Module 6: Congruent and Similar Triangles

Topic 3 Content: Proving Triangles Congruent by Hypotenuse-Leg Theorem Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on how to prove triangles congruent by Hypotenuse-Leg. Your knowledge of congruent figures and congruent triangles specifically and right triangles is going to come in handy for you during this lesson. You ready to get started? Let's go.

Let's recap. Let's review all the different ways that we know of how to prove that we have two congruent triangles, those short cuts. Those short cuts that we can use to avoid having to prove that we have three pairs of corresponding sides that are congruent and three pairs of corresponding angles that are congruent, those ways around it. Those four different methods that you know so far.

I'm going to show you one triangle because I want you to focus on the marks here. See if you recognize them. Notice, I have three sides marked in this triangle. If I had another triangle marked in the same way, I would know that I could use Side-Side-Side to prove that I had one pair of congruent triangles.

Take a look at this one. Got the red dot there. Let's erase the red dot. Let's get that circle out of the way. I've shown you one triangle, marked up like it is. If I had another triangle marked the same way, where I'd have a pair of corresponding sides that were congruent, actually two pairs of corresponding sides congruent and an included angle, a pair of included angles, I would know that I could use Side-Angle-Side for that particular pair of triangles.

Take a look at the third method here. Let's reveal what's behind this circle. If I had another triangle, marked the same way, then I'd have two pairs of corresponding angles congruent and a pair of included sides congruent. For that method, I could use Angle-Side-Angle. Let's keep reviewing these.

There's one more we know of so far. Let's get that out of the way. If I had a pair of triangles marked up in this way, where I'd have two pairs of corresponding angles congruent and a non-included side or a pair of non-included sides that were congruent, I could use Angle-Angle-Side as the method to show that those triangles were congruent.

Look at this next triangle here. I'm actually going to show you a non-example. If you had two triangles marked up this way, the way this triangle is marked, where you had two pairs of corresponding sides that were congruent and you had a pair of non-included angles that were congruent. In a way you could say it's Side-Side-Angle. This is actually not a method that works. This is not a valid strategy to use to show that you have a pair of congruent

triangles. You actually cannot guarantee that if you have two pairs of corresponding sides congruent and a pair of non-included angles congruent, you can't actually show that those triangles are equal, except for if you're dealing with two right triangles. That's the only way that Side-Side-Angle works. Pretend that we've got two triangles marked up this way, two right triangles. If you have two pairs of corresponding sides that are congruent and a pair of non-included angles that are congruent, in this case the right angles, you can actually show that those two right triangles are congruent. We refer to that method as Hypotenuse-Leg.

Review a little bit. Let's review about the parts of a right triangle so you know where that name came from. We're calling a right triangle that the side opposite the right angle is known as your hypotenuse. In the case of these triangles, I have my hypotenuse here. *BC* is the hypotenuse in that triangle and *PL* is the hypotenuse in this triangle. The other sides in your triangle are referred to as legs. *AB* is a leg and so is *AC*, and *PM* is a leg and so is *ML*. the parts of a right triangle, you have two legs and a hypotenuse. When using the hypotenuse-leg method to show that you have two congruent right triangles, as long as you can show that your hypotenuses are congruent and you have a pair of legs that are congruent, a pair of corresponding legs, then you can show, you can conclude that those triangles are congruent.

Now that you're a little familiar with Hypotenuse-Leg, let's work a few examples. I'm going to show you a pair of triangles. Let's give you the directions first. I want you to determine are those triangles congruent by Hypotenuse-Leg. You'll see it abbreviated sometimes as just HL. If they actually are congruent, we're going to complete that congruent statement. Let's take a look.

Here we have two right triangles that actually share a side. We have to figure out: Do we have enough information here to show that these triangles are congruent by Hypotenuse-Leg? Let's get a little more work space here, do a little scrolling. Recall the reflexive property. You see that *RQ*, that side is shared between those two triangles. Regardless of which triangle *RQ* is in, it's going to have the same length in each triangle. It's congruent to itself. I'm going to go ahead and mark *RQ* congruent to *RQ*.

