Module 5: Triangle Relationships
Topic 1 Content: Ordering the Sides and Angles of a Triangle Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on how to order the sides and angles of a triangle. Your knowledge of triangles, of course, and angles, and line segments is going to come in handy during this lesson. You ready to get started? Let's go.

Let's take a look at this triangle, so I can warm you up for what we're going to get into in this lesson. I have theorem up at the top. When we finish analyzing this triangle the way that we're about to, we're going to actually end up at a theorem that helps us answer a lot of questions about triangles. Here we go.

Notice this triangle has sides of varying length and angles of varying size. For example, I can see this blue line segment here, *BC*. This side of this triangle is the shortest side in the triangle.

AB, this side appears to be the longest. If you could visualize it laying on top of that red side, you can see it would be longer than it.

AC, we could call that side our middle side, the side that's not the shortest. It's not the longest. It just falls somewhere in between the shortest and the longest side.

Take a look at the angles of triangle *ABC*. Notice that angle *C* is the largest angle. It opens up the widest among the three angles in this triangle. Angle *A* is the smallest one. It opens up the smallest amount. Angle *B* falls somewhere in between. It's not the smallest angle. It's not the largest angle. It just falls somewhere in between.

I purposely left off degree measures and left off line segments. I just want you to analyze this image to get to the theorem that I want to discuss.

Like we said, you notice how angle *C*, that's our largest angle in the triangle, it has a special relationship to the longest side in the triangle. It actually sits right across from it, or right opposite from it.

What I mean by that is angle *C* is opposite, or across from side *AB*. Their relationship to each other is angle *C* is the largest angle. *AB* is the longest side.

In a triangle, what that theorem is, in a triangle, the largest angle is always opposite the longest side. The converse of that is true vice versa. The longest side is always opposite the largest angle. A similar relationship is true for the shortest side and the smallest angle.

I'm going to erase that arrow just to show you what I mean by the relationship, a lot of tongue twisters here, between the shortest side and the smallest angle. Angle *A*, that's our smallest angle. It's actually opposite the shortest side in the triangle. That's the other part of our theorem.

In a triangle, the smallest angle is opposite the shortest side. The converse of that is also true. The shortest side is opposite the smallest angle. That third side and that third angle, that's in between one that we had mentioned earlier, in this case angle *B* was that in-between angle. It wasn't the smallest one. It wasn't the largest one. It was just the medium sized one, the one that fell between.

It's across from that medium-sized side, the side that's not the largest. It's not the smallest. It's just somewhere in between, right here. There you go.

That's where we're going to focus on in this lesson. The relationship, I'm having a hard time saying that today, between the sides and the angles of a triangle.

All right. Take a look at this just so you can see the theorem formally stated. Given a triangle, the longest side is opposite the largest angle; also the shortest side is opposite the smallest angle. We're going to use those two pieces of information to solve the next few examples.

Here's our first one. We're asked to list the angles of triangle *ABC* from largest to smallest. You notice here, this is where it gets interesting, is I've given you side links, but asked you to order angle measures, or asked you to order angles. You have to use that relationship, between the sides and the angles of a triangle, in order to order the angles for me in this one.

Here we go. Be mindful of the order you're asked to order those triangles. If you're asked to go from largest to smallest, or smallest to largest. Sometimes you'll be asked to order in ascending order, which means from smallest to largest; sometimes in descending order, which means from largest to smallest.

Here, largest to smallest is the way we're asked to order. I might even draw an arrow that way just to show me I'm going from the biggest one to the smallest one. All right. Here we go. The largest angle will be across from the largest side.

In my triangle here, my largest side is side *AB*, this side. This triangle isn't drawn to scale. I'm just assigning these values here. The *AB* is 13 inches. If this is my largest side, that means that angle *C* hast to be my largest angle. I'm going to list that angle first.

The next side, in order, think I'll switch colors here. Let's go to red. Just going down like we said, in size, would be *AC*. It's 8 inches. *AC* is opposite angle *B*. Going from largest to smallest, that would be the next angle in line.

Process of elimination, you know angle *A* has to be the smallest angle. It's the only one left. Just to show you where that's coming from here, I'll use black, but I'm just going to dash the line a bit just to show you a difference.

BC, 6 inches. In this triangle, it's the smallest side. It's opposite angle *A*. That's the smallest angle here. You've got it. You've answered the question. In this triangle, from the largest to smallest, their angles would be named angle *C*, angle *B*, and angle *A*. All right. Good job on that one.

Let's take a look at this one. We're asked to list the sides of triangle *XYZ*, in order from shortest to longest. We're going the opposite direction this time, shortest side to longest side. You notice here we're actually ordering sides and not angles.

