Module 6: Congruent and Similar Triangles

Topic 4 Content: Using Direct Proofs to Prove Triangles Congruent Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on how to use direct proofs to prove triangles congruent. Your knowledge of the five ways to prove triangles congruent is going to come in handy during this topic. You ready to get started? Let's go.

Okay. Before we jump into the direct proofs, let's just review the five ways to prove triangles congruent. Let me make sure I've got my pen. Okay. Looking at this first triangle, if I were given a pair of triangles marked with congruent marks in this way, I know that I could use Side-Side-Side to prove that those triangles were congruent.

Now, take a look at the next one. If I were given a pair of triangles marked in this way, I know that I could use Side-Angle-Side to prove that those triangles were congruent.

Again, this one, given a pair marked this way; I could use Angle-Side-Angle. The next one, given a pair of triangles with those congruent marks in that particular orientation, I could use Angle-Angle-Side.

The last one. Right. If I were given a pair of right triangles marked in this way, I could use HL, or Hypotenuse-Leg.

Now, those are the five ways that you know of how to prove that you have a pair of congruent triangles. Now, what we're going to focus on, on this topic, is we're going to use those five strategies, those five ways that we know, in order to complete direct proofs. Specifically, we're going to work on two-column proofs. We're going to be given a set of information and a figure and a set of statements describing some relationships in our pair of triangles, and we're going to justify each of those statements and work our way towards proving that the triangles are congruent. Okay?

Let's take a look at this first example. Let's start out by reading that given statement. B is the midpoint of segments AD and EC, and we're trying to prove that triangle EAB is congruent to triangles CDB. Let's take a look at our first statement here. B is the midpoint of segments AD and EC.

If you recall, when we dealt with two-column proofs before, the first statement is always our given statement. The given information. Let me still make sure I got my pen. How we know that that first statement is true is that it was given information. That's how we know. Let's go ahead and think about what this means for the figure.

If B is the midpoint of segments AD and EC, that means that we have some congruent parts regarding those segments. Take a look. If B is the midpoint of segment AD, then that has to mean that AB is congruent to BD. In other words, this is telling us B is the middle of this segment, so that means I have two congruent parts along that segment. We're also told that B is the midpoint of EC. If B is the midpoint of this segment, that means that EB is congruent to BC.

Those are basically the two halves along that segment. Okay? We use that given information, we were able to locate some congruent relationships, so let's go back to our two-column proof. The second statement is that AB is congruent to BD, and that EB is congruent to BC. Now, let's look back at those marks that we just made. We saw that AB congruent to BD, and that EB was congruent to BC, and we were able to make those marks because we were told that B was the midpoint.

To justify those congruent statements that were given as our second statement, we're going to say that's because of the definition of midpoint, because basically, we know that if we're told that a point is the midpoint, that it's the center, or it's the middle of a segment. It divides it into two congruent parts.

We know that those relationships are true, those congruent relationships, because we know the definition of a midpoint. Okay? All right. Let's keep going through our proof here.

The next statement is that angle ABE is congruent to angle CBD. Okay, so let's write in, let's get the congruent marks in at these angles, and then let's see if we can figure out what justifies that relationship. Okay. Angle ABE, so ABE is congruent to angles CBD. CBD. Okay.

Do you recognize that angle pair? Those are a pair of vertical angles. How we know that those angles are congruent is because we know that vertical angles are congruent. That justifies the relationship between those angles. Okay? Now we see we're at the last step of our proof. I'm going to actually scroll a little bit here.

Remember, the last line of a two-column proof is always whatever it was that you were trying to prove. Okay? Here we go. Triangle EAB is congruent to triangle CDB. If the last line is that congruent relationship between the triangles, I know that it's going to be one of my five ways to prove triangles congruent that completes this proof. Let's take a look at the relationships we have with these triangles.

I'm going to give you a little bit of a strategy that you can use, as far as your proof. If you focus on just one of the triangles, so I'm just going to focus on the left one here, doesn't matter which one, I'm just going to focus on the left, and I'm going to copy down the congruent marks. If you just focus on one triangle, it may help you be able to determine which one of those five ways to prove triangles congruent that you have here. Let's see. I have two sides that have congruent marks, and I know that they're going to match up with the marks in the other triangle, but like I said, I'm just going to focus on this one, so I have 2 sides that are marked here, and I have the included angle that's marked here. I'm even going to do this, too, just in case this helps you.

