Module 3: Lines

Topic 1: The Distance Formula Transcript

Hi guys. Welcome to Geometry. This topic's going to focus on the distance formula. Your knowledge of the coordinate plane and the Pythagorean theorem is going to come in handy during this topic. You ready to get started? Let's go.

Okay, before I show you exactly how to use the distance formula, I want to show you how to derive it, and that's actually where the Pythagorean Theorem comes into play. Now, we us the distance formula to determine the distance between 2 points, on a coordinate plane. Let's use ... we have just a right quadrant here that we're going to use just to show you how to derive the distance formula. I want to keep everything general, so I'm just going to call this point, this location on the axis, X-sub1, because that's just going to represent any X coordinate, and this is going to be X-sub2. Let's get a little more room here. On the Y axis I'm going to go something similar. I'm going to call this Y-sub1, and this location Y-sub2.

If I had a coordinate or if I have a segment that was horizontal, say something like this, if I wanted to find the distance between these 2 points. I'm actually going to connect them and make a line segment. If I wanted to find the distance between these 2 points, I could really just count, right, because I'm going a horizontal distance. Now, let's say I had a vertical segment. Let's get rid of this. Let's say I wanted to find the distance between this point and this point. I'm going to connect those again. Free-handed it, so get as straight as I can. If I wanted to find the distance between those 2 points, again I could just count.

Now, it becomes a little tricky if your segment isn't horizontal or vertical, if it's diagonal. Say for example between these 2 points. I'll put a point right here on the coordinate plane, and a point right here, and I'm going to connect it. There we go, pretty straight line segment there. Now, if you wanted to find the distance between 2 points and they formed a line segment that was diagonal like this one, you can't actually count diagonally. You can't count in the same way that you could count horizontally or vertically. That's where the Pythagorean theorem comes into play, to help us figure out exactly how we could figure out the distance between these 2 points. How we think about it is, instead of just thinking about it as a little old line segment, we're going to actually think about it as the hypotenuse of a right triangle, so watch me do this. I'm going to drop a vertical segment, I'm going to drop a horizontal, and now I have a right triangle. Now, I do know how to find the length of the hypotenuse of a right triangle, that's the Pythagorean theorem.

Right off to the side, just to jog your memory, the Pythagorean theorem is C squared equals A squared plus B squared. Remember C is your hypotenuse, so in this case that'd be the blue line segment, and A and B represent your legs. It doesn't matter if you reference the horizontal leg as A and the vertical one as b, or vice versa. It doesn't make a difference as long as you know that C is the hypotenuse, and A and B are your legs. Keeping the Pythagorean theorem in mind to find the length of this blue line segment here ... Let's switch to ... Okay, we've got the black ink here. If I wanted to find the length horizontally of this leg, I would need to subtract those 2 coordinates from each other. I'd say, all right, the length of this line segment here is represented by X-sub2 minus X-sub1, and the length of this vertical segment here would be represented by Y-sub2 minus Y-sub1. I'm going to call the hypotenuse here, the blue line segment, I'm just going to call that D, because I'm trying to find the distance between those 2 points, okay?

Now, keeping the Pythagorean theorem in mind, and now that we have some values on this right triangle, scroll down a little bit just to get a little space here, and let's extend the page because we are going to need some more work space. Okay, I'm going to be moving back and forth for a little bit, so just bear with me. The Pythagorean theorem, I'm actually going to copy that underneath our sketch here on the coordinate plane so that you can keep that in mind. We're working off of, let's get the pen back, C squared equals A squared plus B squared. Now bouncing back to that right triangle, the hypotenuse is D in the case of our example here, so I'm going to replace that C squared with D squared. We said that A and B were our legs, right? I'm going to tall this one A, so I need the length of this side here, and I need to square that. Let's get that back here. Just a couple more scrolls and then we'll be done. I don't need the A. All right, get that pen back. X-sub2 minus X-sub1, squared. We're putting that in the place of a squared. Then B squared we said is our other leg, so we're going to square this length right here. Down here where we're working off of the Pythagorean theorem, we'll have plus Y-sub2 minus Y-sub1, squared.

This so far is actually letting me know the length of the hypotenuse squared. If I actually want to know just the length of the hypotenuse, you have to use your algebra skills, I need to take the square root of both sides of this equation. The square root of D squared equals the square root of that quantity. All right, so now the square root of D squared, that's just D, so that distance equals the square root of X-sub2 minus X-sub1, squared, plus Y-sub2 minus Y-sub1, squared. That is actually the distance formula. We use this formula, again, if we're trying to find the distance between 2 points on the coordinate plane. Like I said, if it's a horizontal segment or a vertical segment, you can kind of take that shortcut and just count, but if it's a diagonal segment, you're going to want to use the distance formula. Or if you have coordinates that are really large and it would be a really tedious process to count, you're going to want to use the distance formula, okay?

Let's take a look at this first example. Actually, let me just formally show you the distance formula before we do that. Given 2 points, any 2 points, C and D, this is another notation. I know geometry is full of notations. If you ever just see 2 capital letters next to each other with no line segment above it, or no line symbol above it, you're being asked to find the distance between those 2 points, or you can think about it as that you're finding the length of that line segment. In this case, given points C and D, here's our distance formula. Now, this is a formula that you want to memorize, okay? It's a good idea that every time you work a problem that requires you to use the distance formula, go ahead and write it out so you can really start to commit it to memory.

