Module 6: Congruent and Similar Triangles

Topic 1 Content: Proving Triangles Congruent by Side-Side-Side Postulate

Transcript

Hi, guys. Welcome to Geometry. This topic is going to focus on how to prove triangles congruent by Side-Side-Side. Your knowledge of congruent figures is going to come in handy during this lesson. You ready to get started? Let's go.

Okay, so before we jump into talking about how to prove triangles congruent, let's first just talk about congruent figures in general. Take a look at these two trapezoids. Now, you can tell from their markings that the corresponding angles and the corresponding sides in these two trapezoids are congruent. What I mean by that is I can tell, for example, by the marks, by these arcs on angle A and angle M that angle A is congruent to angle M, angle B is congruent to angle N, angle C and angle L are congruent, and so are angle D and angle P. In regards to the sides, I can make those same kind of conclusions.

Side CD is congruent to side LP because I can tell that they both have that one slash going through. BC is congruent to NL. AB and NM are congruent, and so are AD and MP. Those are also congruent sides. When you have two figures that have corresponding sides congruent and corresponding angles congruent, you can conclude that the two figures are congruent. By *congruent*, remember all I mean is "equal," that they have the same measure. When we have that relationship, what we can do is actually right a congruence statement about the figures. For example, with these two trapezoids ... Let me make sure I've got my pen ... I could write a statement ... Let's move this a little bit to get some space ... that says trapezoid $ABCD$, and really I can just name this trapezoid by its angles or by its sides depending on how you look at this, $ABCD$, is congruent to ... Now, when I name this trapezoid, I have to make sure that I'm aligning those corresponding sides. What I mean by that is if I name this as $ABCD$, notice a pattern here, that I went along the side with the four slashes, two slashes, one, and then three. I have to name this triangle in that same way so that the corresponding parts line up.

If this is trapezoid $ABCD$, then I have to name this one, $MNLP$. Now, that isn't the only way that I could have written this congruence statement. Depending on how I started, it could have ended up a few different ways. For example, another way I could have named this trapezoid, maybe I started at B. I could have said, "Okay, $BCDA$ is congruent to," so again, if I start at that top left spot, that angle marked with the two arcs, I have to keep that same pattern when I name this one. If this is $BCDA$, then $NLPM$. $NLPM$. There are several other congruence statements that I could have written that would have all been true. Remember, as long as the corresponding parts match, then how you write the congruence statement is okay, as long as those corresponding parts line up.

Now that I've jogged your memory a little bit about congruent figures, and you know now or you remember now that if your corresponding angles and corresponding sides are congruent, then you could conclude that you have two congruent figures. Now, when we speak about triangles specifically, there's a shortcut that we can take. If given two triangles, if we can show that we have three pairs of corresponding sides that are congruent, then we could conclude that the triangles are congruent. We actually don't even have to worry about those corresponding angles as long as we can show that the corresponding sides are congruent. We refer to this relationship as Side-Side-Side. For example, given these two triangles, I could see that they have three pairs of corresponding sides that are congruent.

Okay, let's scroll a little bit; write a few things down here. From how my triangles are marked, I can see that AB is congruent to DE. I can also see that BC is congruent to EF. Get a little more space. I can see that AC is congruent to DF. Because I can show that I have three pairs of corresponding sides that are congruent, then I can conclude that triangle ABC, if I start out by naming that triangle that way, okay, so triangle ABC is congruent to triangle DEF by Side-Side-Side.

Sometimes you'll see Side-Side-Side abbreviated as just SSS. All right, so now that you're a little familiar with Side-Side-Side, let's try an example. Take a look at this. You're given actually two triangles that share a side here, and you're asked: Is triangle ABC congruent to triangle DCB. Well, right now, I know that if I can prove that these two triangles have three pairs of corresponding sides that are congruent, then I can conclude that these two triangles are congruent. From how my triangles are marked up here, I can see that AC is congruent to BD, and I can see that AB is congruent to CD, so I have two pairs of corresponding sides at are congruent. It's just the third side that I have to find. You may see it right now in how my triangles are connected right now, but if you don't, I want to show you a little trick that you can use.

Let's move a few things around here just to get some space going. Move this up here. Okay. When your triangles are connected like this, you see that they share a side, what you can do is separate the triangles. If it's difficult for you to kind of see the relationship as it is right now, just break those triangles up. This is what I mean by that. Get that pen back. Okay, so I'm going to sketch triangle ABC, and it may not look perfect, but I'm just going to get it as close to how it is in the figure as I can, and I'm going to take the markings with it also. AC was marked, AB was marked. Then I have triangle BCD, so let's just sketch that one. Okay, let's get the vertices, and let's get those markings.