I'm going to mark that I have a side marked here, I have a side marked here, and I have an angle marked here. Okay? Two pairs. A pair of sides, I'm just focused on this triangle, and an included angle, I know that I have Side-Angle-Side. That is the method, or that's going to be how I'm able to conclude that these triangles are congruent. Okay? If you look back at your proof, you saw how we justified each statement given in our proof, and we're able to work towards proving that those triangles were congruent. All right?

Take a look at the next one. Okay. Given segment YR bisects angle TRS, and angle T is congruent to angle S. Here we're trying to prove that triangle TRY is congruent to triangle SRY. Okay? We've read through the given and the proof statement. Let's start attacking this proof.

Remember our first statement. In a two-column proof, there's always the given information, so again, it just repeats that YR, segment YR, bisects angle TRS, and angle T is congruent to angle S. We know that because that was given information. Now, let's think about what this means. Let's use this information to get some congruent marks on these triangles, okay?

YR bisects angle TRS. I'm going to switch to red for a second. Okay. Here's segment YR. I'm just going to trace over it in red. It says that it bisects angle TRS. Here's angle TRS. Now, just focus on that red here, and maybe I'll even pull it out of that figure so you can really focus on it.

I have an angle, angle TRS, and I'm told that YR bisects it. Remember that that means when you bisect a segment, or when you bisect an angle, to bisect anything really, you cut it in half. This statement's telling me that YR bisects angle TRS, that's just another way of telling me that that segment split that angle in half. It divided it into two congruent angles. What that means is I can go back to my figure here and I can get some congruent marks in there, right?

First, I'll put them here so you can really see it. If YR bisected angle TRS, then that means angle TRY is congruent to angle YRS. Those are the two halves of that original angle, angle

TRS. Let me get rid of the red up here so it doesn't start to get too crowded, and let's get those congruent marks up here. Here's angle *TRS*.

Sorry. Angle *TRY* congruent to angle *YRS*, okay? We've got that. We were also told that angle *T* was congruent to angle *S*, so let's get that up there. Angle *T* congruent to angle *S*. All right. We've got some congruent marks down.

Let's get back to the two-column proof. The second statement is angle *TRY* is congruent to angle *SRY*. Angle *TRY* congruent to angle *SRY.* Those are just the angles that we marked because we knew that *YR* bisected angle *TRS*. Basically, those are those two halves that that larger angle was split into.

How we knew that those angles had to be congruent, that that angle was going to be split in half, is we were told that *YR* bisected it. In other words, *YR* was an angle bisector. It was just a segment splitting another angle in half. Our reason for being able to justify the second statement is that we know, I'm going to abbreviate here, the definition of an angle bisector. That definition led us to be able to determine that those were two congruent parts now. That angle was split in half.

All right. Now, let's look at the second statement. *YR* is congruent to *RY*. Notice those letters are the same, right? Look at the figure here. *YR* is congruent to *RY*. That's basically telling me that this segment is congruent to itself, right?

If you recall, from your practice earlier with congruent triangles, when you have a segment congruent to itself, the property that allows you to be able to use that as a reason of congruence is the reflexive property. That's how we know that *YR* is congruent to *RY*. Even when those letters are flipped around, they're written in different order, it's still talking about the same segment, right? Okay. Reflexive property for that one.

Notice now we're at the end of our proof, so triangle *TRY* is congruent to triangle *SRY*. Let's use that strategy. I've got to scroll a little bit here to get some space. I'm going to erase our scratch work that we used for the angles. Let's just look at one of those triangles here, because now we want to determine which of those five ways is how we prove these two triangles are congruent.

I'm just going to take one of the triangles, I'm going to take the bottom one, and I'm just going to focus my attention on the marks. Okay. I'm going to use that little strategy of actually writing down, do I have a side marked or do I have an angle marked? Here I have an angle marked. Put a little A for angle. I have another angle marked, and I have a side marked, okay?

If I look, I have two angles that are marked, and a non-included side that's marked, right? If you think about which one of those five ways that is, when you have two angles that are marked and a non-included side, that's Angle-Angle-Side, okay? For this relationship between these two triangles, I can use all of these reasons, all of these justifications to lead me to Angle-Angle-Side as the method to prove these triangles are congruent. All right? Okay. Let's do one more together, and then I'm going to have you do one.

All right. Let's look at this given information here. Segment DE is congruent to segment QR, and angle D and angle R are right angles. We're asked to prove that triangle DEQ is congruent to triangle RQE. All right. We've read through that given information and the statement we're asked to prove. Let's go to the two-column proof.

