Module 6: Congruent and Similar Triangles
Topic 2 Content: Proving Triangles Congruent by Angle-Angle-Side Theorem
Transcript

Hi, guys. Welcome to Geometry. This lesson's going to focus on how to prove triangles congruent by Angle-Angle-Side. Your knowledge of congruent figures is going to come in handy for you during this lesson. You ready to get started? Let's go.

Okay, so before we jump into Angle-Side-Angle, or Angle-Angle-Side, specifically, I want to just discuss congruent figures in general. Giving congruent figures, you know that if you have all your corresponding sides congruent, and your corresponding angles congruent, that you do indeed have two congruent figures, right? You can write several different types of congruent statements to describe that relationship. We're going to show that relationship.

For example, given these two triangles, I can see that I have three pairs of corresponding sides that are congruent, and three pairs of corresponding angles that are congruent. I can write a congruent statement to express that relationship. I can say, "Okay, triangle *FDC* is congruent to triangle *SMN*." Notice that I wanted to make sure that I lined up those corresponding parts, right, so that the congruent statement would be completely true, and each corresponding part would line up in the statement, so that just from looking at the statement, I can see those congruent relationships. I would know that angle *F* is congruent to angle *S*, and angle *D* is congruent to angle *M*. I would know that *FD*, that side is congruent to *SM*, and that *FC* is congruent to *SN*.

That's why it's so important that you make sure your corresponding parts match, so that you really see all those congruent relationships in the statement itself. Depending on how you wanted to start around the triangle, you could have written another congruent statement to describe the relationship. For example, you might have written, "Triangle *CDF* is congruent to triangle *NMS*." Fix that. Just thinking about those triangles, right? *MNS*. Okay, so as long as you match up those corresponding parts, then how you write the statement is okay, because I can still see, even in this statement, that angle *C* is congruent to angle *N*. I can see that side *CD* is congruent to side *NM*. I can see all those relationships from my statement.

Now, when you're talking about triangles specifically, you don't actually have to show that all of your corresponding parts are congruent to show that you do indeed have two congruent triangles. There's a shortcut that you can take known as Angle-Angle-Side. What this tells us, or how we use this, is if we can show that we have two pairs of corresponding angles that are congruent, and a non-included side in one triangle congruent to the corresponding non-included side in the other triangle, then I can show that I do indeed have two congruent triangles. Let's kind of take that apart so you can see what I mean.

You notice angle *A* is congruent to angle *D*. Angle *B*'s congruent to angle *E*. Then, you have a side here, right? *AC*. We call this a non-included side because it's a side that is not included between the two angles that were marked in this triangle. Then, again over here, I have angle *D* marked, and angle *E* is marked. This is the non-included side. Just so you really understand what I mean by non-included, *AB* would be an included side, right, because that side lies on that ... Lies along where those angles are, right? It's between those angles. This would be the included side here. I'll just clean that up a little bit. Over here, this would be included. Included side, non-included side. Not included between those angles.

EF is also a non-included side. It's not included between those angles. *BC* is also a non-included side, not included between the two angles that are marked. For Angle-Angle-Side, let's erase this just to bring it all back, you need two pairs of corresponding angles that are congruent. Lost that angle marking there. You need a pair of non-included sides that are congruent, okay? Keep that in mind while we work our way through a few examples. What we're going to do next is I'm going to give you a few different pairs of triangles. We're going to use what we know about Angle-Angle-Side to determine if those triangles are indeed congruent by Angle-Angle-Side. All right?

Okay, let's look at the first example. I notice I have these two triangles, and they do share a side. Knowing what we do about the reflexive property, we know that *YR*, this side they share, is congruent to itself. Regardless of which triangle it's in, *YR* is congruent to *YR*. *YR*'s congruent to itself. I'm going to go ahead and mark it. Then, let's scoot this out of the way a little bit. Going to move this over to the left. Because if you don't see it yet, if you don't see that relationship, those two triangles connected like that, like sharing that side, I'm going to break them up. I'm going to separate them and see if you can see it that way. All right, so we have a triangle up top, *TRY*. We have this marked, we have this marked, and then we have *YR* marked.

