Module 6: Congruent and Similar Triangles
Topic 1 Content: Proving Triangles Congruent by Side-Angle-Side Postulate
Transcript

Hi, guys! Welcome to Geometry. This topic's going to focus on how to prove triangles congruent by Side-Angle-Side. Your knowledge of congruent figures is going to come in handy during this lesson. Are you ready to get started? Let's go.

Okay, so before we dive into how to prove triangles congruent specifically, I want to just jog your memory a bit about congruent figures, but I am going to actually use triangles to talk about that. We know that, given two figures, if their corresponding sides are congruent and their corresponding angles are congruent, then you can conclude that the figures are congruent. When you name congruent figures, like when you write that congruent statement, you just have to make sure to line up their corresponding parts. For example, if I wanted to write a congruent statement about these triangles, I could say triangle *FCD* is congruent to ... Now, look at how I started with this one, *FCD*. I went along the side that had one congruent mark and then down the side with two, so I have to name this triangle using that same pattern. I'd say triangle *SNM*.

There are several other congruent statements that you could have written about these two triangles, depending on at what point you wanted to start along that first triangle. For example, you also could have written ... I'll scroll a little bit here. You could have said, okay, triangle *CDF* is congruent to triangle *NMS*. That's another statement that I could have written about these two triangles, and it would have also been true because their corresponding parts were lined up. Okay?

Now in the case of triangles, you don't actually have to show that all of their corresponding parts are congruent to show that the triangles are congruent. There's actually a shortcut you can take. One of those shortcuts is known as Side-Angle-Side. If you can show that you have two pairs of corresponding sides congruent, so in this case I have *AB* congruent to *DE*, and *DF* and *AC* are congruent, and if you can also show that you have a pair of included angles that are congruent ... By *included angle*, what I mean is that it's the angle formed by the two sides that are marked meeting in each triangle. *AB* is included between these two sides ... I'm sorry, angle *A* is included between these two sides, and angle *D* is included between these two sides. I have two pairs of corresponding sides congruent, and a pair of included angles are congruent, so I can conclude that these triangles are congruent by Side-Angle-Side. You may sometimes see Side-Angle-Side abbreviated as SAS. Okay?

All right, so now that you're a little familiar with Side-Angle-Side, let's use that to work through a few examples. Okay. Here we're being asked, is triangle *ACB* congruent to triangle *DCE*? Okay, now let's take a look at these triangles. Notice they meet at point *C*. They intersect at point *C*. Now look at their setup here. I notice they have some sides that are marked. I know that AC is congruent to CE. I'm going to go ahead and write that. AC is congruent to CE. I know that DC is congruent to CB, so I have two pairs of corresponding sides that are congruent. What I need are a pair of included angles that are congruent.

Now think back to what you know about angles. Do you remember this? I'm just going to draw just this here. I'm going to draw some intersecting lines. Do you remember what we called, say, that pair of angles, those pair of angles right there and here? Do you remember we referred to those as vertical angles? If you recall, the relationship between vertical angles is that they are congruent. Now look at these triangles. You actually have a pair of vertical angles in those triangles. Even though it's not marked, you know that angle ACB is congruent to angle DCE because angle DCE and angle ACB are vertical angles, so they have to be congruent.

Now, notice that I didn't just say angle C was congruent to angle C. There's a reason I didn't say that. If I just say angle C is congruent to angle C, you don't know if I'm referring to this angle C or if I'm referring to this angle C or this angle C or this angle C. The reason I have to use three letters to name that angle is because there are four angles that have C as the vertex. Okay? That's why sometimes in geometry you'll see angles just named by their vertex, but at other times, you'll see them named by the points that actually are on that angle. I'm going to put it in red so it stands out, and I'm going to say angle ACB is congruent to angle DCE. You have two pairs of corresponding sides that are congruent, and you have a pair of included angles that are congruent, so are these triangles congruent? Yes, they are by Side-Angle-Side. All right? Okay, good job on that one. You're starting to see this in action here.

Let's take a look at this. Here we're asked, is triangle GVR congruent to triangle LEN? Okay, so let's start taking this apart. I think I actually will stick with red, so it stands out a little bit. I don't have congruent marks, but I notice some relationships going on in these triangles, and I can put some congruent marks on these figures. For example, I see that GR and LN are each eight units long, so I'm going to mark those sides congruent. RV and NE are each seven units long, so I'm going to mark those sides congruent. Now what I need is a pair of congruent angles, right, because I'm trying to prove these triangles congruent by Side-Angle-Side.

I notice 80 degrees is the measure of angle N, and it is included between these two sides here. It is the included angle, but I don't actually have a measure for the included angle in triangle GRV, but knowing what I do about triangles, I can figure out what the measure of angle R is, right? Yes, because you know the angles in a triangle have their measures sum to

180 degrees. Right now, I see angle G is 40, angle V is 60. Forty plus 60, that's 100, and then 180 minus 100 is 80, so I do know that angle R is 80 degrees. Then now that I know that, I can see that angle R is actually congruent to angle N. I do have a pair of included angles that are congruent. Are these triangles congruent? They sure are. Okay? Do you see how you could use some numerical relationships to figure out if you do indeed have a pair of congruent triangles?