Again, it starts with the given information. Segment DE is congruent to segment QR, angle D and angle R are right angles. The reason I know that is because that's given. Let's go ahead and get some congruent marks on our figures using these relationships. DE is congruent to QR, and then we're told that angle D and angle R are right angles.

Okay, so we've got that there. Now, triangle DEQ and triangle RQE are right triangles. Let's think about that. How do you know that you have a right triangle? It's the right angle in the triangle, right?

If you're given a triangle that has a right an angle, or a ninety-degree angle, you know that you have a right triangle. That's just the definition of a right triangle. How I know that triangle DEQ and triangle RQE are right triangles, I know that because they each have a right angle, right? The justification, or the reason that this statement is true, is just the definition of a right triangle. If you're given a triangle with a right angle, it's a right triangle. That's it, okay?

Let's look at our next statement here. Segment EQ is congruent to segment QE. EQ congruent to QE. Again, that is a segment congruent to itself, right? We just saw that on our last example, that the property that allows us to say something is congruent to itself, reflexive property. All right. Let's get that down.

We've reached the last line of our proof. We know that now we're at the point that we're going to use one of the five ways that we know how to prove triangles congruent to fill in this blank. Let's just take away one of these triangles. Let's break these apart and just focus on one.

Take a look at the marks here. If I look, I have a right triangle. Now, any time you're working with right triangles, you want to immediately check to see if it's going to be HL. Now, every time you're working with right triangles, that doesn't necessarily mean that the answer's going to be Hypotenuse-Leg, okay? Sometimes it may, sometimes it won't, but don't think, "I've got a right triangle, automatically is HL." That's not, necessarily, true, okay?

Now, for this one, you probably noticed that it is true, but once we finish it, I'm going to show you how it's not necessarily true every single time, okay? Looking here, I do see I have a right triangle, like we said, and we have a hypotenuse marked, and we have a leg marked. In this case, it is definitely HL, Hypotenuse-Leg, that gives us the reason that these two triangles are congruent. Like I said, every right triangle, every pair of right triangles is not necessarily HL.

Take, for example, if you had a triangle marked this way. In this case, you'd have Side-Angle-Side, not necessarily HL, right? If you do have a right triangle, and the hypotenuse is marked and the leg is marked, it is going to be HL. All right? Okay. Good job on that one.

We've reached the time that it's time for you to try one. I'm going to present you with a two-column proof. Go ahead and press "pause" and take a few minutes, and work your way through this example, and when you're ready to check your answer, press "play".

All right. Let's see how you did. Here you were given that segment RT bisects angle STA, and angle SRT is congruent to angle ART. All right. Again, that's repeated here in our given statement. The reason for that, it's given. Now, let's use this information to get some marks on our figure here.

RT bisects angle STA. Here's segment RT, it bisects angle STA, so what that's telling me is that angle STR is congruent to angle ATR, right? Basically, if we were to break it up here, let's do that down here, RT split angle STA in half. It divided it into two congruent parts. Angle STR is congruent to angle ATR, okay?

All right. If I look, that's actually the next statement in my two-column proof. Let's get rid of the marks up here. I don't want this to become too crowded for us, and let's just go ahead and get just that up there. Okay. Again, the reason that we knew that angle STR was congruent to angle ATR, definition of an angle bisector.

All right. We saw that in one of our previous examples. Okay, got that there. Let's mark this other pretty significant information before we get too far. Angle SRT is congruent to angle ART. SRT, angle SRT, is congruent to angle ART. ART.

All right. We've got all of that up there. Now, we've taken care of knowing that our first pair of angles that we marked, definition of an angle bisector, so now third line. RT is congruent to TR. RT, TR, basically, that's telling me that that segment is congruent to itself, right?

We know whenever we have something congruent to itself, reflexive property. That's what allows us to know that that's true. Okay, so we've got that. Now, we're at the last line of our proof. Now we know it's time to figure out which of those five ways to prove triangles congruent, this one is.

Let's get some stuff out of our way here, and let's just focus on one of these triangles and take in the marks. We've got that, that, that. Okay. If we look at this one, I see we have an angle marked, then we have a side marked, and then we have an angle marked. I have two angles that are marked, and an included side that's marked.

For this one, it's Angle-Side-Angle. That's the property, or that's what we're going to use to prove that these two triangles are congruent, and we used all these justifications to work our way to ASA. All right? Good job on that.

Okay. You've reached the conclusion of this lesson on how to use direct proofs to prove triangles congruent. I hope you saw how your knowledge of the five ways to prove triangles congruent, as well as a lot of other properties related to sides and angles, helped you get through this topic. Bye.