linear pair. We know that linear pairs are supplementary. The reason that you know that angle 4 and angle 8 are supplementary is because linear pairs, I'm going to abbreviate this, are supplementary. That's how you can justify that statement.

Alright so let's keep moving through here. Angle 6 is congruent to angle 8. Take a look at your diagram. Angle 6 and angle 8, that's a pair of corresponding angles. How do you know they're congruent? You were told that *t* was parallel to *p*, right? If 2 lines are parallel and they're intersected by a transversal, you know that their corresponding angles have to be congruent. That's the reason that you can justify this statement. We'll just abbreviate that.

We'll say *t* is parallel to *p* because our geometry, or symbolic notation, then corresponding angles are congruent. Alright, that's how we know that angle 6 is congruent to angle 8. Alright, now let's take a look at the last statement. Now when you're working with a two-column proof, the first statement is generally what you've been given, it always starts like that, the given information. The last statement is what you've been asked to prove, because you're done when you've proven it. We've reached the end of this statement or the end of this proof which means that this is going to be our very last justification where it ties in everything together. A lot of times when you get to the last line, it's a good idea to go back and read through your proof so you can use your logic and your reasoning skills to really figure out what that last block should be, that last space.

Let's see. Angle 4 and angle 6 are supplementary, so I'm going to read through everything. I started out with the given statement, okay. Angle 4 and angle 8 were supplementary so I knew these were supplementary, linear pair. 6 is congruent to 8 and then I'm given that angle 4 angle 6 are supplementary. Alright, now look at this, it may not be so clear. I'm going to do a little color coding here. Angle 4, I'm going to circle every angle 4 in red. I'm going to circle every angle 8 in black. Then you notice how it says that angle 6 is congruent to 8 right here? I'm also going to circle that in black. Notice, angle 4 and angle 8 are supplementary, angle 6 and angle 8 are the same thing, so angle 4 and angle 6 I could also say are supplementary.

I've essentially just swapped out angle 8 for angle 6. I've dropped in angle 6 in angle 8's place because they have the same measure. Now that's actually a throwback to Algebra I, that's the substitution property. Okay, so angle 4 and angle 8 are supplementary. Angle 6 is congruent to angle 8, so then I can say angle 4 and angle 6 are supplementary. Okay? Alright, so you've worked your way through this proof with me, see if you can try this next one on your own. Go ahead and press pause, take your time and work your way through this proof. When you're ready to check your answers go ahead and press play.

Alright, let's see how you did with this. The first thing I'm going to read through, given line *m* is parallel to line *n*, we're going to prove that angle 5 and angle 3 are supplementary. The first statement, line *m* is parallel to line *n*, that's given. Angle 2 and angle 3 are supplementary, so let me go to the diagram. Alright well I'm told that line *m* and *n*, those lines are parallel, so that means that my consecutive interior angles are going to have to be supplementary. That's the reason that I can justify this statement. I'm just going to abbreviate that. Line *m* is parallel to line *n*, so if that's true, then consecutive interior angles supplementary. Okay, so I've got that.

Angle 5 and angle 2 are congruent. Alright, so let's look back at the diagram. Okay, so I notice that angle 5 and angle 2 are a vertical angle pair and I know that vertical angles have to be congruent. That's my reason for this one. Vertical angles congruent. Okay my last statement angle 5 and angle 3 are supplementary. Remember when we reach the last statement, it's a good idea to go back and read through your other statements to tie everything together. We were given the lines were parallel *m* and *n*. 2 and 3 were supplementary, those 2 angles. Angle 5 and angle 2 were congruent. Then I could say that angle 5 and angle 3 were supplementary. Did you notice what happened on this one? Similar to what happened on our other example. We do a little color coding to make it standout for you.

Angle 2 we'll circle that in red. 5 is the same as 2, so I'm also going to circle 5 in red. Then I'll circle 3 in black. Angle 2 and angle 3 were supplementary. Angle 5 is congruent to angle 2 so angle 5 and angle 3 are supplementary. You see all we did was switch the places of angle 5 and angle 2 and we could do that because they have the same measure. Again, we use the substitution property. Alright? Okay, good job on that proof.

Before we leave this topic, we're going to try some algebraic problems. Okay? Let's take a look at this next one. Let me move this out of our way. What value of x will guarantee that line c is parallel to line d? Okay, so I see I have my diagram here. I need to figure out what angle pair I'm actually working with here. I notice these angles are in the interior of the line that I'm trying to prove parallel and they're on the same side of the transversal. These are actually consecutive interior angles. Knowing what you do about that angle pair, you know that if you can prove that those angles are supplementary then you'll know that those lines, lines c and d are parallel. We need to figure out what value of x will guarantee that these angles are supplementary. Okay?

To do that we're going to set up an equation to figure that out. We know that this angle measure, plus this one needs to add up to equal 180. Going to switch color here. $8x + 80 + 12x - 40$ should equal 180. Right? That's what we're trying to prove here. Alright, so let's apply our algebra skills, let's combine like terms $8x + 12x$ that's $20x$. $80 - 40$ that's 40 equals 180. Now let's subtract 40 from both sides, and I'm running into this let me just make sure I've got enough room here. Get that out of my way, okay. Cancel out $20x = 140$. Last step, divide by 20 and let's just move this up here so I have room to write my final answer. Move that work up. There we go. Get the pen back. 140 divided by 20 is 7. $x = 7$. That's the value of x that will guarantee line c is parallel to line d. Okay?

Now I want you to apply your algebra skills and tackle this problem. Go ahead and press pause, and take a few minutes to work it out. When you're ready to check your answer, press play.

Alright, so let's take a look and let's see what you got for this one. We want to find the value of x that will guarantee that line y is parallel to line e. Let's first figure out what angle pair we're working with here. I notice I have an angle on the interior and angle on the exterior. When that happens check the positions of your angle, see if they're in matching positions. I see here, this angle's in the bottom right of this group of 4 and I have angle here on the bottom right of this group of 4. Their positions match that means I have a pair of corresponding angles. We know that if corresponding angles are congruent, then our transversal intersects parallel lines. What I need to prove here, I'm going to get a little more workspace. Let's group this together and just make a little bit smaller just to get it out of my way. Okay.

I know that if I can prove that these angles are congruent, these corresponding angles that I can show that these lines are parallel. I'm going to set up the equation $7x + 15 = 3x + 35$. Solve this for *x.* I'll start by subtracting 3*x* from each side so I have 4*x* plus 15 equals and that cancels out, bring down my 35. Now continue subtracting 15 from each side and I see we're going to run out of space all this algebra here. Let's get some of this a little off to the side and bring that up a little bit. Get that out of our way, get our pen back. Alright, back to this equation. Okay, so I'll bring down my 4*x* that cancels there. 35 − 15 is 20. Then the final step, divide each side by 4. Our 4's cancel out on the left. 20 divided by 4, write that over here is 5. If $x = 5$ that will guarantee that line *y* is parallel to line *e*. Okay? Good job on that.

Alright, you've reached the conclusion of this topic on using angle relationships to prove lines parallel. I hope you saw how your logic and reasoning skills helped you get through two-column proofs and proofs using diagrams. Bye.