Module 6: Congruent and Similar Triangles

Topic 5 Content: Proving Triangles Similar by Side-Side-Side Similarity Postulate Transcript

Hi, guys. Welcome to Geometry. This topic is going to focus on how to prove triangles similar by Side-Side-Side similarity. Now, your knowledge of similar figures and a little bit about what you know about congruent figures is going to help you get through this lesson. You ready to get started? Let's go.

Okay, so before we dive in to talking about similar figures, let's review a little bit about congruent figures. Now, you know that if you have some congruent figures, and here we're going to look at these congruent triangles, you know that their corresponding sides are congruent, and their corresponding angles are proportional. You can look here at triangle MLP and triangle TVS, and you're able to tell that these triangles are congruent. Now, this triangle wasn't drawn to scale, so the numbers or the lengths and the measures here I've just assigned for the sake of example. We can tell that ML and TV are each two units long. LP and VS have the same length, and MP and TS. Then we can see angle M and angle N are congruent. So are angle L and angle V, and angle P and angle S.

From this diagram here, we could write a congruence statement. Make sure I've got my pen. We could say triangle MLP is congruent to triangle TVS. We made sure that when we wrote that statement we matched up congruent parts, right? We had, like we said before, angles M and T are congruent, ML is congruent to TV, all of those congruent parts match up. That's congruent figures.

Now, for similar figures, there are some congruent relationships in there, but not everything is congruent. Similar figures have the same shape, but they have a different size. One is actually a dilation or an expansion or reduction of the other. Take a look at these two triangles here. We have triangle MLP, and we have triangle DFE. Now, notice their angles. Notice their angles are congruent. The corresponding angles are congruent. Angles M and D, those are congruent. Let's actually mark them in the figure. These angles are congruent. Angles L and F, they're congruent. Angles P and E ... Oh, let's get two marks there. Angles P and E are congruent. The corresponding angles are congruent. I'm going to actually write that over here. We're going to build our definition of similar figures.

As far as the sides go, notice those aren't congruent, those corresponding sides. When you have similar figures and specifically here similar triangles, your corresponding sides are proportional. What that means is that your ratio of the lengths of the corresponding sides, those ratios will all be the same. Now, I think to really understand that, you need to see it, so let me show you what I mean by that. First, I'm going to write ratios of corresponding sides, just by name. Here, the smallest side in this triangle on the left, triangle MLP, is LM. It corresponds to the smallest side in the triangle on the right, which is DF. There's one ratio.

Now, because I already know that these triangles are similar because I created this example, I'm going to put an equal sign here because I know this is going to end up being equal. Let's see. The medium side, so what I mean by that, not necessarily the largest side in the triangle, but the next one going up in length, would be LP for this triangle. The medium side in this one, FE. Then last, the largest side in this triangle, MP, it corresponds to the largest side in this triangle, which is DE.

Here I've written a relationship. It's actually an extended proportion, to show me what the relationships are between these two triangles. Now, I'm going to replace the names of these segments with their actual lengths so that I can tell you what I mean by the corresponding sides are proportional. LM, 2 units. Let's switch color here. Let's go to red. DF, 4 units. LP, 5 units. FE, 10 units. MP, 6 units. DE, 12 units.

You see, I have three ratios here, right? I'm going to simplify these so we can see really that these are equal. I know looking at them right now, you're probably like, "Hmm, how do I know that those are equal?" Remember what you know about fractions. You can reduce. Let's reduce these. $\frac{2}{4}$ I know will reduce to $\frac{1}{2}$. $\frac{5}{10}$, that'll reduce to $\frac{1}{2}$. $\frac{6}{12}$, that'll also reduce to $\frac{1}{2}$. You were able to tell right here that the ratios of the lengths of those corresponding sides were proportional.

In other words, you see that the ratio that we got was $\frac{1}{2}$. You could interpret that to mean that triangle LMP is $\frac{1}{2}$ the size of triangle FDE, or, you could interpret that to mean triangle FDE is twice as large as triangle LMP. Whichever way you wanted to interpret that would be fine. That number there, that ratio that we got for each of those sides when we simplified those lengths, that's known as our scale factor. It's $\frac{1}{2}$. The scale factor of that smaller triangle to that larger one is 1:2. That smaller one is half the size of the larger one, or you could look at it as that larger one is twice the size of the smaller one.

Because we were able to tell that the corresponding angles were congruent and that the corresponding sides were proportional, now we know that these two triangles are similar to one another. You can write what's called a similarity statement. What that would look like is let's say I started out by naming this triangle PML. I could say triangle PML is similar to triangle EDF just to match up those parts. We would call that a similarity statement. Just like congruent statements, depending on where you started when you named that first triangle, you may end up with a different kind of similar statement at the end.

For example, you could have said that triangle LMP is similar to triangle FDE, and that would have been true also. Now that you've got a little background on similar figures, let's talk about how to prove triangles are similar. It's actually a shortcut, kind of will those shortcuts that we used to prove that triangles were congruent. Given a pair of triangles, if you can show that their corresponding sides are proportional, you don't even have to worry about the relationships between their angles. As long as you can show that their corresponding sides are proportional, then you could conclude that you have two similar triangles by what's called side-side-side similarity because now we're dealing with similar figures.

Let me show you what I mean as far as the lengths of those corresponding sides. Now, we're going to take, for example, we're going to assume that the way these triangles are oriented, their corresponding sides are already aligned. Formally, just so you can see this written out, if AB to JK is equal to BC to KL, to that relationship, and also as AC is to JL, then what that means is that triangle ABC is similar to triangle JKL by, I know it's a lot to write here, Side-Side-Side similarity. Basically, if you can prove the corresponding sides are proportional, you could conclude that you have similar triangles by Side-Side-Side similarity.

