Module 4: Symmetry and Transformations
Topic 3 Content: Rotations Transcript

Hi guys. Welcome to geometry. This topic's going to focus on rotations. Your knowledge of transformations and the coordinate plane is going to come in handy during this lesson. You ready to get started? Let's go. To review a little bit, because I know you've heard about rotations in middle school and probably a little bit in elementary school. A rotation is a turn. The original location is known as the pre-image and what's happened or where the image lands after you've performed the rotation is known as the image. A rotation is a transformation that's known as isomorphic because the size and shape of the image remain the same. It's a position of it or the location of it that changes. You've probably heard a little bit about this like I said before. What's going to make it different in geometry is we're going to perform rotations on the coordinate plane.

Take a look at this. I want to review some of the most common rotations that we're going to use so beginning here we're going to call triangle ABC our preimage where the figure's going to start. Here's a 90-degree clockwise rotation about the origin. What that means is that a turn to the figure once to the right. Here triangle ABC, the blue triangle that was my pre-image and the red triangle, you notice the apostrophes at each of the vertices or each vertex, that's a notation to let us know that a transformation occurred and this is the image and we note or we refer to this as A prime, B prime, and C prime. Triangle ABC started here and after a 90-degree clockwise rotation about the origin, when in other words, one turn to the right about the origin, our triangle will land here. When I say about the origin, what I mean is that the origin is our fixed point. If you visualize turning this triangle or turning this entire coordinate plane, the origin will always stay exactly where it is. That's your line up point right there.

Take a look at this one. Here's a 180-degree clockwise rotation about the origin. 180 degrees, that's two turns and you notice here you have two apostrophes so we would refer to those as A double prime, B double prime, C double prime. Here, this is a 270-degree clockwise rotation about the origin. That's 3 turns to the right and you notice here I have A triple prime, B triple prime, and C triple prime. Here we have 360-degree clockwise rotation about the origin and if you notice, we actually end up right where we started from. You ever heard the expression somebody say they did a 360 or in skateboarding when you do a 360 spin? That means from where you started, you completely turned around and landed back at your starting point. Same thing with the transformations in geometry.

All right now. Let's try a few examples here. What I'm going to do, I'm going to give you an image or pre-image graphed on a coordinate plane and we're going to look at the transformation that we're asked to perform and then we're going to graph the image. We're going to graph where that image lands after the rotation. All right. Here's our first one. Going to do a bit of scrolling here to make sure you can see everything, but to start off we're

going to be rotating triangle *ABC* 270 degrees counterclockwise about the origin. Before I actually scroll down and show you that triangle, let's focus on a few key thing here. We're going to do a 270-degree counterclockwise rotation. Remember counterclockwise, like a clock. Clockwise would be like we said to the right the way you would follow the numbers around on a clock. We say counterclockwise because it's going backwards direction around a clock. We know that this is a turn to the left because that would be reading the clock backwards.

Then about the origin. We know the origin is our fixed point. When we rotate this triangle, we're always going to line the coordinate plane back up at the origin. Here's our triangle and what I would advise you to do is on some graph paper, go ahead and graph this triangle and then you can actually perform the rotation with me as I do it. If you like, pause me for a little bit go ahead plot that triangle. Graph that triangle on a coordinate plane so you can perform the rotation with me. All right, now that you've got that graphed, I'm going to actually go here. I've used the magic of technology to copy and paste a little image where my triangle is and I shrunk it a little bit so I can perform the rotation. Remember 270 degrees counterclockwise. That's three turns to the left. Here we go. Here's one turn. Two turns. Three turns. There we go.

If you turn that graph paper with me remember you'd end up right here after your three turns and we have the coordinate plane, here's our origin. Now how we're going to figure out where vertex is right now is we're going to treat the coordinate plane as if we were looking at it right side up. What I mean by that, because we know we've turned it several times now, but we're going to treat it as if this were sitting right side up. *X*-axis. *Y*-axis and here would be 1, 2, 3. −1, −2, −3 and so on and so forth. Let's jot down where each vertex is right. Reading the graph as if it were right side up. *B* is −2, 1. Going to make a note of that here. *A* is (−2, 3) and *C* is at (−4, 1). These are actually the vertexes or the vertices of the image. That's my *A* prime, my *B* prime, and my *C* prime. Now what I'm going to have to do, I'm going to have to flip back and forth a little bit, is I'm going to graph my image on that original coordinate plane because this was my scratch work. Me having a little picture to rotate here on the screen.

I'm actually going to go back to my previous slide and graph these points on the coordinate plane. Let's see, *A* prime's at (−2, 3). Let's get that. (−2, 3). −2, 1, 2, 3. Here's *A* prime. Let's flip back again. *B* prime is at (−2, 1). *B* prime, (−2, 1) and let's see. *C* prime at (−4, 1). 1, 2, 3, 4, (−4, 1). There's *C* prime. Then we'll connect the dots and get our right triangle. There we go. Here's our image after 270-degree counterclockwise rotation about the origin. Good job on that. Let's try this next one. Flip through our scratch work. Here we go. Now, we're going to rotate triangle *ABC* 180 degrees about the origin then translate the triangle one unit

right. Here we actually have a combination of transformations that we have to perform. First a rotation, 180 degrees about the origin. Then a translation one unit to the right. If you remember, a translation is a slide.