Now, notice that BC is in both triangles. Those triangles actually share that side. You may have seen it here, and you could also see it if you separate those two triangles. Now, I don't know for sure what the length of BC is, but I know that regardless of which triangle it's in, it's going to have the same length in each of my triangles, so what I can say is BC is congruent to itself. That side is going to be equal to itself regardless of the fact of which triangle it's in. Let's just clean that up a little bit. Our marks got a little crazy over there. All right. Let's just kind of get that back. There we go. Okay. I can see that BC is congruent to itself. Now, if you saw that before separating the triangles, don't feel like you have to separate them, but that's just kind of an extra little strategy you can use when you're solving these type of problems.

I'm actually going to throw that mark up here too. Now, if you take a look, you see that you do have three pairs of corresponding sides that are congruent. Going back to our original question, if I scroll back up here, is triangle ABC congruent to triangle DCB? Yes, it is. I'm going to say yes. Because we were able to show that BC was congruent to itself. Remember, the property that says that that is possible is the reflexive property. Okay, let's take a look at this one. Prove the triangles are congruent by Side-Side-Side. You see now we're on a coordinate plane, so we're going to have to use our coordinate methods to figure out ... Or, we know that they are congruent, but to show that they are congruent.

Okay, so I'm going to start with this triangle on the left and show you how you can attack this problem. Now I know if I'm trying to prove the triangles are congruent by Side-Side-Side, that I need three lengths. I need to know what the length of each side of this triangle is. I notice MN is horizontal and LN is vertical, so I can just count the lengths of those sides. MN is 2 units long. LN, 1, 2, 3, 4 units long. Now, remember for diagonal lengths, you can't count those like we just did with the horizontal and the vertical length. We're going to have to use the distance formula to figure out what the length of that side is.

Remember to use the distance formula, you're going to need to know the coordinates of these two endpoints, and then we're going to go ahead and get that formula down and start using that. Okay, so let's see. L is at -1, 2, 3, 4, so -4; 1, 2, 3. There's L. M is at -1, 2, 3, 4, 5, 6, and then -1. I'm going to use the distance formula to figure out the length of LM. Now, remember that distance formula, it is a formula that you want to memorize. You see that it pops up from time to time in different problems. Here we're going to want to leave that like that just for a second.

Remember, the distance, square root of x sub 2 minus x sub 1 squared, plus y sub 2 minus y sub 1 squared. Remember our points that we have here ... We're going to do a little shifting back and forth for a second. It doesn't matter which one of these you label x_1 and y_1 and x sub 2, y sub 2; just keep it straight. I'll call this one x_1 and y_1, x_2 and y_2. Then I'm just going to start substituting into my formula to get the length of *LM*.

All right, so bear with me for a second with the scrolling. It's going to have to happen for a minute. Okay. x sub 2, let's see what that is. It's −4. −4 minus −6, squared. Plus y sub 2 is 3, so 3 minus −1, squared. I immediately notice those double negatives, so I'm going to clean those up. Then let's start simplifying this here. −4 plus 6, that's 2. This is 2 squared. 3 plus 1 is 4, so that's 4 squared. Keep scrolling, get some more workspace.

Okay, so 2 squared, that's 4. 4 squared is 16, so I could say, "Okay, this is the square root of 20." I'm just going to leave that exact, as an exact answer. I'm not going to actually take the square root of 20 because it's not a perfect square. I'm just going to leave it just like this, and I'm going to back up to my triangle, and I'm going to ... It's starting to get crowded here, so let me start erasing a few things.

I don't need this anymore. We've used that. We've used that. We've used those to figure out that *LM*, its length is the square root of 20 units. Now that I know the length of each side in this triangle, I just need to figure out the length of each side in the other triangle, and then I'll be able to show that they are indeed congruent by Side-Side-Side. That's where I'm going to step back a bit. I'm going to pause, and I'm going to let you figure out the length of each side in that triangle. Remember our steps. We counted the vertical and the horizontal lengths, but you had to use the distance formula to figure out that third length. I'll go ahead and leave this distance formula up here for you. We'll erase the work, though, because I don't want you to get that mixed up with the other problem. Let's get that out of our way.

I will leave that distance formula up for you while you're working that part of the problem. Press pause, take a few minutes, work your way through solving that other triangle. When you're ready to check your answer, press play.

All right. Let's see how you did on that one. Let me get my pen back. Okay, so the vertical length, I can count. *TU* is 2 units. Horizontal length I can count, 1, 2, 3, 4 units. Again, I just have to use the distance formula to figure out the length of *TB*. Now, you may have already noticed, you could kind of use logic to conclude that if the lengths of the sides of this triangle, the vertical and the horizontal length are the same as the length of the horizontal and the vertical length, or we should say the corresponding lengths in the other triangle, that you should get the same answer for *TB*. You know we like to see the work in math, so let's go ahead and show it algebraically that it is indeed, that those lengths are the same.