Here, you may have noticed, at first, that you have measures for angle *X* and angle *Y*, but not for angle *Z*. You can't actually answer the question until you know the measure of angle *Z*. Throw back to your prior knowledge here from elementary and middle school, also.

You know the angles in a triangle sum to 180 degrees. Their measures will always add up to 180. Even though I don't know what *Z* is, I could figure it out, because I know the measures of its other two angles in this triangle. Here we go.

60 plus 70, that's 130. The two given angles add up to 130 degrees. 180 minus 130 is 50. That means angle *Z* has to measure 50 degrees. That's the only way the angles of this triangle will add up to 180 degrees. Their measures will.

Now that we have that piece of information, we can order the sides. Let's just get that work out of our way. Just get a little work space. Get the pen back. Here we go.

Remember, we're going shortest to longest. That means we need to start with the smallest angle. In this triangle, angle *Z* is our smallest angle. It's opposite from side *XY*. *XY*, or you could call it *YX*, either one's fine. That's the shortest side in this triangle.

Notice that I wrote *XY* with that line segment on top. It's another notation for geometry, which just shows you that you're talking about a line segment. In this case, it's part of a triangle.

For a line segment, you name it by its endpoints, in whichever direction you choose. Like I said, we could have called this side *XY* or *YX*. Both are acceptable. All right. Let's keep going through this one.

We know the smallest side's *XY*, because it's across from the smallest angle. The next angle measure, going up, would be angle *X*, 60 degrees. It's opposite from *ZY*. That's the next side, or *YZ*. The largest angle, angle *Y*, is opposite from *XZ*. There you go.

Notice what I was doing here to get through this problem. I'm drawing arrows to show this angle's opposite from this side. This angle's opposite from this side. If you find that confusing, or you're still having a hard time figuring out what angle's actually opposite from what side, here's another strategy that you could use. Let me erase my arrows. Let's get those out of the way.

When working through this problem, we said that angle *Z* was our smallest angle. You could actually take your hand and cover up angle *Z*. You're left with the side opposite from it. In this case, *XY*.

We said angle *X* was the next angle increasing. You could cover it up and then say, "*ZY*, that's the side that's opposite angle *X*." Finally, angle *Y*; we said that was our largest angle. You could cover it up and say, "All right. *XZ*, that's the side opposite angle *Y*."

If you don't like the arrows, or the arrows don't make it as clear as it could be for you, you can, again, just use that cover up method. All right.

Let's take a look at this one. Here we're asked to order the sides of triangle *PMN* in order from longest to shortest. Let me underline that, so I remember that. Let's get that pen here. There we go.

Notice in this triangle, that the only angle measure that I know for sure is angle *N*. I was told, directly, that the measure of angle *N* is 20 degrees. For angle *P* and angle *M*, I don't actually know what the measures of those angles are, but I do know a way to figure that out.

I know that the sum of the measures of the angles of a triangle always equals 180 degrees. I could set up an equation that would allow me to solve for *X*. Then I can use *X*, use that value to figure out what the measure of each of these angles is. Once I know that, I can go ahead and order those sides, in this case, from longest to shortest.

We're going to have to do a little bit of algebra here to work our way through it, but it's all going to fall into place. All right. Let's move a few things around here to get some room. I'm going to scroll back and forth for a while. I'm going to start by setting up my equation.

I know that these three angles, these three angle measures have to add up to equal 180 degrees. 6*x* plus 40 plus 4*x* plus 60 plus 20 equals 180. I'm going to combine like terms here. Get some more space. Go ahead and solve for *x*.

6*x* plus 4*x*, that's 10*x*. 40 plus 60 is 100 plus 20 is 120 equals 180. Then I'll subtract 120 from each side. 10*x* equals 60. Divide each side by 10. *x* equals 6. Now that I know that *x* equals 6, I can use that value to figure out what the measures of angle *P* and angle *M* actually are.

Let's scroll back up here. Bear with me for a little bit with the scrolling. All right. I'm going to move that out of our way. All right. I know that x is 6. The measure of angle *P*, 6 times 6 plus 40. I'm substituting 6 in the place of *x* since now I know that *x* equals 6. 6 times 6, that's 36 plus 40. 36 plus 40, that's 76. I know that angle *P* actually has a measure of 76 degrees.

I'm going to keep that in mind. Going to get some more workspace here. Let's get this algebra out of our way, this equation, that'll work for that. We've got what we needed there.

Like I said, on your notebook paper, feel free to keep it up there. You've got a lot more space than I do. That's why I just erase the work just to get some more workspace.

The measure of angle *M*, that would be 4 times 6 plus 60. 4 times 6 is 24 plus 60, that's 84. I know that the measure of angle *M* is 84 degrees. Now that I know that, I'm going to redraw this triangle with those measures that I know now.

