Module 6: Congruent and Similar Triangles

Topic 5 Content: Proving Triangles Similar by Side-Angle-Side Similarity Postulate Transcript

Hi, guys. Welcome to Geometry. This topic is going to focus on how to use Side-Angle-Side Similarity to prove triangles are similar. Your knowledge of congruent figures, similar figures, ratios, lengths is really going to come in handy for you during this topic. You're ready to get started? Let's go.

Okay. Before we dive in to talking about similar figures, let's review a little bit about congruent figures. Taking a look, here we have a pair of triangles; you know that if you have three pairs of corresponding angles congruent, three pairs of corresponding sides congruent, you know that you have a pair of congruent triangles.

Take a look here, we see that angle M is congruent to angle T, angle L is congruent to angle V, angle P is congruent to angle S, we have our three pairs of corresponding angles congruent. We have ML congruent to TV, LP congruent to VS and MP congruent to TS. We have our three pair of corresponding sides congruent. Then, we can conclude that we have a pair of congruent triangles and we can write a congruent statement. We could say, "Okay, triangle MLP is congruent to triangle TVS. Because we were able to justify because of the angle and the side relationships that these were a pair of congruent triangles.

In regards to similar figures, in specifically triangles, if you have three pairs of corresponding angles congruent and three pairs of corresponding sides proportional, then you know that you have a pair of similar triangles. By *similar*, what I mean, is that the triangles have the same shape, but they have a different size. They're dilation of one another. You can look at it as one's an enlargement of the other or you can look at it as one's a reduction of the other. The size is different but the shape is the same.

Let's look at these few triangles here. I can see that angle M is congruent to angle D, angle L is congruent to angle F and angle P is congruent to angle E, those are my angle relationships. As far as my side relationships, when you have similar figures, in this case similar triangles, your corresponding sides are proportional. What I mean by that is that the ratios of the lengths of your corresponding sides is going to be the same.

Take a look at what I mean by that. In triangle MLP the shortest length is side LM. I'm going to start out by writing the name of the side here. LM. In triangle DFE, DF is the smallest side. I'm going to put an equal sign in here because I already know the ratios is going to be the same. LP, that's like our medium side, it corresponds to FE, our medium side there. MP, that's our largest side, it corresponds to DE, our largest side there.

If we replace the names with the lengths of these sides, LM is two units, scroll a little bit here, switch colors. DF is four units. Again, these triangles are not drawn to scale, I kind of assigned these lengths just for the sake of the example. Let's here, LP is five, FE is ten, MP is six, DE is 12. I know that you're probably looking at that and saying, "Those ratios aren't the same." Remember, you can always reduce. When you can reduce, do reduce.

$\frac{2}{4}$, that'll reduce to $\frac{1}{2}$. $\frac{5}{10}$, that'll reduce to $\frac{1}{2}$ and $\frac{6}{12}$ will reduce to $\frac{1}{2}$. You see? Those ratios are the same. We call this number here, that simplified ratio, our scale factor. So how you can use that scale factor to describe the relationship between these triangles is you could say that triangle MLP is $\frac{1}{2}$ the size of triangle DFE. You could look at that as triangle DFE is twice the size of triangle MLP. Doesn't matter which way you like to look at it, unless you're told to look at it in one particular way, but that's how you use that scale factor to describe the relationship between those triangles.

And because we saw that the scale factor was the same, that that ratio of those lengths of those corresponding sides was the same, we can conclude that the corresponding sides are proportional and that these triangles are similar. We can write what's called a similarity statement. We could say, "Okay, triangle MLP is similar to triangle DFE." That's our symbol for *similar*, for *similarity*. You see here how we use those side relationships, those angle relationships, to show that we had a pair of similar triangles here.

There's a shortcut that you can take where you don't have to figure out the relationship between all the angles in the triangles and those sides also. That shortcut is Side-Angle-Side Similarity. Let me write out what it means formally, then I think you'll be able to tell really what's going on here.

If the ratio of AB, this side, to JK equals the ratio of AC to JL and the measure of angle A is equal to the measure of angle J, then, you can conclude that triangle ABC is similar to triangle JKL, by Side-Angle-Side Similarity. That's it right now, formally. Let me show exactly what I mean by that.