Notice here, I'm going to highlight our keywords or underline them. We're rotating 180 degrees about the origin and then we're translating one unit right. It didn't tell us whether we were rotating 180 degrees clockwise or counterclockwise because either way, with a 180 degree rotation, whichever rotation you turn, you end up in the same spot. That's why when you see 180-degree rotation, you won't be given a direction because whether you turn two turns to the right or two turns to the left, you actually end up in the same spot. For this one, first thing we need to do is perform that rotation. I'm going to have to do some flipping back and forth again. I've already taken a little screen shot so that I can rotate. Let's perform 180-degree rotation and I'm going to go clockwise this time. Like I said, doesn't matter which direction you go. I'm going to go two turns to the right. Here's one turn and two turns. That's the first transformation of this combination that I've performed.

Before I do the second one, I'm going to make a note of where the triangle is right now after the rotation. Let's get the coordinates. I'm going to slide this over a little bit, the coordinates of those vertices. It's upside down, but we can handle this. A prime now would be at −1, 2, 3, 1, 2, 3. The pen back here. A prime's at (−3, −3). B prime is (−1, −1) and C prime is at (−2, −1). Before we perform that translation, let's go ahead and graph the image after that first transformation. Here I go flipping back and forth for a second. Bear with me. A's at (−3, −3) now. A prime. (−3, −3), so here's A prime. Flip again. B prime's at (−1, −1). Scroll. (−1, −1) that'd be here. Here's B prime and C prime (−2, −1). Scroll a little bit. (−2, −1). Here's C prime. There's the triangle after the first transformation, but remember you were also asked to perform a translation. You have to translate the triangle one unit to the right.

Now that we know the triangle is right here for right now, we've got to shift it. We have to slide every vertex one unit right. I'm going to switch colors for that one to make it stand out. B's right here right now. Now it will be here and we're going to notate that as B double prime to show that it was the second transformation. C's here, it'll actually be here where B prime was. There's C double prime. A's here or A prime's there I should say. Scroll a little bit. It'll be here. A double prime. After you've completely finished both of those transformations, the image of the triangle will be right here. This one in red with the double prime notations. All right. Good job with that one. Let's slide through the work here and it's your turn.

What I'm going to do similar to what we just did. I'm going to show you a pre-image. Going to ask you to perform a transformation or it might even be a compound transformation and go ahead and graph that image. Now make sure you still have that graph paper handy because you're definitely going to need it perform those rotations. Here you go. Pause and take a few minutes to work through this and when you're ready to do over it, press play. All right. Let's see how you did. You were asked to perform a 90-degree clockwise rotation about the origin and then asked to translate the triangle 4 units down. I've already a little screen shot of this one so I can perform the rotation. Let me get it in my mine 90-degree clockwise about the origin. That is one turn to the right. Here we go. One turn to the right. My triangle would be right here. Let's make a note of where those vertices are before we perform that translation. Reading the graph as if it's right side up, *A* prime would be at 1, 2, (2, 0). *B* prime's at the origin so (0, 0) and *C* prime is at (0, 3).

Let's get that graphed and then we'll go ahead and do that translation. *A* prime is at (2, 0) and *B* prime's at (0, 0). I think I can remember those at the same time. *A* prime's at (2, 0) and *B* prime is at (0, 0). That actually stayed right there. I'll trace over that. *C* prime, let's get that is at (0, 3). Let's actually let's move this down a bit. Get it out of our way so we can get *C* up there. Let's erase those lines. There's a lot of work we have to do for this one. Let's get that back and let me remember, I believe it was (0, 3). Yup. *C*'s at (0, 3), 1, 2, 3, so *C* prime. There we go. After the rotation, the first transformation, the triangle is here. You were also asked if I scroll down now to translate the triangle 4 units down. We're going to take each vertex and move it 4 units down. Now we've got to move some things around again. Let's move the directions up to get those out of our way and then I'm sure we're going to have to slide our graph a few times to get everything in view.

Every point 4 units down. Let's start with *C* or *C* prime. 1, 2, 3, 4 *C* prime will be here. *B* prime. 1, 2, 3, 4 let's move this a bit. *B* prime will be here. *A* prime 1, 2, 3, 4 and actually I need double primes right? That's my second transformation. 1, 2, 3, 4 *C* 1, 2, 3, 4 now we have *C* double prime. *B* prime to be double prime 1, 2, 3, 4. All right we got it. After that combination of transformations, your triangle should end up right here where we have *A* double prime here, *B* double prime, and then *C* double prime. All right. Good job on that. Flip through the work and here we go. We've reached the conclusion of this lesson on rotations. I hope you saw how you're knowledge of the coordinate plane and of transformations all came together for you here on this lesson. Bye.