Module 5: Triangle Relationships
Topic 3 Content: The Range of the Length of the Third Side of a Triangle
Transcript

Hi, guys. Welcome to Geometry. This topic is going to focus on the range of the length of the third side of a triangle. Yvour knowledge of the Triangle Inequality Theorem is going to help you get through this one. You ready to get started? Let's go.

Knowing what you do about the Triangle Inequality Theorem, you know that given a triangle, in this case, triangle DEF, three things are true. You know that the length DE plus DF is greater than EF. Again, I have another geometry notation here. Notice that I don't have line segments above the names of these sides. What that means is that I'm not referencing the side itself, but I'm referencing the length of the side here. What that means is, here, the length of side DE plus the length of side DF is greater than EF, if you want to say it all plainly like that.

Based on the Triangle Inequality Theorem, you know that that is true, right? The second thing you know is true is that the length of DE plus the length of EF is greater than the length of DF. Just another combination of sides there, we're taking. Then, also, you know that this is also true. That the length of EF plus the length of DF is greater than the length of DE. All we've done here is taken every combination of pairs of sides. Those lengths added together have to be greater than the third side. We know that from the Triangle Inequality Theorem.

Now, keep that in mind, and take a look at this. We're still going to be working with triangle DEF, but here I've assigned just a couple of side lengths. Here, we're going to say that side DE has a length of 2, and EF equals 5. DF is going to be the unknown side length, and then we're going to use our algebra skills to work through this, where it's going to help us get to the root of what we're going to work on in this topic. Let's move a few things out of the way here.

Take a look at this. The first inequality. DE plus DF is greater than EF. DE is 2. DF is x. Greater than EF, which is 5. I'm just replacing the notation with the values themselves. Now, solving for x here, using our algebra skills, I'd subtract 2 from each side. Let's get a little more space, going to have to scroll back and forth a little bit on this one, or up and down a little bit. Cancel out there. x is greater than 3. I don't know actually what the length of DF is, but what I do know is that whatever it is, it's larger than 3. That's telling us one thing about that side.

Let's take a look at the second inequality. DE plus EF is greater than DF. All right, I'll switch colors here, to kind of break it apart a little bit. DE is 2. EF is 5. Greater than DF, which is x. 2 plus 5, that's 7, so 7 is greater than x. Another way you could say that, is you could say,

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"Okay, x is less than 7." Those mean the same thing. Throwing back to Algebra I a little bit. The inequality sign is telling me that 7 is greater than x, so what that also means is that if 7 is greater than x, then x is less than 7. That's all that means here. Just kind of flipped it around a little bit. So far, I still don't know what DF is, but what I do know is that whatever it is, it's larger than 3, but at the same time, it's smaller than 7. It falls somewhere between those values.

Let's see what the third inequality will tell us. Scrolling a bit. EF is 5. DF is x, and DE is 2. Let's get a little work space here. Algebra skills. Subtract 5 from each side, cancel them out there. x is greater than −3. You notice we have a negative value. Not exactly a negative value assigned for x, but if x is greater than −3, that means that it's saying it could include negative values. It could include −2, −1. There could be some negative values in there. Now, because we're talking about this triangle, and the length of its side, we know that negative values cannot happen. Algebraically, the work we did is correct, but applying it to what we know about geometry, it's a solution that we have to throw away. We're calling it an extraneous solution, because algebraically, it makes complete sense.

We solved that inequality correctly, but because that inequality is connected to this triangle, and describing a length of a side, we know negative values are not going to work. We're going to throw away that last solution, and we're going to focus our attention on these two descriptors, which are telling us that this side falls somewhere between 3 and 7. It's greater than 3. It's smaller than 7, and that's really all I know right now. What I could do, actually, let me get a little more room here, is I could write this a couple of ways. I could say, "All right, I don't know what x is, but I know that it's greater than 3, and I know that it's smaller than 7."

You could write it like that, but how you're probably going to see it more often is written as a compound inequality. What that means is, you'll see it written like this. That x falls between 3 and 7. Taking it apart, I'll cover this part up first, this is telling us that 3 is less than x, which means that x is greater than 3, while at the same time, x is less than 7. It's just putting those two restrictors together, and telling us that x falls somewhere between 3 and 7. It can't be 3. It can't be 7, but it falls somewhere in between those values.

All right. Let's keep moving through this. Just so you can see it written out formally, get that out of the way, given two side lengths of a triangle, the third side, which we'll refer to as x in this case, is the absolute value of the difference of the 2 sides. Now, can you guess why we've thrown in those absolute value bars? That's to make sure that you always have that third case thrown away. That case that gave us that negative value, that x was, let's flip back, that x was in this case greater than −3, that included negative values for that length, and we had to say that wasn't going to work. That's why you'll always have the absolute value bars, to force a positive answer for that difference.