All right, now let's get to that example. We're given 2 points, A at (-3,6), and B at (4,2), and we're going to use the distance formula to find AB. It's telling me to round to the nearest tenth, so that's a signal to me that I'm probably not going to end up with a number that's a perfect square at the end of this problem. The first thing I'm going to do to start it is copy down my points. I have (-3,6) and (4,2). Before I dive into using the distance formula, I'm going to go ahead and label these coordinates. I'll call this X- sub1, Y-sub1, and X-sub2, Y-sub2. Now if I had reversed those and called the first point X-sub2, Y-sub2 and the second one X-sub1, Y-sub1, it couldn't have made any difference in the answer that I'll end up at the end of this problem, so don't feel like you have to use this order all the time. Just make sure you have your X-sub1, Y-sub1 together for one ordered pair, and then X-sub2, Y-sub2.

Now that my points are labeled, I'm going to go ahead and write the distance formula, because remember that's something we want to try to memorize as soon as we can. Because I know I have points A and B, and I'm trying to find the length of AB, that segment, I'm going to go ahead and just call it AB equals the square root of X-sub2 minus X-sub1, squared, plus Y-sub2 minus Y-sub1, squared. Now that I have that written down, I'm going to go ahead and substitute my coordinates into the formula.

Let's get a little more space. Again, square root. X-sub2 is 4, and X-sub1 is -3, so 4 minus -3, squared plus Y-sub2 is 2, and Y-sub1 is 6, so 2 minus 6, squared. Okay, let's get some more space. Scroll a little bit so we can still see that part.

Okay, now let's start to simplify this expression. The first thing that jumps out at me is that I have a double negative here in this first group, so I'm going to go ahead and clean that up first. Now that will become a plus. Now I'm going to go ahead and continue to simplify. I've got the square root. 4 plus 3 is 7, so this is 7 squared. 2 minus 6 is -4, so this is -4 squared. Again, I'm going to continue simplifying. I'm not taking any shortcuts right now and writing out all the steps, but if you start to feel comfortable and feel like you can skip a step here or there with doing the calculations, that's fine. Just make sure you keep everything accurate.

Now I have the square root. 7 squared is 49, and negative 4 squared is 16. Remember, even though you're squaring a negative number, a negative times a negative is a positive, so -4 times -4, positive 16. Now I have to add 49 plus 16. Scroll a little bit more here. 49 plus 16, that's 65, so the square root of 65.

If I was asked to give an exact answer to this problem, I'd stop right here and tell you the length of segment AB is the square root of 65, but because I was asked to round to the nearest tenth, I'm going to go to the calculator, and I'm going to approximate the value of this square root. I'm going to switch to my calculator. I've already cleared my memory, so I'm going to go ahead and just start the calculation. I need the square root of 65, and that is approximately, if I'm rounding to the nearest tenth, that's about 8.1, okay? I'm going to go back to my work. Let's go back full screen here. The square root of 65, get a little more space, that is approximately 8.1, and you're all done with this problem. Just to make it formal, because you want to in geometry attach units to something, to lengths anytime you can, I would say that AB ... I wasn't told if it was inches, or that on the coordinate plane this represented meters, or centimeters, or anything like that, so I'll say AB is approximately 8.1 units, and I'm all done.

All right, now keep that in mind, that process that we just did, and I want you to try this one. Press pause, look back at the other example again even if you need to do that, and take a few minutes to work through this problem. When you're ready to check your work, press play.

All right, let's see how you did here. The first thing I'm going to do, I'm going to copy down my points, (5,7), (11,-1), and then I'm going to label my coordinates, so X-sub1, Y-sub1, X-sub2, Y-sub2. Now that I have that, I'm going to go ahead and write the distance formula. All right, scroll down here. FT equals square root of X-sub2 minus X-sub1, squared, plus Y-sub2 minus Y-sub1, squared. Now I'm going to substitute my coordinates into the formula. X-sub2 is 11 and X-sub1 is 5, so 11 minus, squared plus Y- sub2 is -1, and Y-sub1 is 7, so -1 minus 7, squared. I'm going to keep scrolling and get all the work in here. Okay, so I have 11 minus 5, that's 6, so 6 squared, and -1 minus 7, that's -8, so -8 squared.

Now I'm going to keep simplifying. 6 squared, that's 36, and -8 squared, that's 64, so now I'm just left with these 2 values to add together and then take the square root. Some more root. Okay, 36 plus 64 is 100, and I'm remembering I wasn't told to round to the nearest tenth, or hundredth, or anything like that in this problem, so I know I'm going to get a perfect square value at the end. Unless you had been told to just give an exact answer, remember if you end up with a number that's not a perfect square, go ahead and round to some value. The square root of 100, that is 10, and you're all done, so FT equals 10 units. All right, good job on that.

Now we've reached the conclusion of this lesson on the distance formula. I hope you saw how we could use the Pythagorean theorem to derive the distance formula, and how that formula helps us find the length of segments on the coordinate plane. Bye.