Module 6: Congruent and Similar Triangles

Topic 5 Content: Proving Triangles Similar by Angle-Angle Similarity Postulate Transcript

Hi, guys. Welcome to Geometry. This topic is going to focus on how to use Angle-Angle Similarity to prove that you have a pair of similar triangles. Your knowledge of angle relationships is going to come in handy during this topic. You ready to get started? Let's go.

Before we dive in to similarity, let's review a little bit about congruent figures. You know, specifically here, we've got two triangles. You know that if you have three pairs of corresponding angles congruent and three pairs of corresponding sides congruent, you have a pair of congruent triangles, right? If we look at our triangles here, we see that angle M is congruent to angle T, angle L is congruent to angle V, angle P is congruent to angle S, so there are three pairs of angles. ML and TV are congruent. LP and VS are congruent and MP and TS are congruent, so there are three pairs of sides. We can conclude that these two triangles are congruent. We can write what's called a congruent statement. Depending on where I start in this triangles, it will affect where I start in this triangle as far as writing that congruent statement.

For example, I could say triangle MLP is congruent to triangle TBS. Just being sure to match up those congruent parts, right? You've dealt with congruent figures, now let's go ahead and move on to similar figures. Similar figures, their corresponding angles are congruent and the corresponding sides are proportional. Basically what that tells us is that our figures have the same shape, but they have a different size from one another. One is a dilation of the other. One is going to be larger or smaller basically is what that means. The shape of the figures is the same. Taking a look here, we see we have angle M congruent to angle D, angle L congruent to angle F, and angle P congruent to angle E, so we have three pairs of corresponding angles congruent. As far as being able to show that our corresponding sides are proportional, what that means is that the ratios of the lengths of your corresponding sides is going to be the same.

Take a look here. ML is our smallest side here, and it corresponds to DF our smallest side there. The ratio of those lengths is going to be the same as the ratio of LP to FE. The medium side to the medium side. Also the same as MP to DE. Let me prove that by showing you numerically. Get some more space up here. Let's switch to the black ink, get a little more room. All right. ML is two units. They can have assigned values, these triangles are not drawn to scale. DF is 4 units. LP is 5 units. FE is 10 units. MP is 6 units. DE is 12 units. If I reduce these ratios, I get the same value. $\frac{2}{4}$ reduces to $\frac{1}{2}$, $\frac{5}{10}$ also $\frac{1}{2}$, $\frac{6}{12}$ also $\frac{1}{2}$. This ratio that we got when we simplified those lengths of those corresponding sides, we call it our scale factor. What that means is that you could describe the relationship between these two triangles by saying that triangle MLP is one half the size of triangle DFE, or you could also think of it as triangle DFE is twice the size as triangle MLP. That's how you can use that scale factor to describe the relationships between these triangles.

Because we were able to show that the corresponding sides were proportional by setting up those ratios and proving that they were equal, and we can see that our corresponding angles are congruent just from looking at the measures, you can conclude that you have a pair of similar triangles. You can write what's called a similarity statement. I could write that triangle MLP. Let's get that right up here, let's leave it red so it stands out. Triangle MLP is similar to triangle DFE. That's our symbol for similarity. Now that you're a little familiar with similar figures, I'm going to show you a shortcut that you can use to prove that you actually have a pair of similar triangles.

There are times that you don't have to go through all the work of showing that your corresponding angles are congruent and that your corresponding sides are proportional. Here's that shortcut. If you can prove that you have just two pair of corresponding angles that are congruent, then you can conclude that you have a pair of similar triangles. All you have to focus on for this shortcut, the Angle-Angle Similarity shortcut, is proving that you have two pairs of corresponding angles that are congruent. Here, just to write it out formally, if angle A is congruent to angle J, and angle B is congruent to angle K, then I can conclude that triangle ABC is similar to triangle JKL by Angle-Angle Similarity. This one only focuses on angle relationship and specifically, you just need two pairs, right? Just two pairs of corresponding angles that are congruent.

Let's use what we know about Angle-Angle Similarity to complete the next few examples. We're going to see if we can find those angle relationships that we need and if we can, we'll be able to show that those triangles are similar, and then we'll go ahead and complete our similarity statement. Take a look at this first example here. The first thing that jumps out at me as that angle N and angle P have the same measure. They're each 85 degrees. I immediately know that angle N is congruent to angle P. It was just given to me right in the figure. Now I don't have a second angle measure given to me. In each of my triangles. I can't figure out numerically if I do have angles that have the same measure. What I can do is use what I know about angle pairs in order to show that I do have another pair of congruent angles here.

Take a look at the center. Notice you have a pair of vertical angles in your triangle. What you can write here is you can say, angle NWY is congruent to angle TWP. You know that because they're vertical angles so they have to be. We don't know exactly what their measures are but we know that whatever they are, they're going to be the same. If we look here, we have two pair of corresponding angles that are congruent, so we do indeed have a pair of similar triangles. Are these similar? Yes, they are. Let's go ahead and complete our similarity statement. We have triangle NYW, and just looking for a pattern here so we started at eighty five degrees, went to the angle that we didn't mark and then to the vertical angle. Let's follow that same pattern when we write this triangle's name. We'd have to call it PTW. Those triangles are similar by Angle-Angle Similarity.