Then on the bottom, we have triangle *YRS*. We have *SRY*. Marking here, marking here, and I'm marking there. Everything's matched up. Now, let's see you see this. Notice that we have a pair of corresponding angles, right? Angle *T*'s congruent to angle *S*. Those are congruent. Now, the triangles are separated now, right? They're separate, so it looks like angle *R* is congruent to angle *R*, but we know in the figure, where they're connected, I'd have to name angle *R* with three letters. Just to focus on the marks for right now, two angles are marked here, and I have a non-included side. To angles are marked here, and I have a non-included side. Are these two triangles congruent by Angle-Angle-Side? Yes, they are. You do indeed have two pairs of corresponding angles congruent, and a pair of non-

included sides that are congruent. Are they congruent by AAS, or Angle-Angle-Side? Yes, they are. All right? Okay, good job on that.

Let's look at this one. These triangles do not share a side, but they are reflections of each other, right? We're trying to see, are these triangles congruent by Angle-Angle-Side? Well, just giving them a first glance, I see that I have a pair of corresponding angles congruent, and I have a pair of corresponding sides that are congruent. That's really all I have. I don't have enough information. I don't have another angle relationship, right? That's what I need. I need another angle relationship that would show I have two pairs of corresponding angles congruent, and a pair of non-included sides congruent. I don't have that, and there isn't any geometry I can use. There's no property that I could use to prove that there are indeed some angles here that are congruent. Are these triangles congruent by Angle-Angle-Side? They're not. I actually can't show that they're congruent at all, okay? Example two is a no.

Now, it's your turn. Let's see what you can figure out here. Are these triangles congruent by Angle-Angle-Side? Go ahead and press pause. Take a few minutes, and work through each of these examples. When you're ready to over them, go ahead and press play.

All right, let's see how you did. I'm going to go over these one by one, so first one's first. What I noticed first of all is I noticed in this figure, I have a pair of vertical angles. Recall vertical angles are formed by intersecting lines, right? They're those non-adjacent angles, and then here also. In this figure, it has kind of an hourglass kind of shape, the way these two triangles meet, or intersect at Y. I know that angle NYX is congruent to angle FYT. Because those are a pair of vertical angles. Let's actually use two angle markings to mark that because I already have the one angle mark used.

You may already see it with the triangles meeting at Y, but just in case you want to separate them, let's get this out of the way. We'll get those separated. The triangle up top and XY, and we have angle X marked. That side is marked. Then, let's use the red there. Then, we have our bottom triangle, FTY.

That side's marked. This angle, and here we go. All right, so let's kind of break this down. I do notice that I have two pairs of corresponding angles that are congruent, and I have a non-included side in the triangle on the bottom congruent to a non-included side in the triangle on the top. Are these two triangles congruent by Angle-Angle-Side? Yes, they are. You have your two pairs of corresponding angles congruent, and you have your one pair of included sides that are congruent. This one is yes. Okay?

Now, let's look at that second one, see how you did with that one. Here, my triangles do not share a side, they are reflections of one another, but that's all I can really say. I'm trying to prove them congruent by Angle-Angle-Side. I see angle S is congruent to angle H, and angle G is congruent to angle M. I have two pairs of corresponding angles that are congruent, but what I'm missing is that non-included side, right? I don't have a non-included side in the triangle on the bottom congruent to a non-included side in the triangle on the top. There's no geometry that I can use to show that any of these sides are congruent to each other. Right? For number two, that one's a no. These triangles are not congruent by Angle-Angle-Side. In fact, I can't prove they're congruent at all. I don't have enough given information. Okay? All right, good job on that.

All right, you've reached the conclusion of this lesson on proving triangles congruent using Angle-Angle-Side. I hope you saw how your knowledge of congruent figures really helped you get through this lesson. Bye.