Just in case you don't see it with the triangles connected like that, adjacent, sharing that side, let's go ahead and separate them. Let's break them apart, get a little more space. Got to scroll. Let's get the triangle up top, *QTR*, and let's get the marks. Let's get the triangle on the bottom, *QSR*, and let's get the marks. Let's take this apart here. We're looking for a pair of congruent hypotenuses and a pair of corresponding legs congruent. Let's see. I know the

side across from my right angle's the hypotenuse, so I do have *RQ* congruent to *RQ*, because that's the hypotenuse in the other triangle. The hypotenuse part's taken care of. The H part of HL. Let's look here. I also have *TR* congruent to *QS*, so I have a leg in one triangle congruent to a leg in another triangle.

I do have enough information to show that these triangles are congruent by Hypotenuse-Leg. I've got what I need. Are they congruent? Yes, they are. Let's go ahead and complete our congruent statement. It's started for us by naming the first triangle as triangle *TRQ*. Let's look at the pattern that's started. *TRQ*, we started at the right angle, down the side with one mark, up the side with two. We need to keep that same pattern. Let's start at the right angle, so *SQR*. Triangle *SQR*. Good job on that. You saw how we took that apart, we figured out that we did have what we needed for Hypotenuse-Leg and we wrote out our congruent statement.

Example two. Let's see here. Again, we have two triangles that do share a side, and we want to know do we have enough to prove the triangle's congruent by Hypotenuse-Leg? Like we said initially, they do share a side so I can go ahead and mark, make sure I still have the pen, that *KM* is congruent itself. Regardless of which triangle *KM* is in, it's going to have the same length.

Let's go ahead now and break those triangles apart, just so we can see what's going on a little easier. I got to scroll for a little bit. We've got *KMJ*, and let's get the marks. I wanted that in red. There we go. We have *KML*. *KML*, and let's get the marks. Let's see what we have here. I notice that the hypotenuse in each triangle, I don't have a pair of congruent hypotenuses here. What I have is two pairs of legs that are congruent. Really what you have here, if you really think about what you know about congruent triangles, is you have two congruent triangles but they're congruent by Side-Angle-Side. You have two pairs of corresponding sides that are congruent and you have one pair of included angles that are congruent. Are the triangles congruent? Yes, but by Side-Angle-Side, not by Hypotenuse-Leg. I can't complete this congruent statement because they're not congruent by Hypotenuse-Leg. I can change it. We can go ahead and do that. I can write that they're congruent by Side-Angle-Side because they are, but not by Hypotenuse-Leg.

If we follow the pattern that was set for us here. Triangle *JMK*, so *JMK* congruent to triangle *LMK*, keeping that pattern, but not by Hypotenuse-Leg, by Side-Angle-Side. Good job on that one. I do believe that it's your turn to try one.

Again, I'm going to give you a pair of triangles and I want you to see are they congruent by Hypotenuse-Leg? If they are, go ahead and complete that congruent statement. Here you go. Press pause. Take a few minutes, and when you're ready to check your answer go ahead and press play.

Let's see how you did on this one. Here you were given a pair of triangles, and what immediately jumps out at me in this example is that my two triangles share a side. They share TR. I know that regardless of which triangle TR is in, TR is going to have the same length in each triangle. TR is always going to be congruent to itself by the reflexive property.

I'll separate those triangles, just to make sure that you know what I mean by that. Let's get triangle STR, you have a right angle here, we have that mark. I'm going to have to scroll up and down a little bit just for the sake of workspace. Let's copy down that triangle on the bottom. We have right angle TCR. Notice TR is in each triangle. TR is always going to be congruent to itself. I'm going to mark TR congruent to TR. Now if I look at the congruent parts in these triangles, have a pair of right triangles and I have a leg in one triangle congruent to a leg in the other triangle. I have a hypotenuse in one triangle congruent to the hypotenuse in another triangle. Are these triangles congruent by Hypotenuse-Leg? Yes, they are.

Because I know these triangles are congruent, I can go ahead and write my congruent statement. I'm going to go ahead and throw those congruent marks on that given figure also. Let's see. Our statement starts out triangle STR, so STR. I'll need to use that same pattern to complete this statement. Triangle STR is congruent to triangle CTR by HL, by Hypotenuse-Leg. Good job on that one.

All right guys, you've reached the conclusion of this topic on how to prove triangles congruent by Hypotenuse-Leg. I hope you saw how your knowledge of congruent triangles and right triangles helped you get through this topic. Bye.