Good job on that. Let's take a look at this example. I bet the first thing you noticed is that you had no angle measures given to you what so ever. You actually do have enough information to figure out if you have a pair of similar triangles. What I'm going to do first, we're going to take a few different strategies to get to the bottom of this. I'm going to separate these triangles, I'm going break them apart. We have our triangle up at the top, triangle DAK. Then we have this larger triangle, SAT. Notice here that angle A is in each triangle. Regardless of which triangle angle A is in, it's going to have exactly the same measure. Angle A is always going to be congruent to angle A. It's always going to be congruent to itself. That's the reflexive property in action right there. I'm going to go ahead and mark angle A congruent to itself.

Now, did you notice these marks here on DK and ST? I'm going to go ahead and throw them in over here also. Now it's a throwback, you've got to kind of think back to what you learned about parallel lines and the angles formed by a transversal intersecting parallel lines. What we have here kind of hidden in this picture, let me kind of move it around a little bit so I can show you what I really want you to see. Group that together, I'm going to make that a little smaller because I want to show you something. What we have here, if we were to visualize extending this segment here, I'm going to extend this one, and I'm going to extend this side. Now I'm going to move the triangle, I'm going to shift it out of the way for a minute. Do you recognize just that particular diagram, that particular figure? What you have here is you have parallel lines intersected by a transversal.

You'll recall all the different angle pair relationships that happened when you had this particular set up. You had corresponding angles congruent. You had consecutive interior angles supplementary. You had alternate exterior angles congruent, and you had a few more. What I need here is to use what I know about those angle relationships in order to prove that I have a pair of congruent angles in each of my triangles. What I notice, I'm going to switch colors again, is I notice this angle right here and that top triangle, so basically right there. Then I notice this angle down here in that larger triangle, so basically right there. There's a way that I can show that angle ADK is congruent to angle AST. We've got to move that triangle out of our way again.

Module 6: Congruent and Similar Triangles

Topic 5 Content: Proving Triangles Similar by Angle-Angle Similarity Postulate Transcript

Take a look at what we have here. If you recall, we call these corresponding angles because their positions match. You consider this group of four angles here, it's in the top right of this group of four, and this angle here is in the top right of this group of four. You know that if you have parallel lines intersected by a transversal, the corresponding angles are congruent. That's that second angle relationship that we need to prove that these triangles are similar. That's what we needed and we got it. We have angle A congruent to angle A, reflexive property. We have angle ADK congruent to angle AST, because corresponding angles are congruent when you have a transversal intersecting parallel lines. Are these triangles similar? Yes they are. We can go ahead and complete our similarity statement. Triangle KDA, we'll look over here to get my pattern. Triangle KDA is similar to triangle TSA by Angle-Angle Similarity. Good job on that.

It's your turn to try an example on your own. Go ahead press pause, take a few minutes and work your way through this problem. You're going to use kind of the same strategies that we did on those other two examples. See if you can find the angle relationships that you need to prove that you have a pair of similar triangles. If you can, go ahead and complete that similarity statement. Press pause and take a few minutes and press play when you're ready to check your answer.

All right, let's see how you did. The first thing I notice is I have a pair of right angles. I know all right angles are congruent because they all measure 90 degrees. I'm going to go ahead and just write to keep myself straight that angle E is congruent to angle P. I see here I was given angle H is 40 degrees and here I was given that angle V is 50 degrees, so I know that angle H isn't congruent to angle V, but maybe there's a way I can figure out if angle G may be congruent to angle L or maybe there's a way to figure out just what the measure of angle G is. That might help me figure out some angle relationships that I might have between these two triangles.

Basically what I'm trying to get you to see is that you want to just use the information that you have to figure out something that you may not know. A lot of times the problem just starts to come together right along that way. Let's do what we can here. I don't know the measure of angle G but I can figure it out because I know that the sum of the measures of the angles of a triangle is always 180. Here I have 90 plus 40, that's 130. 180 minus 130, that's 50. Angle G, that's 50 degrees. Immediately I see angle G is congruent to angle V. Just in case you didn't take that route, I'm going to go ahead and calculate what the measure of angle L is also.

Here I have 90 plus 50, that's 140. 180 minus 140, that's 40 degrees. Let's write that in here, put in red just so it stands out. Here I notice also that angle H is congruent to angle L.

Module 6: Congruent and Similar Triangles
Topic 5 Content: Proving Triangles Similar by Angle-Angle Similarity Postulate
Transcript

You may have gone either route. Maybe you calculated angle L and noticed that it was congruent to angle H, or maybe you calculated the measure of angle G and then realized it was congruent to angle V. Either one would have been fine because you only needed one more pair of corresponding angles congruent. Rather it was angle G and angle V or it was angle H and angle L, either one would have been fine. The end result, are these triangles similar? Yes, they are. I'm going to go ahead and complete my similarity statement. Triangle GEH is similar to triangle VPL by Angle-Angle Similarity. Good job on that.

All right guys, you've reached the conclusion of this topic on how to prove triangles similar using Angle-Angle Similarity. I hope you saw how your knowledge of angle relationships helped you get to the end of this lesson. Bye.