Now, let's use this to work through a few examples. We're going to look at a few different pairs of triangles, and we're going to try to determine are those triangles similar by Side-Side-Side similarity. If they are, we're going to complete the similarity statement that's given. Let's take a look at what we have here. We have two triangles given, and we've got some lengths, so we need to determine are these corresponding sides proportional. Are those ratios of those lengths of those corresponding sides the same?

Let's scroll a little bit, get some space. I'm going to show you a little strategy that kind of helps you kind of keep everything straight. If you go ahead and in your triangle and mark the smallest side, the medium side, the largest side. 3 is the smallest side, 7 is the medium side, 9 is the large side. Then in my larger triangle, 9 is the smallest side, 21 is the medium, 27 is the large. It'll just kind of help you keep straight that what your ratios are that you should be putting together, what should line up with what, because sometimes your triangles are given to you kind of in line where everything kind of matches up as far as position, but sometimes one triangle may be sitting upright, another one may be rotated, and it may get to become difficult to see what actually corresponds to what. That's just a little strategy that you can use in order to line up the smallest one to the smallest one, medium to the medium, largest to the large.

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Let's get going with this. I'm going to start with the smaller triangle. I could start with the larger one. It really doesn't matter which one you start with. Just stay consistent. If I start with the smallest one over here, 3 corresponds to 9, those lengths. I need to determine if 3 to 9 ... Okay, so 7, the medium side, corresponds to 21 over here. Oops, I was about to write that 21 first. Let's fix that. Okay, so 7 to 21. Then largest to largest, so 9 to 27. Notice I didn't put the equal sign in here yet because I don't know that these triangles are similar. I've got to simplify these and see if I get the same ratio every time, the same fraction.

Get some space. Okay, $\frac{3}{9}$, that'll reduce to $\frac{1}{3}$. $\frac{7}{21}$, that'll reduce to $\frac{1}{3}$. $\frac{9}{27}$ will also reduce to $\frac{1}{3}$. If I look here, those three ratios are equal, so that means that those corresponding sides are proportional. Are these triangles similar? Yes, they are. Because I know that they are, I'm going to go ahead and scroll up a little bit more and complete my similarity statement. It started out for me here JCB. Triangle JCB, so if I look, they went along the smallest side and then down the largest one, so I have to match that pattern, YGX. Triangle YGX. I'm going to actually go ahead and write by Side-Side-Side similarity. You saw how we worked our way through this one. It's really just a matter of testing out those ratios of those corresponding sides to be able to determine if they're the same. If they are the same, then you have a pair of similar triangles.

Let's keep going. Let's get onto our next one. Again, we have two triangles here. We're going to use that same relationship, go over that same strategy. We're going to see are those ratios of corresponding sides the same. Here I'm going to mark that this is my smallest side, this is my medium, this is my large in this triangle, and here, this is my smallest side, this is my large side, this is my medium. Notice the little flip around on this one.

Let's do this here. All right, so my smallest side in this triangle, 12. Smallest side in this triangle, 6. Medium side in this triangle, 20. Medium side in this triangle, 8. Largest side in this triangle, 24. Largest side in this triangle, 10. All right, so let's simplify these. Let's reduce these fractions. Get a little ... there we go.

Okay, $\frac{12}{6}$, that'll actually reduce or simplify to a whole number. That's just 2. $\frac{20}{8}$, that's not going to be a whole number, so let's use our algebra skills here. We could divide each of these, the numerator and the denominator by 4, and we would have 20 divided by 4 is 5, 8 divided by 4 is 2. Okay. Let's take a look here, 24 and 10, that won't simplify to a whole number, but we could divide them both by 2, so 24 divided by 2, that's 12. 10 divided by 2, that's 5. Look what we have here. We actually have 3 different scale factors or 3 different ratios here. 1 happens to be a whole number, which is fine. You can have whole numbers. Then I have $\frac{5}{2}$ and $\frac{12}{5}$. All of these are not the same. Are these triangles similar? No, they are not.

Because they are not similar, I cannot complete this similarity statement, because it'd be false; it's not true. Notice on this one we used our strategy of marking the smallest, the medium, and the largest side.

We tested out our ratios, and they were not the same. These triangles were not similar. I do believe that it's your turn to try one. Go ahead and press pause. Take a few minutes and work your way through this example. You're going to do the same thing that we did in those previous two. Test out those side relationships and see if you have corresponding sides that are proportional. If you do end up having that, then your triangles are similar. Complete that similarity statement. If you don't, make sure you say that they aren't, okay? Press pause and take a few minutes. Press play when you're ready to check your answer.

All right. Let's see how you did. Let's mark what we have here. I'm going to make sure I have my pen. All right, so in this larger triangle, here's our smallest side, medium side, large side. Over here, smallest side, medium side, large side. Let's get some workspace here so I don't have to scroll up and down for a little bit. Okay. All right. Here, 30 to 6. Those are the smallest to the smallest. Then 35 to 7, and 40 to 8. Let's simplify this.

30 divided by 6, 5. 35 divided by 7, also 5. 40 divided by 8, also 5. Here, we saw that our corresponding sides are proportional. Are these triangles similar? Yes, they are. We'd say, yes, and then let's go ahead and complete our similarity statement. Triangle HYZ, so let's see here. HYZ. We went along the smallest one and then the medium side. That would be similar to triangle LBK but Side-Side-Side similarity. All right, good job on that. Okay, you've reached the conclusion of this topic on how to use Side-Side-Side similarity to prove that triangles are similar. I hope you saw how your knowledge of congruent figures and similar figures and ratios all came in handy for you during this lesson. Bye.