Okay, let's take a look at this example here. We're asked to prove these triangles congruent by Side-Angle-Side. You'll notice that this time we have triangles that are graphed on the coordinate plane, so we're going to have to use our coordinate methods in order to prove that these two triangles are congruent. Okay, so let's dive in. Now, if we're trying to prove the triangles congruent by Side-Angle-Side, I know that we're going to have to first start out by finding out the lengths of at least a pair of sides on each triangle. I'm going to start out by working on triangle LMN. Now, the easiest lengths for me to find are the lengths of MN and LN because this is a horizontal length and a vertical length. I could just count those lengths. Let's see. MN, that's two units. LN, that's one, two, three, four units. Okay.

Now to figure out an angle relationship in this problem, we're going to have to throw back a bit to Algebra I. Now, if you recall, when we were talking about slope in algebra, we know that the slopes of parallel lines are the same, and we know that when dealing with perpendicular lines that our lines will have opposite reciprocal slopes if those lines are perpendicular. Also, remember that special case, that every horizontal line is perpendicular to every vertical line. Recall perpendicular, what that means is that the lines will intersect to form a right angle.

Here, I can use that relationship in order to determine an angle measure in my triangle. Let's look back right here at triangle LMN. MN is horizontal, right? That's a horizontal segment. LN is a vertical segment, so that means that where they intersect, I have a right angle. MN is perpendicular to LN. Okay? Now let's take a look here at our other triangle, and we see here ... Let's get this vertical length. VU, one, two, three, that's four units. UT, that's one, two units. Again, you'll notice you have a horizontal length and a vertical length. Every horizontal segment is perpendicular to every vertical segment, so that means I have a right angle right here at U.

Now, if I go back and compare my lengths here, I do have two pair of corresponding sides that are congruent, and I have a pair of included angles that are congruent. I've shown here that these triangles are congruent by Side-Angle-Side. I'm going to go ahead now, and I'm going to complete my congruent statement. Triangle NML, so let's follow that, NML. We

started at the right angle, went out, and then up to the top. That triangle is congruent to triangle UTV. Again, we started at the right angle, went out, and then up to the top.
All right, it is your turn. Here are the directions for the problems I'm going to show you right after. You're going to be given a pair of triangles, actually two pairs of triangles, and I want you to determine if those triangles are congruent. If they are, I want you to go ahead and complete that congruent statement. Okay? I'm going to go ahead. I'm going to flip to the next slide. Take a few minutes. Press pause, and work your way through these problems, okay? When you're ready to check your answer, go ahead and press play.

All right, let's see how you did here. Now, I'm actually going to solve these one by one. I'm going to move on to the next slide here. I'm going to show you how I answered this one. Okay, so I see here I do have two pair of corresponding sides that are congruent. I have TE and EY that are congruent, and I have KT and GY that are congruent. I do have a pair of vertical angles, right, that I can mark that are congruent, but if you look at this closely, notice that those vertical angles are not the included angles in my two triangles. Okay?

Remember what I mean by that. I'm going to separate these triangles to show you what I mean. I'm just going to roughly sketch them apart and just make sure you keep all the marks. I'm going to even make sure I keep that red, so everything just kind of looks the same here. I have E, G, and Y. Let's get those marks. Okay. Notice in these triangles, here are the two ... Oops, let's keep those marks straight on that one. That one is an extra mark. Okay. Here are the two marks, right?

The included angle on this triangle is angle T. On this triangle, the included angle is angle Y, but there's no way that I can show that those angles do have the same measure. I can't assume, and there's no geometry I can use to prove that those angles are congruent. Are these two triangles congruent by Side-Angle-Side? They are not. I cannot actually complete the congruent statement. I can't show you these triangles are congruent by Side-Angle-Side because I can't prove that I have a pair of included angles that are congruent. Okay? So the answer to that first one was a no.

Now, let's take a look at that second one. Okay, so here I have a pair of triangles. I notice immediately that they share a side. Recall that if you have two triangles that share a side, in this case AZ, that side is going to have the same measure regardless of whichever triangle it's in, right? I can go ahead and mark, by the reflexive property, that that side is congruent to itself, that AZ is congruent to AZ, regardless of which triangle that it's in. If you can't see it now, I'm going to go ahead and separate these because you may be able to tell with them connected what's going on, but sometimes it just helps to look at them separately. Okay, I'm going to mark that in red. Let's get the bottom triangle here, *BAC*, and let's mark that.

Okay. I see I have two pairs of corresponding sides that are congruent, *JA* and *BZ*, *AZ* and *AZ*, and I have a pair of included angles that are congruent. Now, looking at the triangles separately, it looks like you could say, "Okay, angle *Z* is congruent to angle *A*," but looking at the angles in this triangle, we know we'd have to name them with three letters, that angle *JAZ* is congruent to angle *BZA*, right, just so you can really show specifically which angle you're talking about, because there's more than one angle with a *Z* and *A* as the vertex. Okay?

If I look, I do have two pairs of corresponding sides that are congruent and a pair of included angles that are congruent, so yes, these triangles are congruent by Side-Angle-Side. To complete my congruent statement, I can say triangle *ZAJ*, so Z-A-J ... I started at *Z*, went along the side with one mark, up the side with two, so that would be *AZB*, triangle *AZB*. Triangle *ZAJ* is congruent to triangle *AZB* by Side-Angle-Side. All right, good job on that. Okay, you've reached the conclusion of this lesson on how to prove triangles congruent by Side-Angle-Side. I hope you saw how your knowledge of congruent figures helped you get through this lesson. Bye.