The third side is greater than the difference. The absolute value of the difference of the two sides you were given, but it's smaller than the sum of the two sides that you were given. If I flip back again, just to show you where that difference in that sum came from, if you look, 3 is actually the difference of those two sides. 5 minus 2 is 3. Now, 2 minus 5 is −3, and that's why we've got those absolute value bars in there. Then, 2 plus 5, that's 7, and that's where we got that sum. Again, flipping back to the formal rule written out, x will be between the absolute value of the difference of the two sides, and the sum of the two sides.

Let's work through a couple of examples. Given a triangle with side lengths 7 inches and 10 inches, what's the range of the length of the third side? Make sure I've got the pen. All we need to do here is find the absolute value of the difference, so the absolute value of 10 minus 7, and then the sum, so 10 plus 7. When we find that out, we've got the range. 10 minus 7 is 3, and the absolute value of 3 is 3, so 3 is less than x, and x is less than. 10 plus 7 is 17, so what that means is that third side is somewhere between 3 inches and 17 inches. That means it could be 3.5 inches long, 5 inches long, 12 inches long, anywhere in between 3 inches and 17 inches. It can't be 3 inches. It can't be 17 inches, but anywhere in between.

Take a look at this next one, and it's actually a practical problem. It's a real-world problem. We're going to read through it. I know practical problems sometimes can throw you a bit, because they're very wordy. We're just going to take our time, read through it, highlight the key information, and then we'll get to solving this. The locations of school, the gym, and the library form a triangle. I need to know that.

That's important to know. Form a triangle. The school, the gym, and the library. The distance between the school and the gym is 6 miles. The distance from school to the gym, 6 miles. The distance between school and the library, 4 miles. School and library, 4 miles. What is the range of possible distances between the gym and the library?

Here we go. Now, in geometry, I'll always, always, always stress that if you're given a practical problem, and you have no picture, no image to help you through it, draw a picture. 9 times out of 10, that's half the battle that you're solving the problem. Get some kind of image that will represent what's going on here, and that will really help the problem kind of all come together for you, so that we can get to the answer. I'm going to take first, here, the

school, the gym, and the library form a triangle. Let me get my pen. I'm just going to roughly sketch a triangle. Some work space. We're going to be scrolling.
The school, the gym, and the library. I'll just call it *SGL*. Just using letters to abbreviate. School, gym, library. They form a triangle. Let's go back, get some more info. The distance between the school and the gym is 6 miles. School and gym. That's 6 miles. The distance between the school and the library is 4 miles. Let's see here, school and library. Let's get a little more space. There we go. Don't want to start running off the edge. 4 miles, and then we're asked, "What's the range of possible differences between the gym and the library?" That's *x*. That's what we're trying to figure out here.

Basically, what we have here is a triangle. We've been given two links, and we're asked to find the range of the third side. Yes, it's a real-world problem. Yes, these vertices represent locations of places, but really, we're using that Triangle Inequality Theorem to get to the answer. We need the absolute value of the difference, and then we need the sum. 6 minus 4 is 2, and the absolute value of 2 is 2. 6 plus 4 is 10. What that means is that possible distances between the gym and the library are anywhere between 2 miles and 10 miles. The gym and the library may be very close together. They may be just slightly above 2 miles apart from one another, or they could be less than, or up to 10 miles apart from each other. They can't be 2 miles apart exactly, and they can't be exactly 10 miles apart. Anywhere in between those distances would work for this situation. Good job on that one.

We've reached a problem where it's your turn. You've been given two lengths, 12 centimeters and 19 centimeters, and you're asked to determine which of these would work for possible lengths of the third side. Use that Triangle Inequality Theorem to get that inequality that will give you those boundaries of where that third side will fall in between, then you can figure out which lengths will work. Press pause, take a few minutes, work through that. Press play when you're ready to go over it.

Let's see how you did, here. The lengths that we had were 12 centimeters and 19 centimeters. We'll get a little work space here. Make sure I've got my pen. I need the absolute value of the difference, so 19 minus 12, and then I need the sum, 19 plus 12. 19 minus 12 is 7, and the absolute value of 7 is 7. 19 plus 12, I'll use a little mental math here, that's 31. That means that the third side of this triangle will fall anywhere between 7 centimeters and 31 centimeters.

Let's see which values work. Would 5 work? It's too small, right? We know that third side has to be greater than 7. 5 won't work. Will 7 work? That third side has to be greater than 7. It can't actually be 7. 16 centimeters. Would that work? Yes, it would, because that does fall between 7 centimeters and 31 centimeters. How about 20 centimeters? Yes, that would work, because it does fall between 7 centimeters and 31 centimeters. Now, how about 34 centimeters? You're right, that's not going to work. It's too large, because you know that the third side has to be smaller than 31 centimeters. That one's not going to work. Here, you had two values that would work. 16 centimeters and 20 centimeters. Good job on that one.

You've reached the conclusion of this topic in geometry. I hope you saw how the knowledge that you have of the Triangle Inequality Theorem helps you determine the range of the third side of a triangle. Bye.