We have *PNM*. *P* is 76 degrees. *M*'s 84 degrees. *N* is 20. Just look back at the directions, scrolling again. We were asked to order the sides from longest to shortest. Longest to shortest.

The largest angle in this triangle is angle *M*. It's 84 degrees. Across from angle *M*, that's side *PN*. Remember if you don't like to draw those arrows, or if it's not really clear to you which side is opposite which angle from the arrows, you can always just cover up the angle.

If angle *M* is our largest angle, cover it up. *PN*'s the longest side. Going down, 76 degrees, angle *P* would be next in line. It's opposite *MN*. If you don't like the arrow, like I said, just cover up angle *P* and you can see *MN*. Angle *N*, smallest angle and opposite from it, *PM*. That's our shortest side.

Again, if you don't like that arrow, just cover up angle *N* and you can see *PM* is the shortest side. From longest to shortest, the sides of this triangle, *PN*, *MN*, and then *PM*. I do believe it's your turn to try one. I'm actually going to have you try two. I want you to try this one first.

Press pause. Take a few minutes. Read through this. Work your way through this example. When you're ready to go over it, press play.

All right. Let's see how you did. Make sure I've got my pen. All right. You were asked to order the angles from smallest to largest. Let's find the smallest side, or the shortest side first. That will help us locate the smallest angle.

Here, our smallest side is *SR*. It's 6 cm. Like I said, this is not drawn to scale. I'm just assigning values here. Let me show you another method on this one also.

We know that *RS* is the smallest side, so I could draw that arrow and say, "All right. That means angle *T*, that's the smallest angle." You could cover up that side and see that angle *T* is opposite from it that way. Again, if you don't like drawing those arrows, cover up that side and then you see angle *T* is the angle that's opposite from it.

All right. Going in increasing order, *ST* is the next side. That's across from angle *R*. Again, I drew an arrow. If you wanted to, I'll use my pen this time, you could just cover up that side and see that *R* is the side that's opposite from it. *R* is the angle that's opposite from it. All right.

Finally, the longest side here is 12 cm. It's across from angle *S*. I drew the arrow, but you could cover it up if you wanted to. Cover up that whole side and then see the angle that you're left with. All right. That's how we got through the first one. All right.

Let's take a look at this one. I want you to read through this problem, or we can actually do that together. You're asked to find out which is the longest side of triangle *JFL*. You don't actually have to figure out the sides, all of them in order. I just want to know which one is the longest.

Use those algebra skills that we reviewed, so you could determine which is the longest side of triangle *JFL*. Press pause. Take a few minutes. Work though that. Press play when you're ready to check your answer.

All right. Let's see how you did on that one. First, I notice that I only know the measure of angle *L*. I need to figure out the measures of the other angles. In this case, it's going to take an equation to figure that out. Let's get a little room here. Let's move some things around, so we can get some space to work. Here we go. Get the pen back.

I know the measures of these angles have to add up to equal 180 degrees. x plus 70 plus $3x$ plus 30 plus 60 equals 180. I'll combine like terms. x plus $3x$, that's $4x$. 70 plus 30, that's 100 plus 60, that's 160, equals 180 degrees. Let's get some more space. All right.

Let's subtract 160 from each side. $4x$ equals 20. Divide each side by 4. x equals 5. Now that I know that x is 5, I can substitute that value into those angles in the triangle.

I'm going to scroll up. Let's move some things around here, get some more room. Move that triangle to the left. x is 5. I'll actually get rid of this work. I believe we're going to run into it if I don't. Let's get that out of the way. There we go.

We do know x is 5 from solving that already. The measure of angle *J*, 3 times 5 plus 30. That's 15 plus 30. That's 45. The measure of angle *J*, 45 degrees. Measure of angle *F*, let's get that one. All right.

The measure of angle *F* is going to be 5 plus 70. Getting ahead of myself. 5 plus 70 is 75. The measure of angle *F*, 75 degrees. I know the measures of each angle in this triangle. I'm going to redraw it with those measures in there.

F, *L*, and then *J*. *F* is 75 degrees. *J* is 45 degrees. *L* is 60 degrees. If I scroll back and look at the directions, I was asked to find the longest side of this triangle. The longest side is going to be opposite the largest angle. Angle *F*'s the largest angle. *JL* is the longest side.

Either you can tell that by drawing that arrow, or just by covering up angle *F*, that *JL* is going to be your longest side.

You've reached the conclusion of this lesson on ordering the sides and the angles of a triangle. I hope you saw how your algebra skills and your knowledge of triangles and angles, and line segments all came together to get you through this lesson. Bye.