Module 7: Right Triangles

Topic 1 Content: The Converse of the Pythagorean Theorem Transcript

Hi, guys. Welcome to geometry. This topic's going to focus on the Converse of the Pythagorean Theorem. So your knowledge of the Pythagorean is going to come in handy for you during this topic. You're ready to get started? Let's go.

Okay. First let's just review the Pythagorean Theorem and then we'll use that to determine what the converse of the Pythagorean Theorem is, okay? Let me get this out of my way here. This little box covering up the Pythagorean Theorem. If triangle ABC is a right triangle, then $a^2 + b^2 = c^2$. We have an image over here on the left of a right triangle. Now recall what you remember from our logic studies as far as conditional, converse, inverse, contrapositive.

Remember that the converse reverses the hypothesis and the conclusion of a conditional statement. What I mean by that, here we have "if triangle ABC is a right triangle." This part, this phrase immediately following *if*, that's our hypothesis. Then we have "then $a^2 + b^2 = c^2$." This equation immediately following *then*; that's our conclusion. The converse of this statement, which is the Pythagorean Theorem, will reverse the hypothesis and the conclusion. That means that the converse of the Pythagorean Theorem states that if $a^2 + b^2 = c^2$, then triangle ABC is a right triangle.

Now we use the Converse of the Pythagorean Theorem to determine if when we're given three side lengths of a triangle, we use that to determine if that triangle actually is a right triangle, okay? Keep that converse in mind and let's take a look at the next problem.

Okay. Example one, triangle DKG has side lengths 15 inches, 20 inches, and 25 inches. Is triangle DKG a right triangle? Okay, let's keep a few things in mind here. The Converse of the Pythagorean Theorem states that if—and I'm going to write it this way. If $c^2 = a^2 + b^2$, then we know that we do have a right triangle. Now knowing what we do about right triangles, we know that c represents our hypotenuse and that c is always the longest side in our right triangle. If triangle DKG is indeed a right triangle, that means that the length that's 25 inches, that's going to have to be the length of our largest side.

When we're testing out this converse of the Pythagorean Theorem, we're going to let 25 be c and again our legs, doesn't matter how you assign those. We'll call the first one a, we'll call the second one b. Let's use that and let's test out this converse here. If $25^2 = 15^2 + 20^2$, then we know that we have a right triangle. Now notice I didn't put that equal sign in there just yet because I don't know if this will end up that both sides have the same value, that it is indeed equal. For right now, I'm going to leave it blank. We may want to even put a little question mark there because you're not sure just yet if it's going to be equal. Okay? Let's get these values here.

25^2—I believe we've gotten that value before in a previous lesson—but let's go to our calculator just to be sure. Let's clear the memory here, so second, then the plus sign. We've done this a couple of times before. We'll hit *Reset*, *ALL RAM*, and *Reset* again. All right, so let's get 25^2. That is 625. Let's go ahead and just back and get all the values while we have the calculator up. We need 25^2, we need 15^2, and we need 20^2. 25^2, 15^2, and 20^2. Okay, so 25^2, that's 625 ... Let's get that down here, so 625. Then we have 225 plus 400. Let's test this out. 225 plus 400 is 625. We did end up with the left side having the same value as the right side. C^2 did equal a^2 plus b^2. Is triangle *DKG* a right triangle? Yes, it is. We've proved that by testing out the converse of the Pythagorean Theorem. All right? Good job on that one.

Now I want you to go ahead and try this example. Go ahead and press pause, take a few minutes, and work your way through this one. Press play when you're ready to check your work.

All right, let's see how you did here. We have triangle *FGM* with lengths 9, 4, and 8 centimeters. If this is a right triangle, the longest side's going to be the hypotenuse, so that's the length with 9 centimeters. Then we'll call the other sides a and b. We need to determine if $c^2 = a^2 + b^2$. Okay? c is 9. We have 9^2. We're going to test that against $4^2 + 8^2$. 9^2 is 81. 4^2, that's 16. 8 is 64. Get a little space here. Now 16 plus 64, that's 80. We have 81 on the left and 80 on the right. Those are not equal. In this case, c^2 did not equal $a^2 + b^2$. Is this triangle a right triangle? No, it's not. Okay? Good job on that one.

Now we've talked a little bit like we just did about how to use the converse of the Pythagorean Theorem to determine if we have a right triangle. There's actually a few things that we can do to determine if we have an acute or an obtuse triangle as well. Now if you recall from your earlier studies, probably in middle school and probably a little bit in elementary school too, an acute triangle is a triangle that has all acute angles. Every angle in the triangle is less than 90 degrees. An obtuse triangle is a triangle that includes one obtuse angle. There's one angle in the triangle that's somewhere between 90 and 180 degrees. Greater than 90, but less than 180. Okay?

You can determine if you have an acute triangle by using this. If c^2, so if your largest side squared, the length of that largest side squared, is less than $a^2 + b^2$, then you have an acute triangle. If c^2, so if that largest side squared, that length of that largest side squared is greater than $a^2 + b^2$, then you have an obtuse triangle. You can use these facts to determine if you have an acute or an obtuse triangle. Okay, let's work a couple of examples with these.

All right. We're told that triangle *XYZ* has side lengths 5, 6, and 7 centimeters. We're asked to determine: Is it acute or obtuse? Okay? *c* is going to be our largest length here. In this case, that's 7 centimeters. This is *c*, and we can assign *a* and *b* however we want. Let's test out to see the relationship between c^2 and $a^2 + b^2$. Then just to get it on this slide, remember that if c^2 is less than $a^2 + b^2$, then it's an acute triangle. If c^2 is greater than $a^2 + b^2$, then it's an obtuse triangle. Okay?

Let's go ahead and figure out which one we have here. *c* is going to be 7. We'll have 7^2, and then 5^2 plus 6^2. All right, so 7^2, that's 49, 5^2 is 25, 6^2 is 36. Okay? 25 plus 36, that's 61. We have 49 on the left, 61 on the right. 49 is less than 61. That means that this set of lengths fit our first statement here. We have an acute triangle in this case because c^2 was less than $a^2 + b^2$. Okay? Keep that in mind and go ahead and try this one. Press pause, take a few minutes, work through it. Press play when you're ready to check your work.

All right, let's see how you did here. We know *c* is going to be the largest length. In this case that's 10, and then we'll call the other sides *a* and *b*. Let's see, we're testing out the relationship of c^2 against $a^2 + b^2$. All right, switch to black ink here. *c* is 10. 10^2, then we have 4^2 plus 7^2. 10^2 is 100, 4^2 is 16, and 7^2 is 49. 16 plus 49, that is 65, I believe, but let me just double check my mental mind because I know I've been doing a lot over here. 16 plus 49 ... yup 65. That means we have 100 on the left and 65 on the right. 100 is greater than 65, so in this case, c^2 was greater than $a^2 + b^2$. We have an obtuse triangle. All right? Good job on that.

All right guys, we've reached the conclusion of this topic on how to use the Converse of the Pythagorean Theorem to determine if we have a right triangle. You saw that we also learned how to determine if we have an acute triangle or an obtuse triangle as well. Hope to see you back next time for more geometry. Bye.