When we said here, if AB to JK, if that ratio is equal to the ratio of AC to JL, what that means? AB to JK, if the ratios of the lengths of these sides, I'm just going to star them, just so they kind of stand out a little to you, if the ratio of the lengths of those sides is equal to the ratio of the lengths of AC to JL and if the measure of angle A is equal to the measure of angle J, another way to say that, is that angle A and angle J are congruent, then you can show that your triangles are similar by Side-Angle-Side Similarity.

Because that's actually what you've shown is that you have two pairs of corresponding sides, the ratios of the two pairs of those corresponding sides is equal, the included angles are congruent, when you can prove that you can conclude that you have a pair of similar triangles by Side-Angle-Side Similarity. I want you to keep that property in mind as we start applying it to get through a few examples.

We're going to look at a few different pairs of triangles, and we're going to use what we know about Side-Angle-Side Similarity to show if the triangles are or are not similar. If they are similar, we're going to complete the similarity statement.

All right, let's take a look at what we have here. I'm going to use the strategy of marking the smallest side and the largest side, then the smallest and the largest side in each triangle just to help me keep my ratio straight. In this triangle, 30 is the largest, 28 is the smallest, here 5 is the largest, 4 is the smallest.

30 is to 5, that ratio, I want to see if it's the same as 28 is to 4. Let's simplify this. 30 divided by 5, that's 6, 28 divided by 4, that's 7. These ratios are not the same. They don't simplify to the same value. Because of that, I already know these triangles are not similar by Side-Angle-Side Similarity. If you look at the figures, you do have a pair of congruent angles marked, the angle part of Side-Angle-Side Similarity holds true, but that side relationship that I was looking for, those corresponding sides, I mean proportional, didn't work out for this one. Are these triangles similar? No, I cannot complete the similarity statement.

All right, take a look at the next one. Here, again, let's start out by marking our smallest side and our largest side. Smallest, largest, smallest, largest. In the triangle on the left, smallest to smallest, 11 to 33, larges to largest, 12 to 36. Let's simplify these, see if we get the same value, get some space here. 11 over 33, that'll simplify to $\frac{1}{3}$, 12 over 36 will also simplify to $\frac{1}{3}$. The ratios were equal. That shows me that my corresponding sides are proportional. So far, for Side-Angle-Side Similarity, we're looking pretty good.

Let's go back and see if we have that pair of included angles that we need. In this triangle on the left, angle M is marked congruent, there's my included angles over here. Over here, I do have my included angle marked congruent to angles M. I do have angles M and angle L congruent, I have the corresponding included angles congruent, for Side-Angle-Side, it did work out for this one. Are these triangle similar? Yes, they are. Let's go ahead and complete the similarity statement. Triangle SME, you went along the largest side, then the smallest side. That would be triangle ZLH. By Side-Angle-Side Similarity. All right? Good job on that one.

Okay, I do believe it's your turn to try one. You've got the same type of example that we just work through previously, you're going to be given a pair of triangles, I want you to use what you know about Side-Angle-Side Similarity to show if these triangles are or are not similar. If you find that they are, go ahead and complete that similarity statement. Press pause, take your time, work your way through this one. Press play when you're ready to check your answer.

Okay, let's see how you did here. Let's start out, I'm going to mark, that's my smallest side, that's my largest side, smallest side, largest side. Let's scroll down a little bit, get some workspace here. Smallest to smallest, that'd be 3 to 12, largest to largest, that'd be 5 to 20. Let's reduce these fractions, let's simplify these, to see if we end up with the same value. Get a little workspace here. There we go.

3 over 12, that'll reduce to $\frac{1}{4}$, 5 over 20 will also reduce to $\frac{1}{4}$, so I look, these ratios are equal, my corresponding sides are proportional. Let's take a look and make sure we have included angles that are marked congruent. If we look, we do. Angle W is congruent to angle R. Are these triangles similar? Yes, they are. Let's go ahead and complete our similarity statement. Triangle HWT, we went along the smallest side, then the largest side, is similar to triangle CRG by Side-Angle-Side Similarity.

All right. Good job on that. You've reached the conclusion of this topic on proving triangle similar by Side-Angle-Side Similarity. I hope you saw how your knowledge of congruent and similar figures along with your knowledge of ratios and angle relationships came in handy during this lesson. Bye.