For T, we're at 1; 1, 2, 3, 4 units, so (1, 4). B is at 1, 2, 3, 4, 5; 2. Okay, so let's just use the distance formula to show that TB is indeed the square root of 20 units. All right, so let's label x_1, y_1, x_2, y_2, and let's get a little space. Okay, so x_2 is 5 and x_1 is 1.

We have 5 minus 1 squared underneath that radical. Then for our y's, we'll have 2 minus 4 squared. All right. Let's keep simplifying this. All right, so 5 minus 1, that's 4, so 4 squared. 2 minus 4, that's −2, so −2 squared. 4 squared, 16. −2 squared is 4. 16 plus 4 is 20. You see it is indeed the square root of 20, so I'm going to go up to the triangle and write that length in there. I don't need those coordinates anymore. Let's erase that. Let's get the square root of 20 up there. You've got it. You've shown using coordinate methods that these triangles are indeed congruent by Side-Side-Side. They have three pairs of corresponding sides that are congruent. All right, let's take a look at one more example.

Let's see here. What value of x will guarantee triangle ABC congruent to triangle DEF? All right, so we have an algebraic problem here. I notice a few different variables in triangle DEF, but I'm only asked to solve for x, so I'm going to focus my attention on x.

Okay, so looking at my triangles here, I notice that AC involves the variable x, and FD also involves the variable x. I need to just guarantee that those sides do actually correspond, so I'm going to go to my congruent statement here. AC does correspond to DF, so that means that AC equals DF, that they have that relationship. Those are congruent sides. AC equals DF because the corresponding parts of congruent triangles are congruent. If I'm to assume, or if I'm trying to show these triangles are indeed congruent, I have to show that, okay, their corresponding sides must be congruent.

Since I know that AC equals DF, I'm going to set up an equation and solve for x. Let's get some work space. Okay, so AC is $11x$ minus 4. DF is $5x$ plus 2. Let's solve for x here. Let's subtract $5x$ from each side. We have $6x$ minus 4. That's wiped out. Equals 2. Add 4 to each side. Get some more work space here. There's a lot of work involved with these, well, some of them. Okay, so $6x$ equals 6. Then divide each side by 6. x equals 1. Now what I've shown is that if x equals 1, then that will guarantee that AC will be equal to DF and that those sides actually are congruent. All right, so you've seen a few examples now. I want you to go ahead and try one. Go ahead and press pause, take a few minutes, and work your way through this. You've got to use some algebraic methods again on this one. When you're ready to check your answer, press play.

All right, let's see how you did. You were asked to show that if x equals, and that's what you've got to find, and you also have to find what y equals, then that would prove, or that would guarantee, that triangle BRV is congruent to triangle ARV. Now, notice how VR, that side was kind of uninvolved with these algebraic equations that we have to set up, do you know why that is? All right, you might have remembered that. You already know that VR is congruent to itself, right? That reflexive property. Even though you don't see those marks, you know that that has to be true. VR is the same side in each triangle. It's going to have the same length, so it's congruent to itself. Now we can just focus our attention on solving for x and solving for y.

I know that BV is congruent to BA. That means that BV equals BA. I'm going to set up an equation and solve for x. $9x$ plus 3 equals $6x$ plus 9. Then let's just go ahead and solve this. Get some space. Okay, subtract $6x$, subtract $6x$. We have $3x$ plus 3 equals 9. Subtract 3 from each side. I need a little more work space. Okay. It's a little scrolling here.

All right, so $3x$ equals 6, divide by 3. x is 2. Now I've shown that if x equals 2, that will guarantee that BV is congruent to VA. We know the first part of this is x is 2. Now let's just solve for y and we'll have the second part. Let's get this work out of our way here because we're done with that. Scroll down and get the rest of this. Okay.

All right, now let's solve for y here. From the markings in my triangle, I know that BR is congruent to RA, so BR equals RA. You can set up the equation, $4y$ minus 4 equals $2y$ plus 6. Then let's get to solving for y. Let's subtract $2y$ from each side. All right, got that out of the way. $4y$ minus $2y$ is $2y$, minus 4 equals 6. Add 4 to each side. All right, let's get this down. $2y$ equals 10, and divide by 2. We have y equals 5. We have that all solved out. We've guaranteed that if y is 5, then triangle BRV is congruent to triangle ARV by Side-Side-Side. All right. Good job on that. Okay, guys, you've reached the conclusion of this lesson on how to prove triangles congruent using Side-Side-Side. I hope you saw how your knowledge of congruent triangles and just congruent relationships in general helped you get through this lesson. Bye.